Event-based control for discrete-time linear parameter-varying systems: an emulation-based design.

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***Abstract*— In this paper, we present an event-based control technique for discrete-time linear parameter-varying (LPV) systems by employing an emulation approach. A new condition to design an event-triggering mechanism is proposed, and an optimisation procedure is presented to minimise the number of events. The conditions are formulated in the form of parameter-dependent linear matrix inequalities. A polynomial parameter-dependent controller is employed to stabilize the system. The impact of increasing the polynomial degree of the controller matrices on the number of events is investigated. The effectiveness of the method is illustrated through a numerical example.**

I. INTRODUCTION

Communication plays a central role in the efficient and safe operation of networked control systems (NCS). It is a recognized fact that network communication faces various challenges and issues. In dynamic and complex industrial environments the existence of time-delays [1], [2], packet losses [3], sample period assignments issues [4], and the presence of cyber attacks [5] are a few of the many issues that may compromise the efficiency and the reliability of NCS.

A specific issue that may occur in control systems is the excessive transmission of signals in the communication channels. In many cases, controllers send signals at fixed intervals, even when there are no significant changes in the controlled process. This behaviour generates unnecessary network traffic, consuming bandwidth and computational resources, as well as overloading the network, impairing the responsiveness of the control system without providing real benefits for control [6], [7]. Therefore, a significant portion of the energy consumed in NCS is due to data transmission. In this sense, event-based control [8]–[10] can contribute to improving system energy efficiency, reducing network traffic, and decreasing latency. Additionally, this technique also helps address systems that consider actuator fatigue [11].

Event-triggered control (ETC), a contemporary control technique, is distinguished by incorporating a feedback mechanism that determines transmission instants based on the evaluation of states or output measurements of the plant [9], [10]. Essentially, there are two classes of approaches for ETC design: emulation and co-design. Emulation-based approaches involve providing the controller while only designing the Event Triggering Mechanism (ETM) to ensure closed-loop stability. On the other hand, in co-design, both the ETM and the controller are jointly designed. Event-triggered control approach, with an emphasis on emulation or co-design, has been extensively studied and applied in various domains [12], such as power systems [13], advanced manufacturing [14], and robotics [15]. In [16], techniques for ETC for discrete-time linear systems under actuator saturation were presented, and a co-design condition was developed to design both the controller and the triggering function. The technique was extended in [17] to deal with the observer-based control problem. In [18], an event-triggered control technique was introduced for the class of discrete-time linear parameter-varying (LPV) systems. On the other hand, [19] presents ETC techniques for nonlinear rational systems. ETC techniques for a class of nonlinear systems represented by quasi-LPV polytopic models for continuous-time systems are presented in [2], [20], and in [21] for discrete-time systems.

Remarkably, few studies have addressed event-based control for discrete-time LPV systems. Moreover, the influence of polynomial structures on the time-varying parameters has been a gap in the existing literature on ETC. In this sense, this work presents an event-based control technique for discrete-time LPV systems. An emulation approach is considered, i.e., the controller is previously designed. The controller design follows a classical condition adapted from [22]. In the next step, parameter-dependent linear matrix inequality (LMI) conditions are derived to design the event-triggered mechanism. The conditions are obtained by using the Lyapunov stability theory, and an optimisation procedure to minimise the number of events is developed. Finally, the impact of increasing the polynomial degree of the controller matrices on the number of events is investigated through a numerical example. Furthermore, the accuracy of the simulation software was also employed to further reduce the number of events in the considered scenarios.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space,

This work was supported by Science Foundation Ireland (SFI) under contract number SFI/21/FFp-P/10065.

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and $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. The notation $X>0$ ($X<0$) means that X is a positive (negative) definite matrix. The symbol $\star$ stands for symmetric blocks in matrices. For matrices and vectors, $(^\top)$ indicates the transpose.

II. TECHNICAL BACKGROUND

Consider the following discrete-time LPV system

$$x(k+1)=A(\alpha_k)x(k)+B(\alpha_k)u(k), \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector, $u\in\mathbb{R}^{n_u}$ is the control input, and $k\in\mathbb{N}$ is the time instant. The parameter-dependent matrices in (1) can be generically represented as

$$Z(\alpha_k)=\sum_{i=1}^{N}\alpha_{k,i}Z_i, \quad \alpha_k\in\Lambda_N, \tag{2}$$

where Z_i, $i=1,\dots,N$, are the vertices of the polytope and Λ_N is the unit simplex:

$$\Lambda_N=\left\{\alpha_k\in\mathbb{R}^N:\sum_{i=1}^{N}\alpha_{k,i}=1,\ \alpha_{k,i}\geq 0,\ i=1,\dots,N\right\}.$$

In this paper, we assume that the time-varying parameter α_k is measured and can vary arbitrarily fast in the unit simplex. There is no constraint imposed on the rates of variation of α_k.

A. Event-Triggered Control

When employing ETC, the main objective is to analyse the system behaviour and identify the key moment to generate control events. It seeks to update the control input, $u(k)\in\mathbb{R}^{n_u}$, only when a certain condition is met, reducing the overall updates and communication. The decision regarding the transmission or not of the state variable $x(k)$ and the time-varying parameter α_k is based on an event-triggering mechanism (ETM). The ETM is based on a function $f(e(k),x(k),\alpha_k)$, where $e(k)$ is the measurement error of the system which will be defined later. Figure 1 illustrates the event-based control structure for LPV systems.

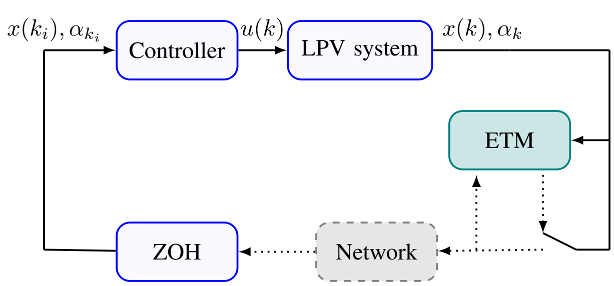

Fig. 1. Event-based control structure.

Notice that a zero-order hold is employed to keep the latest transmitted information by the ETM. Algorithm 1 details the discussed strategy for event-based control in LPV systems, where i is an event counter.

III. MAIN RESULTS

In this paper, an emulation approach is employed. In this case, the controller is designed beforehand and the goal is to provide an ETM that minimises the number of events.

Algorithm 1 Event-based control strategy.

if $k=0$ **then**
  $i\leftarrow 1$
  $k_i\leftarrow 0$
  $K(\alpha_{k_i})\leftarrow K(\alpha_k)$
  $u(0)\leftarrow K(\alpha_{k_i})x(0)$
else
  if $f(e(k),x(k))>0$ **then**
    $i\leftarrow i+1$
    $k_i\leftarrow k$
    $K(\alpha_{k_i})\leftarrow K(\alpha_k)$
    $u(k_i)\leftarrow K(\alpha_{k_i})x(k_i)$
  else
    $u(k)\leftarrow u(k_i)$
  end if
  $k\leftarrow k+1$
end if

A. Design Condition

Our purpose is to evaluate the impact of the polynomial structure of the controller and the Lyapunov matrix on the number of events. For this reason, the following Lemma will be used to design the controller employed in this paper.

Lemma 1 ([22]): If there exist positive definite symmetric matrices $P(\alpha_k)\in\mathbb{R}^{n\times n}$, and matrices $X(\alpha_k)\in\mathbb{R}^{n\times n}$, $Z(\alpha_k)\in\mathbb{R}^{n_u\times n}$ such that

$$\begin{bmatrix} P(\alpha_{k+1}) & A(\alpha_k)X(\alpha_k)+B(\alpha_k)Z(\alpha_k) \\ \star & -P(\alpha_k)^\top+X(\alpha_k)+X(\alpha_k)^\top \end{bmatrix}>0, \tag{3}$$

for all $(\alpha_k,\alpha_{k+1})\in\Lambda_N\times\Lambda_N$, then the closed-loop system

$$x(k+1)=(A(\alpha_k)+B(\alpha_k)K(\alpha_k))x(k), \tag{4}$$

is asymptotically stable, and the parameter-dependent controller is given by

$$K(\alpha_k)=Z(\alpha_k)X(\alpha_k)^{-1}. \tag{5}$$

Proof: Substituting $Z(\alpha_k)=K(\alpha_k)X(\alpha_k)$ into (3) and using the fact that

$$X(\alpha_k)^\top P(\alpha_k)^{-1}X(\alpha_k)\geq X(\alpha_k)^\top+X(\alpha_k)-P(\alpha_k)$$

one has

$$\begin{bmatrix} P(\alpha_{k+1}) & A(\alpha_k)X(\alpha_k)+B(\alpha_k)K(\alpha_k)X(\alpha_k) \\ \star & X(\alpha_k)^\top P(\alpha_k)^{-1}X(\alpha_k) \end{bmatrix}>0. \tag{6}$$

Multiplying (6) on the left by

$$\begin{bmatrix} I & 0 \\ 0 & X(\alpha_k)^{-\top} \end{bmatrix}, \tag{7}$$

and on the right by its transpose, results in

$$\begin{bmatrix} P(\alpha_{k+1}) & A(\alpha_k)+B(\alpha_k)K(\alpha_k) \\ \star & P(\alpha_k)^{-1} \end{bmatrix}>0. \tag{8}$$

By using the Schur complement, multiplying on the left by $x(k)^\top$ and on the right by $x(k)$, we obtain

$$x(k+1)^\top P(\alpha_{k+1})^{-1}x(k+1)-x(k)^\top P(\alpha_k)^{-1}x(k)<0. \tag{9}$$

Finally, considering a Lyapunov function $V(x(k),\alpha_k) = x(k)^\top P(\alpha_k)^{-1}x(k)$, condition (9) can be rewritten as $\Delta V(x(k),\alpha_k) < 0$. Moreover $V(x(k)) > 0$, since $P(\alpha_k)$ is positive definite completing the proof. ■

B. Triggering Mechanism

The choice of the triggering mechanism is inspired by [18]. Consider a discrete-time linear system described by (1), where the control law is defined by

$$u(k) = K(\alpha_{k_i})x(k_i), \quad k \in [k_i, k_{i+1}). \tag{10}$$

The closed-loop system can be rewritten by replacing (10) in (1) as follows

$$x(k+1) = A(\alpha_k)x(k) + B(\alpha_k)K(\alpha_{k_i})x(k_i). \tag{11}$$

Defining the measurement error in the interval $[k_i, k_{i+1})$ as

$$e(k) = x(k_i) - x(k),$$

allows us to write (11) as

$$x(k+1) = \mathscr{A}(\alpha_k)x(k) + \nu(k), \tag{12}$$

where

$$\begin{aligned} &\mathscr{A}(\alpha_k) = A(\alpha_k) + B(\alpha_k)K(\alpha_k), \\ &\nu(k) = B(\alpha_k)K(\alpha_k)e(k) + B(\alpha_k)\Delta K(\alpha_{k_i},\alpha_k)x(k_i), \\ &\Delta K(\alpha_{k_i},\alpha_k) = K(\alpha_{k_i}) - K(\alpha_k). \end{aligned} \tag{13}$$

The following Theorem presents a new condition to certify the stability of the closed-loop system (12) and to design the triggering function that will be employed in the ETM.

Theorem 1: Given a matrix $\mathscr{A}(\alpha_k)$ as in (13), if there exist positive definite symmetric matrices $P(\alpha_k) \in \mathbb{R}^{n\times n}$, $Q_\sigma \in \mathbb{R}^{n\times n}$, $Q_\delta \in \mathbb{R}^{n\times n}$, and matrices $X_1 \in \mathbb{R}^{n\times n}$, $X_2 \in \mathbb{R}^{n\times n}$, $X_3 \in \mathbb{R}^{n\times n}$ such that

$$\begin{bmatrix} H_1 & -X_1^\top + \mathscr{A}(\alpha_k)^\top X_2^\top & X_1 + \mathscr{A}(\alpha_k)^\top X_3^\top \\ \star & P(\alpha_{k+1}) - X_2 - X_2^\top & X_2 - X_3^\top \\ \star & \star & -Q_\delta + X_3 + X_3^\top \end{bmatrix} < 0, \tag{14}$$

hold with

$$H_1 = -P(\alpha_k) + Q_\sigma + X_1\mathscr{A}(\alpha_k) + \mathscr{A}(\alpha_k)^\top X_1^\top,$$

for all $(\alpha_k, \alpha_{k+1}) \in \Lambda_N \times \Lambda_N$, then, the closed-loop system (12) is asymptotically stable under the event-based strategy, where the triggering function is defined as

$$f(e(k),x(k)) = \nu(k)^\top Q_\delta \nu(k) - x(k)^\top Q_\sigma x(k), \tag{15}$$

where $\nu(k)$ is given in (13).

Proof: Multiplying (14) on the right by

$$\mathscr{R} = \begin{bmatrix} I & 0 \\ \mathscr{A}(\alpha_k) & I \\ 0 & I \end{bmatrix}, \tag{16}$$

and on the left by $\mathscr{R}^\top$ results in

$$\begin{bmatrix} G & \mathscr{A}(\alpha_k)^\top P(\alpha_{k+1}) \\ P(\alpha_{k+1})\mathscr{A}(\alpha_k) & P(\alpha_{k+1}) - Q_\delta \end{bmatrix} < 0, \tag{17}$$

where $G = -P(\alpha_k) + Q_\sigma + \mathscr{A}(\alpha_k)^\top P(\alpha_{k+1})\mathscr{A}(\alpha_k)$. Multiplying (17) on the left by $\begin{bmatrix} x(k)^\top & \nu(k)^\top \end{bmatrix}$ and on the right by its transpose, yields

$$\begin{aligned} &x(k)^\top(-P(\alpha_k) + Q_\sigma + \mathscr{A}(\alpha_k)^\top P(\alpha_{k+1})\mathscr{A}(\alpha_k))x(k) \\ &+ x(k)^\top \mathscr{A}(\alpha_k)^\top P(\alpha_{k+1})\nu(k) + \nu(k)^\top P(\alpha_{k+1})\mathscr{A}(\alpha_k)x(k) \\ &+ \nu(k)^\top P(\alpha_{k+1})\nu(k) - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned}$$

The last inequality can be rewritten as

$$\begin{aligned} &(\mathscr{A}(\alpha_k)x(k) + \nu(k))^\top P(\alpha_{k+1})(\mathscr{A}(\alpha_k)x(k) + \nu(k)) \\ &- x(k)^\top P(\alpha_k)x(k) + x(k)^\top Q_\sigma x(k) - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned} \tag{18}$$

By using (12) in (18), results in

$$\begin{aligned} &x(k+1)^\top P(\alpha_{k+1})x(k+1) - x(k)^\top P(\alpha_k)x(k) \\ &+ x(k)^\top Q_\sigma x(k) - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned}$$

Considering the Lyapunov function $V(x(k)) = x(k)^\top P(\alpha_k)x(k)$, which is positive definite since $P(\alpha_k) > 0$, one can write

$$V(x(k+1)) - V(x(k)) < \nu(k)^\top Q_\delta \nu(k) - x(k)^\top Q_\sigma x(k),$$

By ensuring that $V(x(k+1)) - V(x(k)) < f(\nu(k),x(k))$, according to Lyapunov's theory, it is necessary that $f(\nu(k),x(k)) < 0$, thus concluding the proof. ■

The reduction of information exchange between the sensor and the actuator, and the consequent decrease in network usage, is one of the main advantages of event-based control. To achieve this goal, it is necessary to employ optimisation problems in conjunction with the conditions presented in Theorem 1.

C. Event Number Optimisation Mechanism

Consider the triggering function (15). To reduce the number of events, in the worst case, the following condition has to be ensured

$$\nu(k)^\top Q_\delta \nu(k) \leq x(k)^\top Q_\sigma x(k), \tag{19}$$

and multiplying both sides of (19) by $(x(k)^\top Q_\sigma x(k))^{-1}$

$$\frac{\nu(k)^\top Q_\delta \nu(k)}{x(k)^\top Q_\sigma x(k)} \leq 1. \tag{20}$$

Therefore, one should minimise Q_δ and maximize Q_σ for the system to take longer to violate the constraint.

The optimisation problem can be structured as follows:

$$\begin{aligned} &\min: \quad \sigma \\ &\text{subject to:} \begin{cases} (14), \quad \begin{bmatrix} M - Q_\delta & I \\ I & Q_\sigma \end{bmatrix} \geq 0, \ \text{trace}(M) < \sigma, \end{cases} \end{aligned} \tag{21}$$

where $X \in \mathbb{R}^{n\times n}$, and $\sigma \in \mathbb{R}$ are the decision variables.

By applying the Schur complement yields

$$M - Q_\delta - Q_\sigma^{-1} \geq 0,$$

or simply

$$Q_\delta + Q_\sigma^{-1} \leq M. \tag{22}$$

Taking the trace on both sides results in

$$\text{trace}(Q_\delta + Q_\sigma^{-1}) \leq \text{trace}(M). \quad (23)$$

Note that by minimising σ we are also minimising the sum of $Q_\delta + Q_\sigma^{-1}$, which will imply a smaller number of events since we are minimising Q_δ, and maximizing Q_σ.

Remark 1: It is important to highlight that conditions presented in Lemma 1 and Theorem 1 represent an infinite dimension problem, whose approximations (relaxations) in terms of a finite set of LMIs can be obtained by fixing polynomial structures for the optimisation variables. In this paper, this procedure is performed with the aid of the parser ROLMIP [23].

IV. NUMERICAL EXPERIMENTS

The routines were implemented in MATLAB R2015a, by using the packages YALMIP [24], ROLMIP [23] and the solver SeDuMi [25].

A. *Example* 1

Consider the discrete-time LPV system as in (1), borrowed from [26], with matrices

$$A_1 = \begin{bmatrix} 0,9520 & 0,0936 \\ -0,9358 & 0,8584 \end{bmatrix}, \quad A_2 = \lambda \begin{bmatrix} 0,9996 & 0,0824 \\ -0,0082 & 0,6699 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

In this example, Lemma 1 is employed to design polynomial parameter-dependent state-feedback controllers with different degrees. The designed controllers are then used in the emulation condition proposed in Theorem 1 with the optimisation procedure (21) to minimise the number of events. Another conducted test considered that the trigger function will not activate an event only when $f(k) > 0$, but rather when $\text{round}(f(k) \times 10^{16})/10^{16} > 0$. This is because MATLAB has 16 digits of precision, and the “round” function is used to round and take into account MATLAB’s precision. In the sequence, three cases are presented to demonstrate the effectiveness of the proposed method

1) Case 1: $\lambda = 1.0025$, degree 1 controller: In this case, the controller designed with Lemma 1 is given by

$$K(\alpha_k) = \alpha_{k,1}\begin{bmatrix}-0.78477 & -0.13676\end{bmatrix} + \alpha_{k,2}\begin{bmatrix}-0.72810 & -0.20365\end{bmatrix}.$$

By employing the obtained controller, Theorem 1 was applied considering different polynomial degrees for the matrices $X_i(\alpha_k)$, $i = 1,2,3$, and $P(\alpha_k)$. Table I presents the number of events considering or not MATLAB accuracy. The number of events presented was obtained over an average of 500 simulations considering different trajectories for the time-varying parameter α_k within an interval of 200 instants. Note that the number of events in all scenarios outperforms the periodic approach that would use 200 events.

To illustrate the efficiency of the method, Figure 2 depicts the evolution of states x_1 and x_2, when using degree 2 for the matrices $P(\alpha_k)$ and $X_i(\alpha_k)$, $i = 1,2,3$ in Theorem 1. Different trajectories for the time-varying parameter α_k are considered. It is noteworthy that the system, which uses an average of 40 events in its time response, can satisfactorily stabilize the system around $k = 20$, even without sending the state vector signal at every instant. Figure 3 presents the evolution of the control signal.

TABLE I
CASE 1: $\lambda = 1.0025$, DEGREE 1 CONTROLLER.

Degree	Number of events	Number of events considering the accuracy of MATLAB
0	178	51
1	138	41
2	133	40
3	131	40

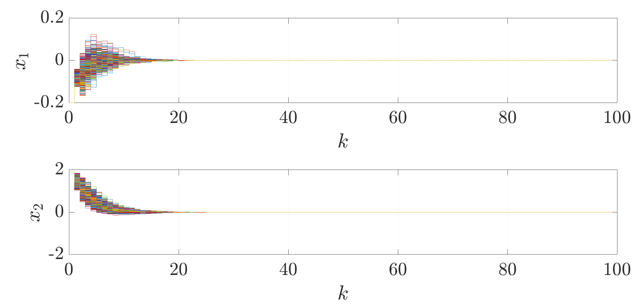

Fig. 2. State trajectories with initial condition $x(0) = \begin{bmatrix}-0.20 & 1.93\end{bmatrix}^\top$ and different trajectories for the time-varying parameter α_k – Case 1.

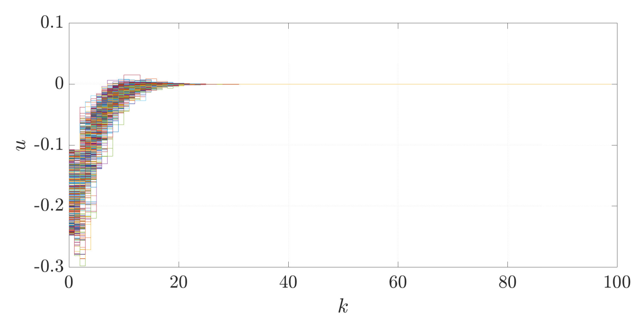

Fig. 3. Control input u for the different trajectories of the time-varying parameter α_k – Case 1.

Figure 4 shows the time interval between events for the case with degree 2 from Table I. In this case, the number of events, considering the MATLAB accuracy is 40, whereas without considering precision, it is 133. It is important to emphasize that this figure was obtained with a random trajectory for the parameters α_k. It can be noticed that the longest time interval between events was 9 time instants, and from approximately instant 90, no more events occurred, in contrast to the case neglecting MATLAB’s accuracy, which obtained 133 events.

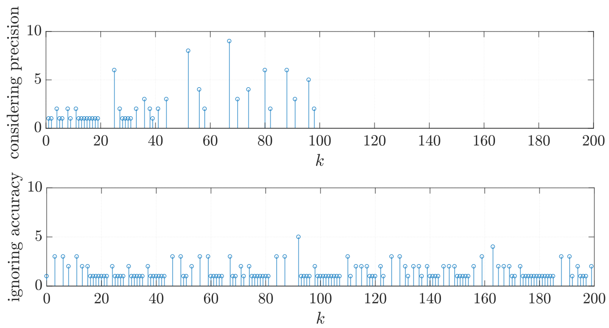

Fig. 4. Time interval between events considering MATLAB's accuracy × Time interval between events without considering accuracy.

2) Case 2: $\lambda = 1.0025$, degree 2 controller: In this scenario, the controller designed with Lemma 1 is

$$K(\alpha_k) = \alpha_{k,1}^2 \begin{bmatrix} -0.83663 & -0.1387 \end{bmatrix} + \alpha_{k,1}\alpha_{k,2} \begin{bmatrix} -1.7411 & -0.42424 \end{bmatrix} + \alpha_{k,2}^2 \begin{bmatrix} -0.7281 & -0.24582 \end{bmatrix}.$$

Applying Theorem 1, the numbers of events found, considering or not the MATLAB precision, according to the degree of $X_i(\alpha_k)$, $i = 1,2,3$, and $P(\alpha_k)$, are presented in Table II. The analysis of the number of events was conducted over an average of 500 iterations for different α_k within an interval of 200 instants. It is observed that, with an increase in the degree of the polynomial matrices, the number of events tends to decrease. However, in comparison with the results presented in Table I, the number of events has shown an increase.

TABLE II
$\lambda = 1.0025$, DEGREE 2 CONTROLLER – CASE 2.

Degree	Number of events	Number of events considering the accuracy of MATLAB
0	180	53
1	145	44
2	137	42
3	136	42

3) Case 3: $\lambda = 1.0025$, degree 3 controller: For this case, the following controller was obtained with Lemma 1

$$K(\alpha_k) = \alpha_{k,1}^3 \begin{bmatrix} -0.86583 & -0.14388 \end{bmatrix} + \alpha_{k,1}^2\alpha_{k,2} \begin{bmatrix} -2.8786 & -0.61992 \end{bmatrix} + \alpha_{k,1}\alpha_{k,2}^2 \begin{bmatrix} -2.5669 & -0.75683 \end{bmatrix} + \alpha_{k,2}^3 \begin{bmatrix} -0.71617 & -0.30049 \end{bmatrix}.$$

Applying Theorem 1, the numbers of events found, considering or not the MATLAB precision, according to the degree of $X_i(\alpha_k)$, $i = 1,2,3$, and $P(\alpha_k)$, are presented in Table III. The number of events was analyzed over an average of 500 iterations for different trajectories for the time-varying parameter α_k within an interval of 200 instants.

Once again, an increase in the degree of the polynomial matrices is observed to correspond with a decrease in the number of events. However, it is noteworthy that the controller with degree 3 exhibits the poorest performance in terms of the number of events. This outcome is attributed to the fact that for controllers of higher degrees, the term $\nu(k)$ in (13) increases more faster due to the presence of additional terms in $\Delta K(\alpha_{k_i}, \alpha_k)x(k_i)$. This source of conservativeness is further associated with the fact that the controller and the trigger function are not designed simultaneously.

TABLE III
$\lambda = 1.0025$, DEGREE 3 CONTROLLER

Degree	Number of events	Number of events considering the accuracy of MATLAB
0	180	54
1	150	48
2	143	46
3	141	46

V. CONCLUSIONS

In this paper, we have presented an event-based control strategy for discrete-time LPV systems by employing an emulation approach. To design the state-feedback controller, a method from the literature, which does not consider the presence of events was employed. A new condition to design an event-triggering mechanism was proposed. Moreover, an optimisation procedure has been introduced to minimise the number of events in the control strategy. It was possible to conclude that the polynomial degree of the matrices $X(\theta_k)$ and $P(\theta_k)$ also contributed to event reduction. However, it was observed that increasing the controller's degree did not always result in a decreased number of events. This can be explained by the presence of the term ΔK in the function $\nu(k)$, which is part of the event-generating function. Thus, an increase in $\nu(k)$ contributes to an increase in the number of events, which explains the inefficiency in reducing the number of events when increasing the controller's degree. In future research, the authors are exploring the design of the triggering function and the state-feedback control through co-design.

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