THE CONCEPTUAL DEVELOPMENT OF POPULATION AND VARIATION AS FOUNDATIONS OF ECONOMETRIC ANALYSIS

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THESIS ABSTRACT

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Economics is a time-bound science. The analytical tools of statistical description and inference, however, were first developed for static comparisons of differences rather than formulation of processes of change. This thesis offers an historical perspective on the dichotomy of logical variation and temporal variation. I examine the interaction of statistical technique with the needs and concepts generated in the study of political arithmetic, observational errors, social physics, natural selection and economic motion.

Through these interactions the concept of statistical population changed. There was a shift in emphasis from the assumption of equivalence of constituents and from the mean as a manifestation of truth and divine order to the assumption of deviation and the mean as a typical value in motion. In Darwin's theory of natural selection, differences within a population were the source of evolutionary variation of a species. The quantitative techniques of correlation and regression were developed to test theories of evolution and inheritance

The problems of reconciling logical variation and temporal variation were most prominent in the application of correlation and regression to economic time series data. Differencing observations and calculations of deviations from moving averages were suggested as solutions. The most significant steps were taken in the the formulation of stochastic processes and in the development of errors-in-equations models. With the latter, the statistical properties of residuals rather than of series of observations became important. In building on some of these historical examples I suggest that acknowledgement of complementary statistical populations may enable us to further reconcile logical and temporal variations.

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Chapter One

INTRODUCTION

To weave a history of thought and method is to work to a final design while accomodating the spontaneous texture generated in the process. This tapestry is a juxtaposition of variation as a process of change and variation as a static comparison of differences. The warp is a history of statistical description and inference. The weft comprises the concepts of variation and population cultivated in European natural and social sciences. The pattern, based on the interdependent development between the revolutionary paradigms of 18th through early 20th century science and the tools of quantitative methods, creates an uncomfortable vibrancy that raises doubts about measurement without history in time series analysis. It also raises hopes that acknowledgement of complementary populations can reconcile the long run of economics with the long run of statistics.

Political economy and statistical method were enjoined in the late 1600's and again in the late 1800's. The path from the political arithmetic to econometrics is not a continuous one. In fact an historical examination begs the question of why two centuries separate these vital interactions of economic subject and statistical technique. This thesis develops the similarities and differences that historically link these two interactions. The links are constructed from the examination of the modification of statistical concepts with their application to various subjects.

The subjects that comprise the discrete layers of the weft are:

Political Arithmetic	c. 1660's
Games of Chance	c. 1710's
Errors in Measurement	c. 1780's
Social Physics	c. 1840's
Natural Selection	c. 1870's
Economic Motion	c. 1890's

With each of these there was refinement of statistical method and the creation of new perspectives and parameters. Although there is a chronological order one did not inevitably flow onto the next. The connections between them are created in the hindsight of a history. Each subject contributed to the conceptual development that we see as the intellectual foundation to econometrics.

The conceptual threads that comprise the warp include:

Quantity, Population, Mean, Deviation, Change.

As with the weft, there is a hint of chronological order in the emphasis given to each in the development of statistical method. These are, however, more continuous threads. For example, the concept of mean is important in most of the applications and it develops from the perfect manifestation of Divine Design to the changing value of the typical.

This history ends with the publication of Udny Yule's article on serial correlation in 1927. It is a search, however, for historical clues and perceptions that can throw light on contemporary practice. It is an examination of the past for the purpose of understanding the conceptual roots of the present and the possibilities for future developments in econometrics.

Despite little reference to the economic and social history of the time, this historical approach is not an idealist one. There is no presumption that one idea lead inevitably to the next nor that the writings examined comprised the dominant ideology of the day and determined the direction of material development. In many instances the writings were obscure, esoteric tracts that had little contemporary influence. Their usefulness was only realized decades or centuries later when a productive application was made.

For example, in his 1717 preface, to the Doctrine of Chances, Abraham de Moivre, argued that his method of calculating the probabilities of events in play would be useful for:

-the knowledge for gain or curiousity of those engaged in 'play'

-for a cure of a 'kind of superstition, which has been long standing in the world, vis. that there is in Play such a thing as Luck, good or bad.'

-a 'due comparison between Chance and Design,'

-the pleasure of discovery of general and simple truths

-serving 'in conjunction with the other parts of the Mathematiks, as a fit introduction to the Art of Reasoning.'

The genius and universality of de Moivre's derivation of the normal distribution was not recognized by him nor by others in his time. In fact even after the normal curve was applied to many other phenomena beyond the realm of play, either Carl Friedrich Gauss or Pierre Laplace was acknowledged as the author of the equation for the curve of errors. Karl Pearson(1923), after an exhaustive archival search, demonstrated that the credit should have gone to de Moivre. In the historical context in which he wrote de Moivre's Doctrine of Chances had little importance except to curious, wealthy gamblers. His work is significant in this history because games of chance are one of the historical analogies which we use to validate and comprehend econometric technique.

The intervals in this thesis at which the material context takes on importance are those at the beginning and end. The reasons for this are that political arithmetic and early econometrics were both designed and recognized as policy tools. They were responses to and reflections on the mass phenomena of the social and economic base of the nation.

The writings of John Graunt and William Petty were directed to the sovereign. Their arithmetic was applied to the counts of births and deaths in the Kingdom to address such questions as: Can frequent wars be sustained? What is the productive and taxable capacity of the country? Udny Yule in the first application of regression to economic data also focused on state policy. His concern was whether the administrative policy of some localities to give relief to the poor outside the workhouses actually led to an increase in pauperism.

There is the question of why political economy did not continue on from political arithmetic to empirically address issues of policy. There is also the question of how the quantitative concepts of population and variation matured outside the realm of economic and social policy. Finally, what were the problems and potentials created with the renewal of the dialogue between political economy and matured quantitative method.

Population

Population is a key concept in the logic of statistical method and in theories of political economy. The act of defining a population is premised with the assumptions of equivalence of individual constituents, of an order and relationship binding the constituents, and of a manifestation of the whole. The historical context in which these assumptions were first relevant to nation-states is also the context in which statistics had its origins.

Equivalence of all inhabitants like equivalence of all outcomes is a relativley modern notion. Karl Marx in volume one of Capital argues that Aristotle failed to understand the relationship of value in exchange because his ideas were the product of a slave society where equivalence between laborers had no meaning. Geoffrey Kay and J. Mott (1982) have pointed out that the Doomesday Book of the eleventh century did not count people but listed fiscal units to which groups of people of varying status were attached. The counting of William Petty six centuries later assumed political and fiscal equivalence of citizens and their labor as a source of value. Population thus became "a symptom for wealth, a cause of wealth and indeed wealth itself." (Kay 1982 p87)

The acknowledgement of equivalence and the consideration of many individuals led to a recognition of mass phenomena. Not only was the whole different from the sum of the parts, but it also displayed a stability and certainty in stark contrast to the attributes of individual constituents. Populations could thus be characterized by summary parameters and analytical images. This was an achievement in the goal of the scholars of the enlightenment to find uniformity amidst variety, and order amidst chaos.

The search for an order displayed in analytical parameters and images led to modifications of the concept of statistical population. This thesis traces the development of the concept in its application to various fields. The choices of what to examine and to emphasize are determined by the desire to comprehend the analogies that serve as the logical and historical bases of econometrics.

Variation

The concept of population assumes equivalence in some respect, and a relationship binding individuals. The concept, however, is not relevant if individuals are identical in all respects. Recognition of difference within the bounds of the relationship has been as important as acknowledgement of equivalence in the development of statistical method.

Recognition, measurement, comparison and explanation of variation are all essential steps in the development of statistical technique. Within the context of technique, the variation is one of difference. For the fields of social physics, natural selection, and economic motion, however, variation also means a process of change. The divergence of these two ways of perceiving variation is similar to the comparison of logical time and historical time made by writers on economic methodology.

Comparison of different observations on a variable or of different coordinates is often substituted for analysis of change over time. Joan Robinson (1980, p. 228) pointed out, "In a theoretical model, time can be frozen, but it is a common error to confuse a comparison of static positions with a movement between them." As an example, Robinson gave an explanation by Paul Samuelson: "When a mathematician says 'Y rises X falls' he is implying nothing about temporal sequences or anything different from 'when X is low, Y is high'."

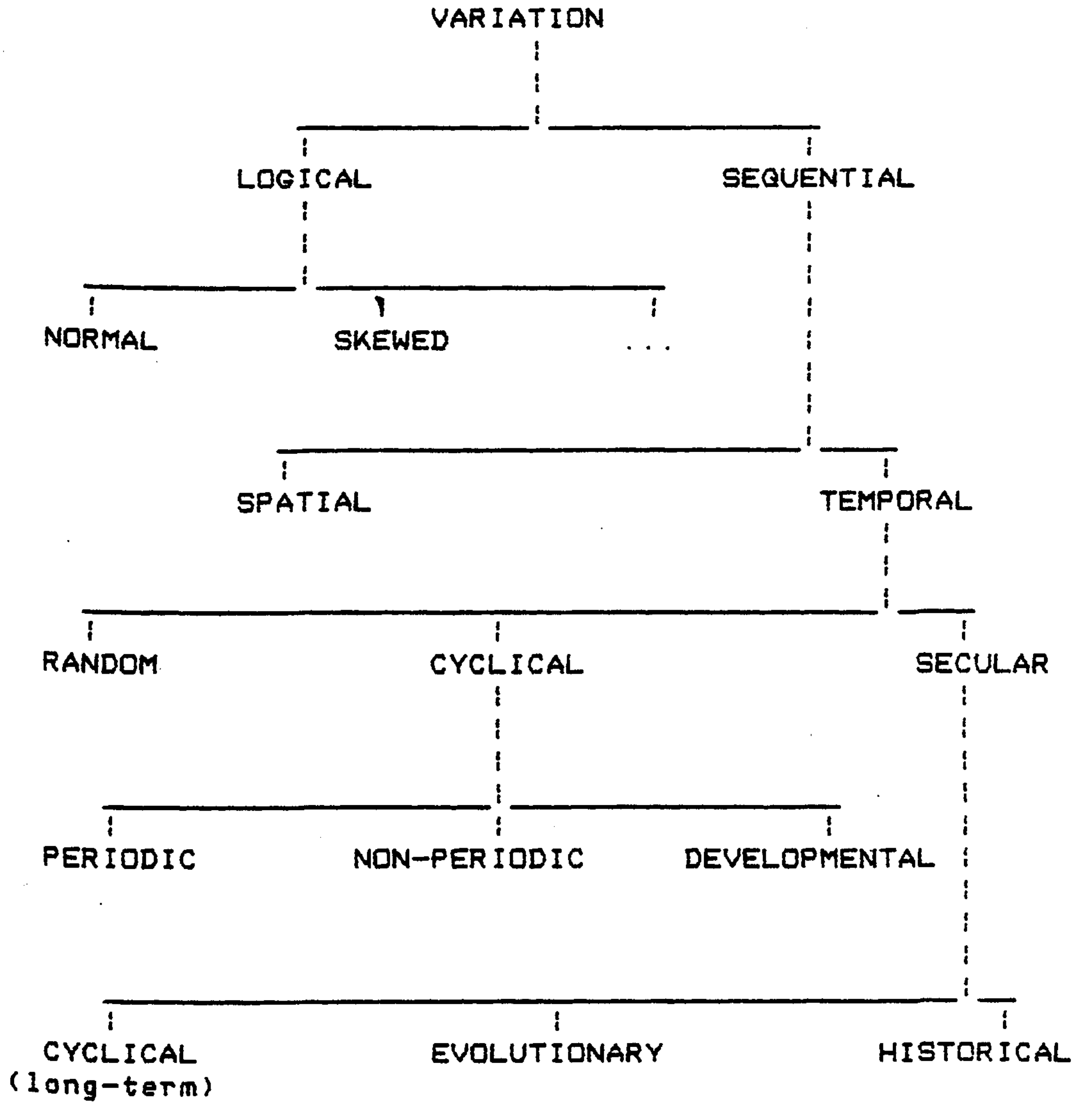
The comparison of different points and a jump from one point to another gives a sense of logical time, but this is not identical with historical time as we experience it. The two-dimensional diagrams and the assumptions of equilibrium in neo-clasical economics deal with logical time and are often inadequate for explaining and predicting change. The confusion of logical time with historical time is very similar to the confusion of "varieties" with "to vary". In the statistical context of populations and distributions, variation is deviation from a mean, not a process of change.

Likewise, the regression line fitted to observations is like the line connecting the points of the neo-classical diagrams. Our eyes move along such lines and give a sense of movement, a sense of analysing change. This sensation should not blind us to the fact that the models are out of time and thus cannot handle change. The visual sensation of movement along the lines is in time, the relationship depicted by such lines is out of time. The models or lines depict sequences, but these sequences are determined by one of the variables increasing in size. As Ludwig von Mises pointed out:

> Logic and mathematics deal with an ideal system of thought. The relations and implications of their system are coexistent and interdependent. We may say as well that they are synchronous or that they are out of time. A perfect mind could grasp them all in one thought. Man's inability to accomplish this makes thinking itself an action proceeeding step by step from the less satisfactory state of insufficient cognition to the more satisfactory state of better insight. But the temporal order in which knowledge is acquired must not be confused with the logical simultaniety of all parts of this aprioristic deductive system. (Von Mises, 1949, p. 99)

The distinction between logical time and historical time is insufficient for classifying the many notions of variation discussed in statistical and philosophical writings over the past three centuries. The chart in Image 1 indicates the breadth of exposition on different concepts of variation. These include:

Image 1


VARIATION
LOGICAL
SEQUENTIAL
NORMAL
SKEWED
...
SPATIAL
TEMPORAL
RANDOM
CYCLICAL
SECULAR
PERIODIC
NON-PERIODIC
DEVELOPMENTAL
CYCLICAL (long-term)
EVOLUTIONARY
HISTORICAL

LOGICAL VARIATION- concentrates on static comparisons and differences. The ordering of the observations for comparison is not based on temporal sequence, but on a logical ordering from smallest to largest.
Bi-directional.

NORMAL VARIATION- yields a frequency distribution of bell-shaped symmetry. first analysed in studies of games of chance and observational errors.

SPATIAL VARIATION- either compares differences or looks at changes from one point in space to another. Mulitdirectional. First analysed in surveying, astronomy, and investigations of terrestrial magnetism, all of which were associated with development of theories of errors in measurement.

DEVELOPMENTAL VARIATION- process of change over a life-cycle. Ontogenic. Unidirectional. First analysed in studies of growth and social physics.

EVOLUTIONARY VARIATION- change of population from one life-cycle to another. Phylogenic. Unidirectional. First analysed in geology and biology.

PERIODIC VARIATION- pattern of change repeated at regular intervals. Unidirectional, but cyclical. Analysed in social physics, meteorology and political economy.

HISTORICAL VARIATION- process of irrevocable change from one point in time to another in time. Unidirectional. First analysed in political economy.

These concepts of variation are related in the chart of Image 1. Statistical method is grounded in logical variation, but it has been used to analyse all of the other types of variation. This thesis examines the successes and failures of these applications and the limitations of logical variation in explaining sequential variation. Of particular importance is the question of what is unique about the sequential variation in economic theory and data.

Most of these attempts have involved separating data into samples distinguished by their relationship to time or to another variable. The method of least squares, regression polygons and early work on seasonal variation are examples of this. In essence, the entire data set on one variable is treated as if it came from separate populations. This approach reached its most sophisticated form in the errors in equations assumption of econometrics.

A significant problem in the application of analytical tools of logical variation to explain sequential variation is the determination of population boundaries. Theory and history should play important roles in this determination. Likewise the acknowledgement of complementary populations can be a powerful catalyst for reconciling logical time and historical time.

Complementarity

The substitution of static comparison of differences for process of change is a common one. As Nicolas Georgescu-Roegen points out:

> Change is the most baffling concept in philosophy...to explain change is the highest aim of any special science, even though we usually proclaim that science can study only what does not change.
> (Georgesu-Roegen, 1976, p. 39)

In addition to the problem of how to apply logical variation to analyse temporal variation there is often a need to distinguish different types of temporal variation. Economics shares this problem with history. As Fernand Braudel eloquently described it:

> It is the problem confronting every historical undertaking. Is it possible somehow to convey simultaneously both that conspicuous history which holds our attention by its continual and dramatic changes and that other, submerged history, almost silent and always discreet, virtually unsuspected either by its observers or its participants, which is little touched by the obstinate erosion of time?

Braudel accepts this problem as a challenge.

> This fundamental contradiciton which must always lie at the centre of our thought, can be a vital tool of knowledge and research.... History accepts and discovers multidimenisonal explanations, reaching as it were vertically from one temporal plane to another. And on every plane there are also horizontal relationships and connections. (Braudel 1972, p. 16).

Braudel divided his history of the Mediterranean world in the Age of Philip II into three separate parts: geographical time, social time, and individual time. Each of these time frameworks is important, even esssential, to understanding the whole, yet they can not be comprehended simultaneously; they are three distinct histories of the same 'age', of the same area.

Several economists have argued for the acceptance of the contradiction that one cannot simultaneously comprehend nor calculate relationships that are essential parts to understanding a phenomena (Marshall (1920) 1961, Shackle 1965, Georgescu-Roegen 1971, Young 1982). They argue against compulsion for synthesis and for the acceptance of complementarity.

According to Shackle, the problem with constructing a general unified model of economic society or of phenomena is that each insight requires "mutually incompatible interpretations of the word 'time'" (Shackle 1965, p. 195). We cannot add up each relationship in one equation, we cannot logically combine all models. "Instead we have to strive for an insight which fuses informally and, if you like, non-logically a number of strands which, in their formal aspects, mutually repel each other." (Shackle 1965, p. 2). Shackle, like Braudel, argues that this is challenging, rather than discouraging.. The contradiction "can be coped with only by a continuing dialectic, an endless resort to first one tool and then another. The ultimate riddle is not for us to solve but to administer."(Shackle 1965, p 196).

The contradiction that these writers speak of was vividly highlighted in the study of light. In a lecture presented in 1932 Niels Bohr (Bohr 1958) argued that this example from the limited domain of physics could influence our views and methods in other domains. Light could be defined as electromagnetic wave , no different from radio transmissions except in frequency of vibration and wave length. The wave character of light explains colour and optical phenomena. Experiments on interference patterns of light from one source "offers so thorough a test of the wave picture of light propogation that this picture cannot be considered as a hypothesis in the usual sense of this word, but may rather be regarded as the adequate account of the phenomena observed". The light-as-a-wave concept, however, was questioned in modern physics when the atomicity of energy transmission of light was discovered.

The two paradigms account for phenomena observed, neither can be discarded without losing some explanatory power, yet they cannot both be tested and studied in the same experiment.

> Indeed, the spatial continuity of our picture of light propogation and the atomicity of the light effects are complementary aspects in the sense that they account for equally important features of the light phenomena which can never

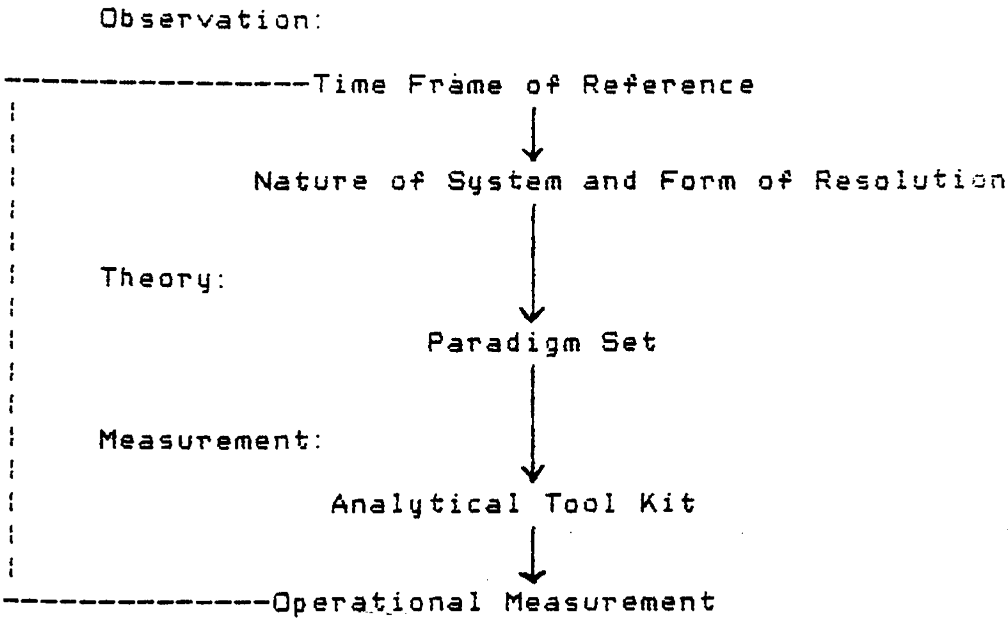
be brought into direct contradiction with one another, since their closer analysis in mechanical terms demands mutually exclusive experimental arrangements. (Bohr, 1958, p. 45).

Bohr's Principle of Complementarity is useful in understanding the difficulty posed by time in economic analysis. Different theories and paradigms can coexist, although we cannot combine them logically into one model, equation or 'experiment'. Warren Young argues that "while economics can be considered an inductive science it is intrinsically and inherently time bound....the observer's choice regarding time frame of reference determines both what is observed and how it is observed."(Young, 1982, p. 175). Young suggests a procedure such as Image 2 for concept formation in economics.

The application of the Principle of Complementarity to econometrics would involve recognition that only one time frame of reference and thus only one paradigm can be analysed in a study. Development in econometrics has gone towards synthesis: including more variables in an equation, including more equations in a structure, modelling an entire national economy. Few seem satisfied with the results, but too often the solution is to attempt further sophistication in the same direction. The problem of time is reduced to the length of the data series (too short a one yields multicollinearity, too long a one, structural change) and decisions on lags.

Calendar time covered by a sample is important, but

Image 2



A Suggested Model for Concept Formation in Economics

(Young, 1982)

so are choice of intervals between observations. These choices in constructing data series determine the relationships that can be revealed in analysis and they exclude the possibility of analysing other relationships simultaneously. A specification of calendar time covered by a sample and of the interval between each observation limits the analysis of change.

The Principle of Complementarity holds out the potential for reconciling the analysis of logical variation for the purpose of understanding temporal variation. It suggests that we can break a time series into samples from distinct populations. This could in some cases do away with the necessity of focusing on errors in equations, and allow a return to a focus on the distributional qualities of the variables themselves.

Structure of the Thesis

A historical treatment of the development of ideas of statistical population and variation can be used in understanding the unique problems of econometrics and possible new directions. The weave of this history is modeled in Image 3. In Chapter Two I examine the maturing of statistical method from its first interaction with political economy to its application to social physics. The development of the concepts of mean and probability distribution and the algorithm of least squares are highlighted.

Image 3

	QUANTITY	POPULATION	MEAN	PROPERTIES OF LOGICAL VARIATION	SEQUENTIAL VARIATION	LOG. VAR. & SEQ. VAR. RECONCILED
Political Arithmetic	Numbers of births, deaths	National and Regional Populace		Stable Ratios		
Games of Chance	Numerical outcomes	Possible Combinations and Permutations	Mathematical Expectation Divine Order	Law of Large Numbers		
Errors of Measurement	Celestial, Terrestial Dimensions	Measurements of one object	True Value	Law of Errors	Spatial	Algorithms for com-binations of observations
Social Physics	Measurements of Human Phenomena	Attributes of Social Groups	l'homme Moyen Social Centre of Gravity	Law of Accidental Causes	Developmental Periodic	Separate Samples
Natural Selection	Dimensions of organisms	Attributes of Species	Typical Ancestral	Natural Random Variation	Evolutionary	Regression Polygon Normal Surface
Economic Motion	Measurements of Stocks, Flows, Rates	Series Differences Residuals	Weighted Average Moving Average	Central Limit Theorem	Cyclical Historical	Deviation from trend Serial Cor-relation in Stochastic Processes

In Chapter Three I look at the important step in the conceptual development of population: the change of focus to deviation from the mean. The work on evolution and inheritance serves as an excellent demonstration of the confusion and ingenuity that arose in attempts to reconcile logical and temporal variation.

Political economy, even more than biology, is immersed in the nuances of temporal variation. In the attempts to go from the classical assumption of change as flow to the modern analysis of change as motion, old and new statistical tools were developed. In Chapter Four I examine the early steps in the development of econometrics.

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CHAPTER 2

FROM POLITICAL ARITHMETIC TO SOCIAL PHYSICS

The dialogue of the study of mass phenomena and probability from 1662 to 1842

"The powers of man are limited. Nature is unbounded. The Supreme Being can alone see events proceed in accordance with his laws. To him TIME IS NOTHING, and all imaginable combinations may be realized in succession. These apparent differences are only found within the sphere of man and spread a remarkable variety over all the events in which he is concerned. This variety, which is in part his work, has however narrow limits, and cannot alter the general order of things." Adolphe Quetelet (1849)

This century has witnessed a tremendous increase in the use of statistics and statistical analysis for economic theory and state policy. In the context of a history of statistics this phenomena appears as a rebirth. The earliest examples of published statistical inference concerned life tables, sex ratios, urban migration, taxation, calculation of a monetary valuation of life, estimation of national expenditure and wealth (Graunt 1662, Petty 1662). The original connection of statistics (then labelled political arithmetic) and the state and post World War II development of national income accounts and macro econometric models reinforces the image of rebirth. John Graunt, often called the Father of Statistics, emphasized the importance of his quantitative analysis to the state:

> I conclude, That a clear knowledge of all these particulars, and many more, whereat I have shot at rovers, is necessary, in order to good, certain and easie Government, and even to balance Parties and Factions both in Church and State. But whether the knowledge thereof be necessary to many, or fit for others than the Sovereign and his Chief Ministers, I leave to consideration. (Graunt (1662) 1975, p.79)

Many have remarked that the labelling of "statistics" to the distinct branch of knowledge we recognise as such was an act of scholarly theft (e.g. Pearson 1978). The name was first applied to the mixture of constitutional history and statecraft exemplified by Gottfried Achenwall in the 18th century. It application to "political arithmetic", however, was not a total misnomer. Sir John Sinclair explained his reasons for the appropiation of the term in his 1798 Statistical Account of Scotland, Vol. XX:

> Many people were at first surprised at my using the new words, 'Statistics' and 'Statistical' as it was supposed that some term in our own language, might have expressed the same meaning. But in the course of a very extensive tour through northern parts of Europe, which I happened to take in 1786, I found that in Germany they were engaged in a species of political inquiry to which they had given the name of 'Statistics', and though I apply a different meaning to that word, for by 'Statistical' is meant in Germany, an inquiry for the purpose of ascertaining the political strength of a country or questions respecting matters of state; whereas the idea, I annex to the term is and inquiry into the state of a country, for the purpose of ascertaining the quantum of happiness enjoyed by its inhabitents and the means of its future improvement.... (quoted in Pearson 1978, pp. 8-9)

Although the concern with "good" government and the use of numerical data link the original statistical studies and current econometric ones, there is a major difference. In the three centuries between, statistical analytical techniques evolved. Unfortunately "statistics" applies both to data and technique. This confusion is probably due to the historical development of an approach orginally limited to numerical summary, eventually combined with mathematical analysis of probability and adapted for inferences on classification, association, causality and change. The stimulus to this development was not always political economy, that role has been assumed by, among other things, the games of the leisure class, astronomy, criminology and biology.

In examining the development from numerical summary of mass phenomena to analytical inference of difference or of change it appears that most of the early writers dealt exclusively with logical variation. Their method of observation and analysis was to ignore the before and after of an event and to ignore the causal sequences of individual occurances. Each outcome studied, whether it be the measurement of a stellar position or of numbers of murders per year, was treated as just one face of a many-sided dye that could possibly be thrown in a game of chance. Processes of change were not studied, rather each possible outcome was compared to all others.

The exceptions to this concentration on logical variation were some references to spatial variation in examples of astronomy and surveying and attempts by social scientists to link logical variation with developmental variation. John Graunt in his studies of mortality figures in London made a brief observation on how an epidemic swells, seems to subside a bit, then surges forward again with severe consequences. The fullest treatment, however, by pre-Darwinian theoreticians on variation as a process of change was that of Adolphe Quetelet. He directed his science of observation toward the development of what he labeled social physics. The substance of physics is force and motion. The application of a comparative method out of time to the understanding of social forces, motion and change seems ackward, if not futile. For Quetelet, however, and many others, if time is nothing for the original creator, then it must be so for those that would try to comprehend the original, grand design

Political Arithmetic

A typical approach in teaching econmetrics is to build up to regresssion analysis through the following concepts:

quantitative measurements, numerical aggregation
proportions, ratios, indices
probability
frequency distributions
arithmetic mean

variance

errors of observation, stochastic properties

association and correlation between variables

hypothesis testing

estimation

This logical progression is similar to the chronological development of statistical inference techniques. There was, however, no unique source of this development; it was a product of dialectical interaction between subject matter and tool, theory and application in a variety of disciplines, places and times.

The theme linking the above concepts is their interaction with a social quantitative approach to comparison and prediction. In contrast with an individualistic mind-set this approach tackles questions of what defines, describes and limits a group, how do groups compare, how do they change? These questions are of interest to, for example, govenors of populations (eg. prime ministers, corporate managers, deans), and to scientists who wish to create order out of chaotic individuality. Arithmetic manipulation of numbers generates the population summaries, the qualitative distinctions, the order.

Numerical recording of human populations has a much longer history than statistics. The origins of the latter are usually attributed to John Graunt and Wliam Petty writing in the latter half of the seventeenth century (Pearson 1978, Kendall 1960). Both Graunt and Petty manipulated numbers to distinguish and summarize catagories, to estimate unknown quantities and make comparisons and generalizations that would be useful to the state. Both were aware that their social,quantitative approach was novel.

> The Method I take to do this is not very usual; for instead of using only comparitive and speculative Words, and intellectual Arguments, I have taken the course (as a Specimen of the Politcal Arithmetic I have long aimed at) to express my self in Terms of Number, Weight, or Measure (Petty, (1690) 1899 p. 244).
>
> That whereas the art of governing, and the true politicians, is how to preserve the subject in peace and plenty, that men study only that part of which teacheth how to support and overdoeth one another, and how, not by fair outrunning but by tripping up each others heels, to win the prize.
>
> Now the Foundation or Elements of this honest harmless policy is to understand the hand and the Lands of the Territory, to be governed according to all their intrinsick and accidental differences..... (Graunt (1662) 1975 p. 78).

Graunts observations were based on the weekly Bills of Mortality continually posted from 1603 to monitor the plague. The plague was a mass phenomena. The breakdown of the sense of the individual was vividly described in Daniel Defoe's description of the 1665 London plague:

> I went all the first part of the time freely about the streets, though not so freely as to run myself into apparent danger, except when they dug the great pit in the churchyard of our parish of Aldgate. A terrible pit it was... about forty feet in length, and about fifteen or

> sixteen feet broad, and, at the time I first looked at it, about nine feet deep; but it was said they dug it near twenty feet deep....
>
> Into these pits they had put perhaps fifty or sixty bodies each; then they made larger holes wherein they buried all that the cart brought in a week, which by the middle to the end of August came to 200 to 400 a week... They had supposed this pit would have supplied them for a month or more when they dug it..... the pit being finished the 4th of September, I think, they began to bury in it the 6th and by the 20th which was just two weeks, they had thrown into it 1114 bodies, when they were obliged to fill it up, the bodies being then come to be within six feet of the surface....
>
> The cart had in it sixteen or seventeen bodies; some were wrapt up in linen sheets, some in rags, some a little after that naked, or so loose that what covering they had fell from them in the shooting out of the cart, and they fell quite naked among the rest..... to be huddled together into the common grave of mankind, as we may call it, fore here no difference made, but poor and rich went together, there was no other way of burials...It was supposed by way of scandle upon the buriers that if any corpse was delivered to them decently wound up, as we called it then, in a winding-sheet tied over the head and feet, which some did, and which was generally of good linen; I say, it was reported that the buriers were so wicked as to strip them in the cart and carry them quite naked to the ground. (Defoe (1720) 1904 pp 66,67,71)

The mass experience of the plague and the weekly posting of the numbers of and causes of death,and of the number of christenings generated a 'statistical' way of thinking. Defoe describes how people checked the newly posted bills and compared present values with averages and previous values.

> This turned the peoples eyes pretty much towards that end of town, and the weekly bills showing an increase in burials in St. Giles parish more than usual, it began to be suspected that the plague was among the people at that end of

town.... it was observed that the weekly bills in general increased very much during these weeks, although it was a time of the year when usually the bills were very moderate.

The usual number of burials within the bills of mortality for a week was from about 240 or thereabouts to 300. This last was esteemed a fairly high bill; but after this we found the bills succesivly increasing, as follows:

	Buried	Increased
Dec. 20th-27th	291	
Dec. 27th-Jan. 3rd	349	58
Jan. 3rd-10th	394	45
Jan. 10th-17th	415	21
Jan. 17th-24th	474	59

This last bill was really frightful, being a higher number than had been known to have been buried in one week since the preceding visitation of 1656. (Defoe (1720) 1904, pp. 3-4)

Graunt noted the bills were made little use of other than to note weekly increases and decreases for general conversation, and in times of plague, 'that so the rich might judge of the necessity of their removall, and Trades-men might conjecture what doings they were to have in their respective dealings.' (Graunt (1662) 1975, p. 17). Graunt went far beyond this weekly comparative approach and

--reduced data from all weekly bills from 1603-1660 into a few "Perspicious Tables"

--investigated a priori opinion as to associations with and severity of the plague

--examined inconsistancies, unaccuracies, biases, and limitations of data observed.

--grouped observations into distinct catagories eg. causes of death due to acute or to chronic diseases

--made comparisons between regions and over time

--calculated a life table

--patterned a life-cycle of disease

--estimated the population, and death, birth and growth rates for London

--Concluded that:

* polygamy should not be allowed since the sex ratio at birth of males to females was greater than one.
* population was unevenly distributed in existing parishes
* 1603 was the worst plague year
* wars can be easily waged and colonies settled without destroying the due proportion of males to females
* trade in the city of London was moving westwards
* the world was not more than 5610 years old

Graunt's study gained him membership in the Royal Society, and his thorough method established a precedent for future statistical analysis. Graunt's observations on the Bills of Mortality and Williams Petty's Essays on Political Arithmetick, written a few years later were studies of society intended for the state. In the cover letter to the Lord Privie-Seal Graunt presents his Observations, "hoping (if I may without vanity say it) they may be of as much use to Persons in your Lordship's place, as they are of little or none to me" (Graunt (1662)

1975 p. 4).

One of the few differences between Graunt's approach and 20th century econometric studies is that in relating any two variables, Graunt assumed constant ratios. Prediction, comparison, and conclusion were based on this assumption. There was no application of frequency distributions, assumptions of errors or inferential methods of estimation that are fundamental to most studies now. The "Rule of Three", whereby a fourth unknown value can be calculated from three known values if the relationship is a proportional one, dominated comparison and prediction from Graunt to Darwin.

Graunt's study is, however, a landmark in the history of statistics. Not only did he study mass phenomena with quantitative analysis, he did so with the use of ratios. The most significant observation Graunt made was on the stability of some of the ratios over years:

> That among the several casualties some bear a constant proportion into the whole number of Burials; such are the Chronical Diseases and the Diseases whereunto the City is most subject; as for Example, Consumptions, Dropsies, Jaundice, Gout, Stone, Palsie, Scurvy. (Graunt (1662) 1975 p36)

Graunt thought that the ratios in the city were more constant than those in the rural provinces because the "airs" were more variable in the latter. He did not recognize the increase in stability that comes with an increase in sample size, nor did he look for stability in most accidental deaths. Graunt's observations, however, were the first celebration of the stability of ratios of measured mass phenomena.

In calculating chances of death from a specific cause, Graunt used the ratio of total deaths in 20 years from that cause to total deaths in 20 years. A modern approach would be to calculate the ratio for each year, construct a frequency distribution, and calculate a mean ratio and probability limits. Graunt acknowledged that his method of constant ratios could only safely be used for inference in cases in which logical variation was slight and the probability of death from a certain cause was not dependent on individual occupation, season or region:

> We shall say nothing of the numbers of those that have been drowned, killed by falls from scaffolds, or by Carts running over them, &c. because the same depends upon the casual Trade, and Employment of men, and upon matters which are but circumstantal to the Seasons, and Regions we live in , and affords little of that Science and Certainty we aim at. (Graunt (1662) 1975 p. 36)

The Science Graunt aimed at was the application of quantitative analysis to population, a social science. Ironically the certainty he aimed at was only achieved when quantitative analysis acknowledged uncertainty and measured it.

Probability and Logical Variation

Death by drowning and other accidents is the result of a multiplicity of causes or as some would see it a random result of chance. For these reasons, Graunt did not bother with an analysis of these mortality figures, but mutliplicity of causes and randomness are the foundation of modern statistical analysis. From the data published in Graunt's General Observations frequency distributions for deaths by drowning or from a variety of accidents as a ratio of total annual deaths yield near-normal distributions.

This incredible pattern formed by observations from accidents was the key to discovering laws, patterns, true values and even causes of many phenomena. Events that seemed so random, so unpredictable such as death by drowning or throwing a seven with three dice became predictable if one observed the outcome of many trials. James Bernoulli in one of the first texts of probablility published in 1713 argued:

> If all events from now through eternity were continually observed (whereby probability would ultimately become certainty), it would be found that everything in the world occurs for definite reasons and in definite conformity with law, and that hence we are constrained, even for things that seem quite accidental, to assume a certain necessity and as it were, fatefulness. (Bernoulli 1956).

The "normal" frequency distribution displayed by the accidental mortality figures is characteristic of observations from a variety of natural and social phenomena and characteristic of errors in measurement of many phenomena. Such a distribution lends certainty to the general in the face of uncertainty to the individual. This two-tiered concept is more than just an acknowledgement of mass versus particular, population versus individual; an additional ingredient is the acknowlegement of probability. A concept that Gibbon described as "so true in general, so fallacious in particular" (Keynes 1921, p. 333)

Petty and to some extent Graunt's approach was arithmetical rather that statistical. As Charles Hull pointed out in his introduction to Petty's economic writings:

> "Statistics demands enumeration. The valididty of its inferences depends upon the theory of probability as expressed in the Law of Large Numbers. Therefore it adds, it does not multiply. Political arithmetic, as exemplified by Petty, multiplys freely; and the value of its results varies according to the nature of the terms multiplied. (Petty (1690) 1899 p. ixvii)

Petty also limited his study to "consider only such causes as have visible Foundations in Nature. In looking at phenomena due to other causes, Petty confessed his inability to "speak satisfactorly upon those Grounds (if they may be called Grounds) as to fortel the cast of the Dye" (Petty (1690) 1899 p. 244). Ironically it was the attempts to fortel the cast of a dye that transformed Petty and Graunt's arthmetic into statistics.

Most histories of probability begin with a history of gambling and games of chance (see Image 1). There is a significant difference within this historical context between a game player that relies on a sense of individual luck, a notion of winning and losing streaks or the invisible hand of a god and one that accepts the certainty of odds over a long time horizon. The strong connection between gambling and statistics is acknowledgement of the uncertainty of any individual outcome and the certainty of the outcome of many repeated events. Gamblers who forsook an individualist approach could become extremely sensitive to the relative frequencies of combinations of dice cast. Galileo was approached by such a gambler and asked to logically confirm the gambler's subjective impression that the probability of a ten being thrown with three dice was greater than that for a nine, although both could be made from the same diversity of numbers. Galileo calculated that the probability of a ten being thrown was .125 that of a nine .116, a difference of only .009. (Galileo 1642)

The approach of these gamblers and logicians was to divorce all expectations of the value of one observation from what was thrown before and what was thrown after. The event is not seen as part of the flow or motion of individual experience. It is one observation randomly picked out of an infinite population of all dice throws. The before and after of one throw can be ignored if one has the patience, as men of leisure and professional gamblers did, to observe many trials. One requires time to ignore time; and one studies logical variation not

ANALYSIS OF GAMES OF CHANCE

Image 1

GENERAL CONCEPT	WHEN/WHERE	SPECIFIC CONCEPT OR METHOD	AUTHOR, DATE OF PUBLICATION, PUBLICATION	DISCUSSED IN
Relative frequencies	16th C	Relative frequency of outcomes from throw of 2 dice.	CARDONO, Gerolama 1501-1576 1526 Liber De Ludo Aleae	David 1962 Maistrov 1974
Combinations, Permutations	Italy	Arithmetic triangle to compute number of variations in throwing any number of dice	TARTAGLIA, Niccolo ca 1499-1557 1556 Trattato General Treatist di Numeri e Misure	David 1962 Maistrov 1974
		A & B intend to play until 1 has won 6 rounds. Game must stop with A having won 5; B, 3. How should stakes be divided?	PACCIOLI, Fra Luea dol Borgo ca 1445-1514 ca 1494, Summa de Arithmetica Geometria Proportioni et Proportionalita	Kendall 1956
		Relative frequencies of 216 ways of throwing 3 dice.	GALILEI, Galileo 1564-1642 before 1642 Sopra le Scoperta dei Dadi	David 1962 Maistrov 1974
	17th C France	1st measurement of probability	PORT ROYAL 1662 La Logique, ou l'art de penser	Hacking 1975
Arithmetic rules of probability	17th C France	Solution to Chevalier de Mere's problem of how stakes should be divided if game of chance ends before its time.	PASCAL, Blaise 1623-1662 FERMAT, Pierre de 1601-1665 1679- Letters	David 1962 Maistrov 1974
Towards a calculus of probability	17th C Netherlands	Mathematical expectation, 1st published text on calculus of probability.	HUYGHENS, Christiaan 1629-1695 1657 De Rationcincis in Ludo Alea	David 1962 Hacking 1975 Maistrov 1974
	18th C Switzerland	Limit theorem	BERNOULLI, James 1654-1705 1713 Ars Conjectondi	David 1962 Hacking 1975 Maistrov 1974
	18th C England	The "Normal" curve as a limit to a binomial distribution, measurements of dispersion, mathematics of life contingencies and annuities.	MOIVRE, Abraham de 1667-1754 1718, 1756 The Doctrine of Chances	David 1962 Pearson 1924 Pearson 1978
		Derivation of binomial distribution "curve" and its properties, rule for obtaining the probability that the required probability lies within given limits.	BAYES, Thomas 1702-1761 1763 "An Essay Towards Solving a Problem in the Doctrine of Chances"	Maistrov 1974 Pearson 1978

historical variation.

Archeological evidence indicates that as early as 5000 B.C. the four-sided astragali bone was used as a die and there are many games of chance in a variety of cultures spanning thousands of years. Several histories (Kendall 1956, David 1962, Maistrov 1974) have asked the question: Why, given the long history of dice-throwing and card-playing, was a theory and calculus of probability so long in emerging? Among the reasons considered were:

- imperfections in early dice
- primitive mathematical notation
- absence of combinatorial ideas and algebra
- superstition of gamblers
- absence of notion of chance events
- moral or religious barriers to the development of idea of randomness and chance
- gambling only acquired mass popularity in 17th century
- statistics were the basic stimulus needed for the devleopment of probability theory

Perhaps a more fruitful approach would be to ask why do histories of statistics so often take games of chance as a starting point? Why do most histories of probability attach so much importance to Cardono's and Tartaglia's attempts to fortel the cast of the dice and Pascal's, Fermat's and Huygen's attempts to determine how stakes should be divided should the an idle game be halted before its time? What could an analysis of games of chance contribute to the developing need of European capitalism and nation-state for explanation and prediction of cause and effect and for abstraction, standardization and massification of concrete phenomena?

The outcomes of games of chance were a useful illustrative vehicle for developing the concepts and methods for rendering the uncertain certain. In a letter probably penned in 1803 to James Bernoulli, Gottfried Leibnitz wrote:

> P.S. I hear the the subject of estimating probabilities -which I consider important- has been not a little developed by you. I would like someone to treat mathematically the various kinds of games (in which there are beautiful examples of this subject). This task would be both pleasant and useful and it would not be unworthy of you.

Bernoulli in response repeats thoughts that were later posthumously published in Ars Conjectandi. The ideas he expressed appear in our hindsight as the roots of what we have labeled the Law of Large Numbers and the Central Limit Theorem. The significance of ideas, is determined by the weights assigned by the historian and the timing of the development of a paradigm is determined by the conceptual jumps we make looking back.

An analysis of games of chance gave 18th and 19th century natural and social sciences a structure for creating order out of the chaos of mass phenomena with an economy of information. Its essential contributions to modern statitical theory were:

1. The notion of mathematical expectation.

2. The foundation for laws and patterns from freaquency distributions of logical variation that could

be used to explain and predict mass phenomena that seemed to follow no obvious law or pattern on an individual scale.

3. The law of large numbers.

Christian Huygens was the first of the logicians of the games of chance to formally articulate the concept of mathematical expectation calling it the 'value of the chance' to win a fair game . James Bernoulli in commenting on Huygen's work elaborated:

> The word "expectation" is not meant here in its usual sense in which 'to expect' or 'to hope' refers to the most favorable outcome; although the least favorable may occur; we would understand this word here as the hope of getting the best diminished by the fear of getting the worst. Thus the value of our expectation always signifies something in the middle between the best we can hope for and the worst we fear. (quoted in Maistrov 1974 p57)

Estimating parameters from observational data has a long history: the Babylonians and the Greeks used simple arithmetical schemes to calculate positions of celestial bodies from conflicting observations in 500-300 BC (Plackett 1958 p.12). The arithmetic mean was widely used in commerce before Huygens work was published. Probability theory, however, gave the use of the mean more credence and more applicability. It demonstrated that the mean was a more precise measurement than any single observation, the best summary parameter for a variety of observations and the most probable, expected value in many cases of uncertain outcome. The equating of mathmatical expectation with a summary parameter such as the arithmetic mean linked description and induction, measurment and theory.

A clue to the contribution of the analysis of games of chance in establishing patterns and laws for summary, explanation and prediction is in the use of the word 'chance'. Henri Poincaré (Poincaré 1956) asked how paradoxically can we establish laws of chance when chance seems the antithesis of all law and how can we use a calculus of probability to determine uncertainty when probability is a statement of uncertainty? Poincare's answer was defining chance as something other than the name we give our ignorance or to effects that seem to have no cause. For Poincaré effects have either simple, significant causes or they are the the result of slight, perhaps complex, causes that produce great effects. The latter is labeled chance.

> The greatist bit of chance is the birth of a great man. It is only by chance that meeting of two germinal cells, of different sex, containing preciseley, each on its side, the mysterious elements whose mútual reaction must produce the genius. One will agree that these elements must be rare and that their meeting is still more rare. How slight a thing it would have required to deflect from its route the carrying spermatozoan. It would have sufficed to deflect it a tenth of a millimeter and Napoleon would not have been born and the destinies of a continent would have been changed. No example can better make us understand the veritable characteristic of chance. (Poincaré 1956 p. 1392)

Having defined chance as slight variations in causes having great effect, Poincaré argues that laws of chance are manifested from representing probability by a continous function. Slight variation in cause allows us to assume a continuous analytical function:

> Thus we see why phenomena obey the laws of chance when slight differences in the causes suffice to bring on great differences in the effects. The probabilites of these slight differences may be regarded as proportional to these differences themselves, just because these differences are minute, and the infitesimal increments of a continuous function are proportional to a variable. (Poincaré 1956 p1387)

Out of the complexity of slight variations in multiple causes simplicity is born. In fact ignorance of specific laws and facts of phenomena observed allows us to explain and predict using laws of chance. One could try to study how the force of a throw, the surface of a table, the resistance of the atmoshpere, etc. determines which side of the dye will land face up, but the assumption of equal probability of all sides would yield useful results with an economy of information. If we knew the velocity, paths and laws of motion of each molecule of gas in a container the inextricable calculations would not allow us to explain or predict change, an assumption of random movement does. The intricate knowledge of cause and effect of all lifestyles and of each individual's habits, genetic and chemical makeup would be of little use to insurers for profitable prediction of mortality rate compared to the usefulness of assumptions arising out of an analysis of chance. In these cases the best approach is to ignore the history, the before and after, the cause and effect of individual outcomes and compare possibilities of all outcomes.

What impressed students of chance was the stablility of statistical ratios or frequences if a sufficient number of instances was observed. In many games of chance we know a priori the probability of a particular outcome, because we are aware of the original design. With most phenomena, however, the probability cannot be determined a prioiri. The earlier probability theorists had confined themselves to calculating probable outcomes when the only requirement was the number of cases be equally likely, faces of a dye or a coin. In Ars Conjectandi James Bernoulli introduces the art of conjecturing "measuring its probability... the stochastic art" using "the way which is based on trials". (Bernoulli (1713) 1966 p. 13):

> What you cannot deduce a prioiri you can at least deduce a posteriori i.e. you will be able to make a deduction from many observed outcomes of similar events. For it must be presumed that every single thing is able to happen and not to happen in as many cases as it was previously observed to have happened and not to have happened in like circumstances. (Bernoulli (1713) 1966 p. 37)

Bernoulli was interested in determining a posteriori the probable outcome in order to apply "the art of estimation to civil, moral and economic affairs". Recognizing that common sense dictates that the larger the number of observations available the smaller the risk of falling into error, Bernoulli laid the foundations for the first limit theorem by asking:

> whether the probability of an accurate ratio increases steadily as the number of observations grows, so that finally the probability that I have found the true ratio rather than a false

> ratio exceeds any given probability; or whether each problem, so to speak, has an asymptote—that is, whether I shall finally reach some level of probability beyond which I cannot be more certain that I have detected the true ratio.... if the former is true we will investigate the ratio between the numbers of possible outcomes a posteriori with as much certainty as if it were known to us a priori.
> ....I have found the former condition is indeed the case; whence I can now determine how many trials must be set up so that it will be a hundred, a thousand, ten thousand, etc. times more probable (and finally, so that it will be morally certain) that the ratio between the numbers of possible outcomes which I obtain in this way is legitimate and certain. (Leibnitz 1855)

Abraham De Moivre elaborated on this and proposed that accuracy increases inversely with the square root of the number of observations. Abraham De Moivre also deduced the "normal" curve from the limit to the binomial distribution. These proofs and tools allowed the calculus of probability to be extended beyond the realms of games of chance where expected outcomes were known a priori. Inductive reasoning could be used to determine parameters a posteriori without having to exhaust the population being studied. Sampling could be used to describe and to infer even in the seemingly chaotic uncertainty of chance:

> As upon the Supposition of a certain determinate Law according to which any Event is to happen, we demonstrate that the Ratio of Happenings will continually approach to that Law, as the experiments or observations are multiplied: so conversely, if from numberless observations we find the Ratio of the Events to converge to a determinate quantity, as to the Ratio of P to Q; then we conclude that this Ratio expresses the determinate law according to which the event is to happen (De Moivre 1756, p 251)

The widespread applicability of this analysis was not lost on philosophers of the 17th and 18th centuries. John Abuthnot in his 1692 preface to his translation of Huygens "De Ratiorinis on Ludo Aleao" noted:

> The Reader may here observe the Force of Numbers, which can be successfully applied, even to those things, which one would imagine are subject to no Rules....I believe the calculation of the Quantity of Probability might be improved to a very useful and pleasant Speculation, and applied to a great many Events which are accidental, besides those of Games....all the Politicks of the World are nothing else but a kind of Analysis of the Quantity of Probability in causal Events, and a good Politician signifies no more, but one who is dexterous at such Calculations; only the Principles which are made use of in the Solution of such Problems, can't be studied in a closet, but acquired by the Observation of Mankind. (quoted in Pearson 1978, p. 140)

An analysis of games of chance seemed frivolous, futile and even blasphemous to many contemporaries of the early probability theorists. Did not the irregularities of chance threaten any attempts at establishing laws or looking for divine will? The analysis, however, yielded laws and for some, further illustration of divine intervention. Abraham De Moivre argued that

> Chance very little disturbs the Events which in their natural Institution were designed to happen or fail according to some determinate law....although chance produces irregularities, still the odds will be infinitely great, that in process of Time, those Irregularities will bear no proportion to the recurrence of the Order which naturally results from Original Design. (Abraham De Moivre 1756 , p. 251).

The elements of games of chance studied had equiprobable outcomes that were combined to yield outcomes of discrete probability. Eventually the intellectual jump was made to the foundation for establishing a pattern that described a variety of natural and social phenomena. The concept of a continuous, symmetric curve formalizing logical variation was intellectually appealing and a good 'fit' for a variety of studies. In his correspondence with Leib nitz, Bernoulli discusses the example of fitting a parabola to several points observed in tracking the path of a comet:

> I admit that every conjecture which is deduced by observations of this sort would be quite flimsy and uncertain if it were not conceded that the curve sought is one of the class of simple curves; this indeed seems quite correct to me, since we see everywhere that nature follows the simplest paths. (Leibnitz 1855)

Bernoulli was describing attempts to model sequential variation, paths followed by nature. Chance outcomes do not seem to follow a unique historical path from cause to effect or even from one trial to the next. If sequence is ignored, however, the relative frequencies of possible outcomes from many observations can be fitted to a simple curve that describes logical variation.

The first widely used pattern of this sort has been variously described as the frequency curve of errors, the law of errors, the Gaussian curve of errors, the law of accidental causes and the law of possibilities, and the normal curve. This pattern of logical variation, a visual and mathematical model for comparison out of time of one observation with many others, took the status of a law in attempts to systematically handle inevitable errors in measurment in the empirical sciences.

The Treatment of Observational Errors

Games of chance were the illustrative vehicle of the mathematicians for developing a calculus of probability and life insurance and annuities their main focus of application. Archery was the example alluded to by the mathematicians developing a system for accomodating observational errors, and astronomy and geodosy were the early foci of applications. A major problem confronting scientists of the 17th and 18th centuries was, if the correctness of ideas was to be decided not on authority, or even just logic, but on observation, how did one systematically deal with errors of observation?

Galileo Galilei was very critical of the blind reliance on authority. In his Dialogue Concerning the Two Chief World System, Galileo discussed methods of determining the position of a celestial body (the Nova Stella of 1572). Thirteen observations were made, all of which gave conflicting positions. The problem was deciding which position was the correct one:

> Simplico: I should judge that all were fallacious either through some fault of the computer or some defect on the part of the observations. At best I might say that a single one, and no more , might be correct, but I should not know which one to choose. (Galilei 1630, p. 281)

Galileo's characters concluded:

errors were inevitable: "there is some error in every combination of these observations. This I believe to be unavoidable."

there is no bias to overestimation or underestimation: "they are equally prone to err in one direction and the other."

small errors are more probable than large ones

the size of the errors "must not be reckoned from the outcome of the calculation, but according to the number of degrees and minutes actually counted on the instrument."

These assumptions allowed one to treat each error indifferently and out of time. They also laid the foundation for a symmetric, bell-shaped frequency curve of errors. One important conceptual step to this acknowledgemnt of a pattern of logicial variation was the significance accorded the mean of observations. Adolphe Quetelet in his Theory of Probability noted:

> When we stand in the Presence of Nature, and seek to interrogate her, we are at once struck with infinite variety which we observe of the least phenomena. Whatever may be the limits within which we concentrate our attention we find a diversity as astonishing as it is embarrasing. The most simple appreciations leave a vagueness incompatible with precision which science requires. One single object, measured or weighed several times in succession, notwithstanding every precaution that may be taken, nearly always presents dissimilar results. Our ideas, however, seem to fix themselves, and to settle on a precise number—on some mean which will show the results of the observations made, as free as possible from accidental error....
>
> The theory of Means serves as a basis to all sciences of observation. It is so simple and so natural that we cannot perhaps appreciate the immense step it has assisted the human mind to take. (Quetelet 1849, p38, 39)

ANALYSIS OF ERRORS IN MEASUREMENT

Image 2

GENERAL CONCEPT	WHEN/WHERE	SPECIFIC CONCEPT OR METHOD	AUTHOR, DATE OF PUBLICATION, PUBLICATION	DISCUSSED IN
Assumptions on nature of and frequency distributions of observational errors	17th C Italy	Errors in measurement inevitable, symmetric distribution, small errors more probable than large ones.	GALILEI, Galileo 1564-1642 1630 Dialogue concerning the two Chief World Systems - Ptolemaic and Copernican	Maistrov 1974
	18th C England	Mean of a great number of observations is preferable to any one single observation, 1st consideration of continuous distribution of errors, each observation treated as one face of a many-sided dye	SIMPSON, Thomas 1710-1761 1755 A Letter...on the Advantage of Taking the Mean of a Number of Observations in Practical Astronomy. 1757 Miscellaneous Tracts on Some Curious and Very Interesting Subjects in Mechanics, Physical Astronomy and Specualtive Mathematics	Maistrov 1974 Tilling 1973
	18th C Switzerland	Maximum likelihood	BERNOULLI, Daniel 1700-1782 1778 The most probable choice between several discrepant observations and the formation therefrom of the most likely induction	Kendall 1961
	19th C Germany	Probability distribution of errors	GAUSS, Carl Friedrich 1777-1855 1809 Theoria Motus Corporum Coelestium	Maistrov 1974 Pearson 1923

ANALYSIS OF ERRORS IN MEASUREMENT

Image 2

GENERAL CONCEPT	WHEN/WHERE	SPECIFIC CONCEPT OR METHOD	AUTHOR, DATE OF PUBLICATION, PUBLICATION	DISCUSSED IN
Assumptions on residuals and criteria for combination of observations subject to error	18th C England	Most probable sight of true value of variable in plane of four observations taking weighted mean.	COTES, Roger 1682-1716 1722 Aestimatio errorum in Mixta mathesi, per variationes partium trianguli plani et spherici	Kendall 1961 Eisenhart 1961 Eisenhart 1964 Tilling 1973
	18th C France	Measuring irregularities in observed orbit of planets by extending zero-sum of residuals to multiparameter problem	EULER, Leonhard 1707-1783 1749 Piece qui a remporte le prix de L'Academie royale des sciences, sur les inegalities du mouvement de Saturne et de Jupiter	Kendall 1961 Eisenhart 1964 Tilling 1973
	18th C Germany	Method of averages applied to lunar libration	MAYER, Johann 1723-1762 1750 Abhandlung uber die Umwalzung des Mondes um seine Axe	Kendall 1961 Eisenhart 1964 Tilling 1973
	18th C France	Criteria for line of best fit for pair of observations measuring ellipticity of the earth: Σ of + and - residuals in Y direction = Σ of absolute values of all residuals as small as possible	BOSCOVICH, Rudjer 1711-1787 1757 De Litteraria Expeditione per Pontificiam ditionem ad dimetiendas duas Meridiani gradus	Eisenhart 1961 Eisenhart 1964 Tilling 1973

ANALYSIS OF ERRORS IN MEASUREMENT

Image 2

GENERAL CONCEPT	WHEN/WHERE	SPECIFIC CONCEPT OR METHOD	AUTHOR, DATE OF PUBLICATION, PUBLICATION	DISCUSSED IN
Assumptions on residuals and criteria for combination of observations subject to error (continued)	18th C France	In Y=A+bx, a+b be chosen so as to minimize the absolute value of largest deviation	LAPLACE, Simon, Pierre, Marquis de 1749-1827 1783 Memoire sur la Figure de la Terre	Eisenhart 1964 Whittaker 1944 Tilling 1973
		Algebraic formulation of Boscovich's algorithm for minimizing residuals	1789 Sur les degres mesures des meridiens, et sur les longueurs observees sur pendule	
		Method for minimizing absolute value of maximum residual with large number of observations	1799 Traite de Mecanique Celeste	
	19th C France	Least sum of squared residuals of which the principle of arithmetic mean is a special case	LEGENDRE, Adrien Marie 1752-1833 1805 Nouvelles methodes pour la determination des orbites des cometes	Eisenhart 1964 Seal 1967 Whittaker 1944 Tilling 1973
	19th C Germany	Maximum probability of zero error of estimation. 1st connection of least squares with theory of probability	GAUSS, Carl Friedrich 1777-1855 1809 Theoria Motus Corporum Coelestium	Eisenhart 1964 Seal 1967 Whittaker 1944 Maistrov 1974 Tilling 1973
		Least mean squared error of estimation	1821-1826 Theoria combinationis observationum erroribus minimis obnoxiae	Pearson 1920

The immensity of the step was acknowledged by Simpson writing in 1755:

> ...the method practised by astronomers, in order to diminish the errors arising from the imperfections of the instruments and of the organs of sense, by taking the mean of several observations, has not been so generally received, but that some persons of considerable note have been of the opinion and even publicly maintained, that one single observations, taken with due care, was as much to be relied on as the Mean of a great number. (Simpson 1755, p.83)

Simpson proved that "the taking of the mean of a number of observations, greatly diminishes the chances for all the smaller errors and cuts off almost all possibility of any great ones". He did so by treating an error as if it were the face of a many-sided die: "each die having as many face as the result of one single observation can come out different ways" (Simpson 1755, p. 86). A prerequisite for the application of the mathematical analysis of games of chance to the science of observation was divorcing measurement error from cause and effect, from the individual observer, from measurements that had been taken sequentially before or would be taken sequentially after, from time.

Laura Tilling (1973) in her history of theories of observational errors states that the first algorithms for systematic treatment of errors in measurement were developed ad hoc for specific practical purposes. The most sophisticated work came out of the attempts to measure the ellipcity of the earth to determine whether it was oblate, according to Newton's hypothesis or oblong. Tilling argues that in this area alone there was enough discrepancy between theory and data to warrant a thorough treatment of the effect of errors of observation.

The establishment of a probabilistic theory of errors came more from the mathematician's search for applications of analysis of games then from the scientist's search for mathematical analysis to eliminate the irregularities caused by observational error. By the early 1800's practical algorithm and probabilistic theory were philosophically joined. Pierre Laplace, Carl Friedrich Gauss and Auguste Bravais formulated equations for a surface and a curve describing a law of distribution of errors and the least squares method for determining the best combinations of several observations to minimize error. Their treatment was almost totally mathematical and when mention was infrequently made of applications it was to geometric relations between instrument readings and desired measurement arising in astronomy and geodosy. The key assumption for these developments was that the errors of observation being considered displayed random sequential variation but predictable logical variation.

> Certain causes of error depend for each observation, on circumstances which are variable and independent of the result which one obtains: the error arises from such sources are called irregular or random, and like the circumstances which produced them their value is not susceptible to calculation.. and it is certainly necessary to tolerate them in observations. One can however by a suitable combination of results reduce their influences as much as possible (Gauss 1821 p 2)

The development of a mathemtical treatment of errors in observation was paralled by the developments in graphical analysis of experimental data. Laura Tilling (1975) in a history of experimental graphs asserts that with the exception of Johann Heinrich Lambert graphs were not used for presentation or analysis of experimental results until the mid-nineteenth century. In his studies of temperature, magnetism, evaporation and other physical variations, Lambert interposed smooth curves amongst the observations to discover periodic variation and bivariate correspondence. (e.g. Lambert 1779) Lambert's description in 1765 of his graphical method is one of the first writings on the theory of errors:

> We have in general two variable quantities, x, y, which will be collated with one another by observation, so that we can determine for each value of x, which may be considered as an abscissa, the corresponding ordinate y. Were the experiments or observations completely accurate, these ordinates would give a number of points through which a straight or curved line should be drawn. But as this is not so, the line deviates to a greater or lesser extent from the observational points. It must therefore be drawn in such a way that it comes as near as possible to its true position and goes, as it were, through the middle of the given points. (Lambert Quoted in Tilling 1975 p204)

This perception was rare for the 1700's and it was not until the 1850's that graphical presentation became a popular technique. William Whewell in 1840 explained his advocacy for the unusual use of the 'method of curves' as one of several 'special methods of induction applicable to quantity':

> By this method, thus getting rid at once, in great measure, of errours of observation, are obtained data which are more true than the individual facts themselves. The philosopher's business is to compare his hypotheses with facts, as we have often said. But if we make the comparison with seperate special facts, we are liable to be perplexed or misled, to an unknown amount, by the errours of observation; which may cause the hypothetical and the observed result to agree, or to disagree, when otherwise they would not do so. If however, we thus take the whole mass of the facts and remove the errours of actual observation, by making the curve which expresses the supposed observations regular and smooth, we have the seperate facts corrected by their general tendency. We are put in possession, as we have said, of something more true than any fact itself is. (Whemwell Quoted in Tilling 1975 p209)

Once it was perceived that errors of observation followed a law, the imperfect observations themselves could be used to discover laws of change and of relationships. The importance of the philosophical and algorithmic work on observational errors to the development of econometrics is not only in its further generalization of probability distributions. The work on combinations of observations gave a conceptual and mathematical basis to multiparameter estimation and to statistical dependence of two or more variables.

In several empirical problems of geodasy the true values of the variables were deterministically related through mathematical law. The pairs or groups of observations of these quantities however were subject to observational errors. Algorithms were developed to reconcile discrepancies when the number of observations exceeded the number of unknowns. Gauss searched for the best algorithm in the theory of probability:

> Among the combinations, the most advantageous should be chosen, that is to say, those which furnish values whose standard errors to be expected is as small as possible. This problem is certainly the most important which the application of mathematics to natural philosophy presents. (Gauss 1821 p.32)

Gauss and others obtained multivariate normal distributions of errors of independent variables. It was not until Francis Galton's work on heredity that normal surfaces were constructed for statistically dependent variables. Gauss' work on the principle of least squares, however, did provide a basis for estimation of relationships between variables. It lay the ground for associating a seperate distribution of observed values of a dependent variable for each value of an explanatory variable. The work of Gauss, Laplace, Lambert and others thus gave a conceptual foundation to the use of least squares to crystallize laws of motion and causation. Errors in measurement were eventually generalized to errors in equations.

Works on the theory of errors reinforced the conjunction of population and variation developed in political arithmetic and analysis of games of chance. This reinforcement was augmented by the patterns of logical variation that were revealed in quantitative comparisons of social and biological analysis. The law of error was generalized to a law of accidental causes.

The Law of Accidental Causes

The law of errors was transformed by Adolphe Quetelet into the law of accidental causes. Quetelet's own intellectual development was from mathematics and astronomy to the eventual study of social phenomena. The techniques of observation and the analysis of probability used in the former was applied to the latter. In _Du Systeme Social et des Lois qui de Re'gissent_ published in 1848 Quetelet explained:

> In this new work I show that the law of accidental causes is a general law which is applied to individuals as well as to peoples and which dominates our moral and intellectual qualities as well as our physical qualities. (quoted in Hawkins 1908 p63)

Quetelet left few areas of human physique and social interaction unexplained in his attempts to apply the general law. And in many cases where chance or human free will would seem to defy any law the pattern of logical variation displayed by the curve of errors fit:

> Now, what do these facts teach us? I repeat that in a given state of society residing under the influences of certain causes, regular effects are produced, which oscillate as it were, around a fixed mean point, without undergoing any sensible alterations....
>
> After these last new researches, I conceive

> I may now confidently say that the table of criminalty for different ages, given my published treatise, merit at least as much faith as the tables of mortality and verify themselves within perhaps even narrower limits; so that crime pursues its path with even more constancy than death (Quetelet 1842 p. VII, VIII).
>
> Society includes within itself the germs of all the crimes committed, and at the same time the necessary facility for their development. It is the social state, in some measure, which prepares these crimes, and the criminal is merely the instrument to execute them (Quetelet 1842 p. 6).

In his application of the law of accidental causes to social phenomena, Quetelet took observations, both cross-section and time series, ordered them from smallest to largest and noted the frequency distribution, which in many cases was that of a normal distribution. In using observations taken over time, Quetelet took pains to distinguish logical variation over the day, logical variation over a month and logical variation over a year. In several cases these were treated as separate populations with separate sample frequency distributions. In his scheme of causes this was explained by the action of periodically variable causes. In some cases these were more significant than constant or accidental causes:

> The regular and periodic causes, which depend either on the annual or diurnal period, produce effects on society which are more sensible, and which vary within wider limits than the combined, non-periodic effects annually produced by the concurrence of all the other causes operating on society; in other terms the social system, in its present state, appears to be more dissimilar to itself in the course of one year, or even in the space of one day than during two consecutive years; if we have reference to the increase in population.

> The diurnal period seems to excercise a somewhat stronger influence that the annual period, at least so far as births are concerned. (Quetelet 1842 p. 108)

Quetelet did apply an analysis of logical variation to time series observations, but he acknowledged complementary time frameworks in his study. He also went beyond the law of accidental causes and logical variation to focus on developmental variation. For Quetelet "Man is born, grows up and dies, according to certain laws which have never been properly investigated". In his investigation of these laws and his attempts to develop a social physics Quetelet's method was

1. To always study a group not an individual.
2. To measure attributes, to search for the limits of each group and in particular to determine the characteristics of the average person.
3. To look at human development by examining the average person at different ages.
4. To examine causal factors and progression of human civilization (evolutionary variation) by examining changes in the average and in the limits of a population.

With regards to the first, Quetelet often maintained that the task of science was to abandon study of individual characteristics and development and to treat the individual as just one part of the species, which should be the focal point:

> It should be well understood that social physics never can pretend to discover laws which will verify themselves in every particular in the use of isolated individuals (Quetelet 1842 p. X).
>
> It is of primary importance to keep out of view man as he exists in an insulated, seperate, or in an individual state, and to regard him only as a fraction of the species (Quetelet 1842 p. 5).
>
> It would appear then, that moral phenomena, when observed on a great scale, are found to resemble physical phenomena; and we thus arrive, in inquires of this kind, at the fundamental principle, that the greater the number of individuals observed, the more do individual peculiarities, whether physical or moral, become effaced, and leave in a prominant point of view the general facts, by virtue of which society exists and is preserved.
>
> .... It is the social body which forms the object of our researches, and not the peculiarities distingushing the individual composing it (Quetelet 1842 p. 6,7).

Having established the importance of looking at the social body, Quetelet focuses on the average person. With regards to all characteristics, height, weight, intellegence, he calculates the arithmetic mean. For Quetelet such a parameter is not a purely abstract numerical result, but it is a true mean and has significance because all observations are related. They come from one population. No one individual may have the characteristics of the average man (l'homme moyen) but such a entity should be regarded as society's centre of gravity. The latter concept was held in high regard by Quetelet who often mentioned the usefulness of assuming one point in studying the physical forces.

"The social man whom I here consider resembles the centre of gravity in bodies: he is the centre around which oscillates the social elements - in fact, so to speak, he is a fictitious being, for whom every thing proceeds conformably to the medium results obtained for society in general. It is this being whom we must consider in establishing the basis of social physics, throwing out of view peculiar or anomalous cases and disregarding any inquiry tending to show that such or such an individual may attain a greater or less development in one of his facilities. (Quetelet 1842 p. 8)

The average man indeed is in a nation what the centre of gravity is in a body; it is by having that central point in view that we arrive at the apprehension of all the phenomena of equilibrium and motion. (Quetelet 1842 p. 96)

Examples of Quetelet's investigations into equilibrium and motion included :

population growth and social prosperity

development of stature weight and strength

development of moral and intellectual facilities

development of the social system

In his studies of human development Quetelet dealt with a few types of variation. He was interested in the developmental variation and the evolutionary variation of the average individual, but to grasp this he had to eliminate logical variation among individuals of the same age and historical period. The studies of developmental variation focused on the growth of the average individual from birth to death. This process of change is repeated in every individual, thus Quetelet was able to use cross-section data and logic to look at developmental variation. In constructing his data tables on mortality and frequency of crime Quetelet used data collected in Belgium over a period of three years. The following passage is typical of his interpretations of the patterns observed when cross section data covering thousands of people were organized according to age:

> Towards the age of 24, there is a peculiar circumstance connected with men; namely, a maximum which is not observed in the curve of mortality of women (see Images 3 & 4). The period of this maximum coincides with that when man shows the greatest inclination to crime; it is the stormy age of passion, which occupies a most conspicuous place in the moral life of man. (Quetelet 1842 p. 32)

Quetelet did not question the use of cross-section data to infer growth and change. If one is trying to determine the relationship between height and age of people, does one take a sample of different aged people and for one moment observe various heights and ages, or does one need observations on one person over an entire life? The former is usually the most convenient since the study can be completed in a much shorter time period, but there are other considerations. In the cross-section study one risks the possibility that the relationship has not been the same for all historical time periods covered by the ages of the sample (a period of famine might have affected one age group, short people might live longer). A longitudinal study can highlight historical influences e.g. recent longitudinal studies on US females who are in their 30's and of those who are in their 50's show remarkable differences in relation of age to labour force participation, first marriage, birth of first child, etc.

Image 3
Quetelet 1842

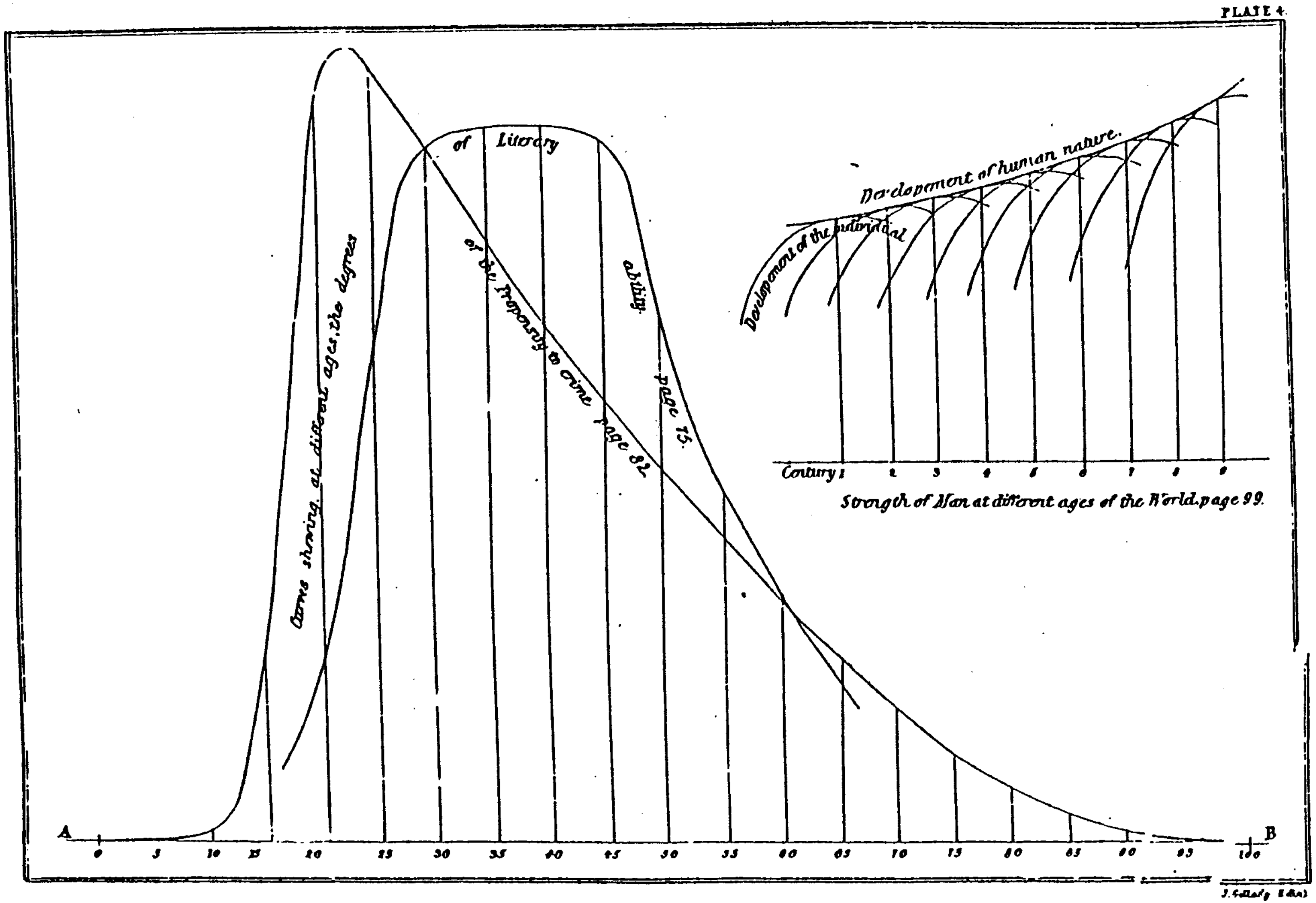
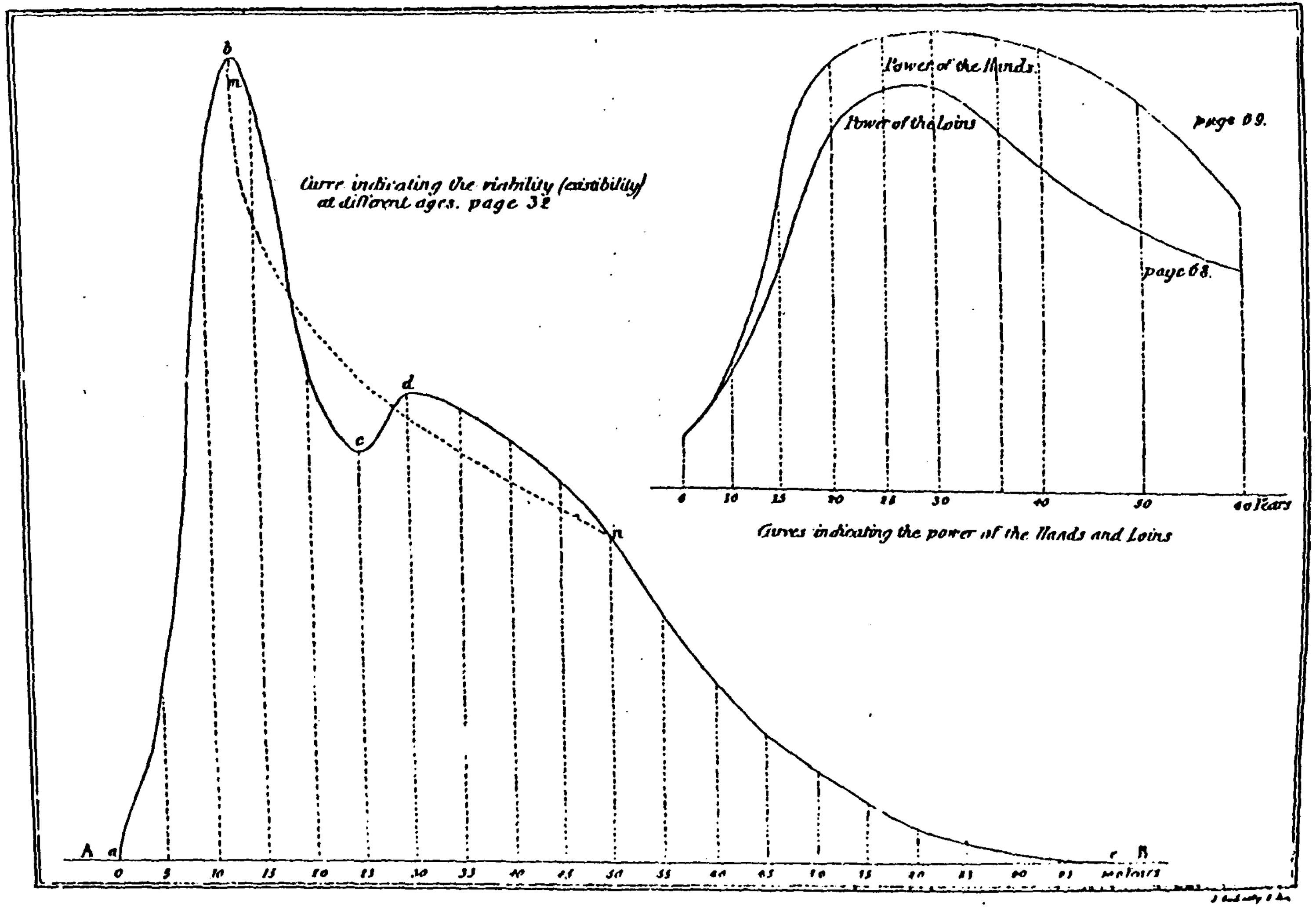


Image 4
Quetelet 1842

PLATE I.



A cross-section study of the relationships between any one of these chararcteristics and age would decribe a pattern that no women from any generation has experienced or will experience.

Empirical investigations in which one of the variables under examination is age are unique: an analysis of change over time can be done with each individual at different stages of their life-cycle or at one point in time people of different ages can be a mass surrogate for one person. It is interesting to note that the first attempts to statistically analyse processes of change were cases where human development was the key factor (Adolphe Quetelet and Henry Pickering Bowditch). For Quetelet the developmental variation exhibitied by humans could well mimic in some cases the evolutionary variation of the species over many generations:

> I have said that, although the law of the development of human nature were not generally the same as those of the average man of any one period, yet these laws might in certain circumstances, be identically the same; and that human nature, under certain circumstances might be developed in a manner similar to a single individual. (Quetelet 1842, p.100)

At each age one observed the average height or the average crime rate. For Quetelet the key to understanding the principles of social physics (the true mechanics of human history) and in particular evolutionary variation was to follow the changes in the average man in many nations from one generation to the next. For Quetelet, as for Aristotle "human qualities become virtues, when they are equally removed from all the excesses into which they may be disposed to fall and confined within due limits, beyond which every thing is vice" (Quetelet, 1842, p. 100). The exhaltation of the moderate, the mean as virtue becomes absolute if the average does not change. For most qual ities Quetelet argued there was little change:

> Now this is what we remark generally concerning most moral qualtities; they admit a type which we may with great probability consider an absolute, so that human nature, considered in reference to these qualities will not be progressive....
>
> It appears to me that science only is truly progressive, and I use this word in its widest sense. All the faculties of man which are not based on science are stationary and their laws of development are constant....The development of science would therefore give the measure of the development of human nature. (Quetelet 1842 pp. 100-101)

For many qualities the average may not change from one age to the next but the limits, the range of logical variation, do:

> I shall conclude this chapter by a final observation which as it were is a consequence of all the preceding viz. that one of the principal fact of civilization is that it more and more contracts the limits within which the different elements relating to man oscillate. (Quetelet 1842 p. 108)

The narrowing of limits became a normative prescription in Quetelet's study of society. After analysing logical variation of data from different time periods, Quetelet constructed a list of phenomena ordering them from least variable to most variable:

human stature

the repression of crime

births

the propensity to crime

deaths

marriages

receipts and expenses of the treasury

the price of grain

Quetelet's recommended that society reduce the variability of the latter in particular:

> Since the price of grain is one of the most influential causes operating on the mortality and reproduction of the human species, and since at the present day, this price may vary within the widest limits, it is the province of the foresight of governments to diminish as much as possible all the causes which induce the great variation in prices and consequently in the element of the social system (Quetelet 1842, p. 108)

Quetelet limited his quantitative investigation to logical and developmental variation. His ultimate goal in the study of social physics was understanding evolutionary variation, but to this study he applied neither data nor an explanation of the process of change in the average and the limits. An explanation of evolutionary variation was later initiated by Thomas Malthus, Alfred Wallace and Charles Darwin and the application of data to the analysis of evolution was initiated by Francis Galton, Karl Pearson and the biometricians. Quetelet's legacy to them was the application of the analysis of frequency distributions to human and social phenomena; extending the uses of laws of chance and errors to natural and social science. This extension allowed Quetelet and those that followed to see laws and patterns in what appeared superficially to be social chaos. In Essay IX David Hume wrote "What depends on a few persons is in great measure to be ascribed to chance or to secret and unknown causes; what arises from a great many may often be accounted for by determinate and known causes."

Conclusion

The development of a science of observation from political arithmetic to social physics allowed causation to be realized in the most unlikely phenomena and provided the graphical and quantitative proof of its timeless existence. In doing so it rewarded the Bacon_ian approach to comprehension with the fulfillment of the goal of the enlightenment to observe uniformity amidst variety. The uniformity and, to some, the Grand Design was manifested in the stability of statistical ratios and the reve_lation from a mass of observations of a unique expected or true value.

A precondition for comprehending mass phenomena is an acknowledgement of the equivalence of individual constituents of a population. The political climate of seventeenth century Britain was consistent with this. In William Petty's fiscal and cameral work, individuals under the sovereign were separate, equal assets. The plagues themselves were levelers. The numbers posted to give the rich the chance to escape were used by Graunt to give the sovereign quantitative answers to questions of state.

Graunt's manipulation and analysis of these numbers and in particular his demonstration of the stability of some of the ratios was a firm beginning to statistical analysis. The stability of ratios in repeated trials was taken further in philosophical treatments of games of chance. Expected values and probability distributions were calculated. In the questioning of the applicability of these to a posteriori reasoning the law of large numbers was formulated.

The work of philosophers and mathematicians on quantitative problems arising from navigation, astronomy and geodosy gave rise to another analogy and application of probability theory. The attempts to eliminate the influence of observational errors to derive the simple mathematical laws of nature led to the discovery of a law of errors. The revealed true value and the probability distribution of errors in measurement closely corresponded to the mean and frequency distribution calculated from problems in games of chance. Also, the attempts to develop algorithms to deal with combinations of observations laid the foundation for multiparameter estimation in problems of statistical dependency.

Adolphe Quetelet's work is a good illustration of the development of the statistical approach from Graunt to Gauss. As Graunt had done, Quetelet drew conclusions from quantified social phenomena. He extended this, however, to include not just death and birth but height, crime, literacy, prices, etc. His tools of analysis were rarely arithmetic and the Rule of Three. Rather he constructed frequency distributions and calculated means in search of l'homme moyen and calculated limits in search of a measurement of immoderation and imperfection.

In his work on the typical development of l'homme moyen Quetelet bridged the work on combinations of observational errors with the biometrician's work on heredity and evolution. For each year of human life Quetelet constructed a distribution and calculated a mean. A smoothed curve connected these parameters of logical variation to formulate developmental variation.

Within the context of early positivism, Quetelet searched for indications in his data of social progress. From him this would be revealed in the narrowing of the limits and thus the reduction of dispersion in the distribution of one generation compared to those of former generations. The idea of evolutionary variation was further devloped by Darwin, Wallace and the biometricians. Ironically, the basis of their theory of evolution was the persistence of random variaiton rather its demise. The mean was to lose its role as the true or perfect value. The concept of statistical population was to fully mature with the new focus on deviation from the mean.

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CHAPTER 3

FROM LAWS OF ERRORS TO NATURAL SELECTION

The contribution of theories of evolution and inheritance to a science of means from 1830 to 1896

"Whether we hold variation to be continuous or discontinuous in magnitude, to be slow or sudden in time, we recognize that the problem of evolution is a problem in statistics, in the vital statistics of populations....every idea of Darwin- variation, natural selection, sexual selection, inheritance, prepotency, reversion seems at once to fit itself to mathematical definition and to demand statistical analysis." (Weldon 1901, pp. 3 and 4)

"In the main evolution has not taken place by leaps, but by continuous selection of the favorable variation from the distribution of the offspring round the ancesterally fixed type, each selection modifying pro rata that type" (Pearson, 1906 p. 306).

Four essential conditions for the development of the statistical methods used in econometric analysis are concepts of:

population

quantitative measurement of observed phenomena

continuous logical variation

stochastic relationships.

Western European society and culture in the 17th and 18th centuries provided fertile ground for the first three. The 19th century was to see further refinement of the concept of population, a generalization of the statistical approach to many fields and the development of the concept of statistical dependency. The logical analysis of co-relationships ironically came about in a search for the truth outside a static framework. The seeds of correlation and regression analysis were sown in the 19th century theories and ideas on the mechanics and process of change. The notions of motion, history, progress, and evolution became essential components of the natural and social sciences. The application of the science of observation to the study of evolution linked univariate probability analysis to estimating and eventually forecasting of multivariate relationships.

The key to understanding the application of an analysis of logical variation, e.g. frequency distributions, to explain and predict a process of change is the acknowledgement of that difference leads to change; the latter is not possible without the former. This is inherent in many theories of change over time, for example the concepts of entropy (change will cease when all matter is in a homogeneous state) and biological evolution (the transformation of a species depends on natural selection confronting different characteristics within the species).

The term 'evolution' was originally applied to growth of the individual; in fact Charles Darwin never used the word in characterising his ideas. The current usage, however, is to describe the history of a population or a system undergoing irrevocable change. An individual is treated as a static component slightly different from other components; only the whole population or system changes. "The tenet in biology is that only an aggregate of similar but not identical individuals, i.e. a species can evolve; an individual never evolves, it only comes into existence, lives and dies."(Georgesque - Roegen 1971 p. 206) A study of logical variation among the individuals of a population at one point in time co-related to a logical variation of a population at a different point in time yields a study of evolutionary change. The application of statistics to such a study required the acceptance of a definitive population aggregate, such as species, economic system, or national populace a committment to observation rather then abstract logic, authority or faith, and a philosophy of change as a process of internal dynamics not an act of divine intervention.

Secondary Causes and the Science of Observation

An excellent example of the 19th century transition from ideas based on religious authority and divine intervention to that based on observation and examination of so-called secondary causes is geology. An eloquent description of that transformation is Charles Lyell's <u>Principles of Geology; or The Modern Changes of the Earth and Its Inhabitants considered as Illtrative of Geology</u>. The book went through 12 editions from 1830 to Lyell's death in 1875. Although Lyell in all but the last few editions rejected the idea of the transformation of the species both Alfred Russel Wallace and Charles Darwin acknowledged Lyell and his work as primary inspiration to their ideas on evolution.

Lyell opens his text with a definition:

> "Geology is the science which investigates the successive changes that have taken place in the organic and inorganic kingdom of nature , it inquires into the causes of these changes, and the influences which they have exerted in modifying the surface and external structure of our planet." (Lyell 1853, p1).

Lyell chronicles ideas on geological change in both Eastern and Western cultures. He argues that in the former and also in the latter up to and including classical Greece, cycles of change in the inorganic world were acknowledged. He quotes Aristotle on such change:

> "As time never fails, and the universe is external, neither the Tanais, nor the Nile can have flowed forever. The places where they rise were once dry, and their is a limit to their operations; but there is none to time" (Lyell 1853, p.13)

Although cyclical change was spoken of, Lyell argues there was no recognition on the immense time span of geological change: "the ancient history of the globe was to them a sealed book, and although written in characters of the most striking and imposing kind they were unconscious even of its existence". Nor was the acknowledgement of the extinction of old species and creation of new ones.

For Lyell the study of changes in the inorganic and organic world took a turn for the worse in the centuries of the Christian era that followed the classical era. All reference to causation was reduced to a first cause/or final cause, divine intervention. What little progress there was, was the result of "constant and violent struggle of new opinions against doctrines sanctioned by the implicit faith of many generations and supposed to rest on scriptural authority." Noah's flood explained the remains of a marine presence on dry land and represented the last of a series of catastrophes that changed the earths surface. These catastrophes had occurred in a very short time span and the creation of the inorganic and organic world took only six days.

Investigations of causes other than divine intervention or of change resulting from internal dynamics was supressed. Lyell cites the fate of the Comte de Buffon who in this 44 volume Natural History raised questions of a common origin of animals, variations within the species and suggestions of an earthly existence of 72,000 years before life appeared. The faculty of Sorbonne in 1751 'invited' Buffon to recant his unorthodox opinions that were 'reprehensible, and contrary to the creed of the church'. The grand principle he was forced to renounce was "that the present mountains and valleys of the earth are due to secondary causes, and that the same causes will in time destroy all the continents, hills and valleys, and reproduce others like them." (Lyell 1853 p. 39)

In such a climate thinking on change in the inorganic and organic world was reduced to fanciful speculation of catastrophes that had occurred in ancient times but had since ceased. Faced with fossils and varying rock strata on the same location the writers were forced to acknowledge a past different from the present, but the inquiry was reduced to a debate between Vulcanists and Neptunists on what type of catastrophe was the instrument of God's work.

Real progress was only possible, Lyell argued, when all attempts to form what were termed "theories of the earth" were suspended in favor of gathering evidence before drawing conclusions. The Geological Society of London was founded in 1807 with the stated task of making and recording observations without a priori speculation.

Lyell was an accomplished practioner of this approach. He traveled widely throughout Europe to observe geological structures and limited his investigation to secondary causes of post-creation change:

> The senses had for ages declared the earth to be at rest, until the astronomer taught that it was carried through space with inconceivable rapidity. In like manner was the surface of this planet regarded as having remained

> unaltered since its creation, until the geologist proved that it had been the theatre of reiterated change, and was still the subject of slow but never-ending fluctuations...
>
> By the geometer were measured the regions of space, and the relative distances of the heavenly bodies; by the geologists myriads of ages were reckoned, not by arithmetical computation, but by a train of physical events--a succession of phenomena in the animate and inanimate worlds--signs which convey to our minds more definite ideas than figures can do of the immensity of time" (Lyell 1853 pp. 60-61)

One of Lyell's contributions to science was the acknowledgement of time. He wrote in a culture that had accepted an Archbishop's date of earthly creation as 4004BC. The acceptance of a geological existence of less than 6,000 years reduced one to using cataclysmic catastrophe to explain change. "How fatal every error as to the quantity of time must prove to the introduction of rational views concerning the state of things in former ages" (Lyell 1853 p 64) The assumption of an existence of hundreds of thousands or millions of years made gradual change and the concept of the earth as a self-sustaining machine feasible notions.

Lyell's theory of gradual inorganic change was soon labled uniformatarianism as opposed to the catastrophism he criticized. Lyell argued that the changes taking place in modern times were of the exact same nature as those that gradually shaped the earth in ancient times:

> By degrees, many of the enigmas of the moral and physical world are explained, and instead of being due to extrinsic and irregular causes, they are found to depend on fixed and invariable laws. The philosopher at last becomes convinced

of the undeviating uniformity of secondary causes... (Lyell 1853 p.62)

Ironically, Lyell's theory of uniform gradual change may have been more of a hinderance than help to Darwins development of evolution (Mayr 1982, Eisely 1958). In rejecting catastrophe as a relevant cause of change, Lyell also rejected the directionalist theory of the progressionists. For example he did not accept the idea of geological or biological eras nor the evidence that complex life forms appeared much later than simpler life forms. Phenomena existed in a steady state condition or changed only in a cyclical manner. Evolution required irrevocable historical variation not cyclical change. Lyell's method of relying on meticulous observation and limiting inquires to secondary causes and his acknowledgment of time, however, were important stimuli to evolutionary thinking and the application of statistical techniques to scientific investigation.

Evolution

Notions of a gradually changing inorganic and organic world had been expressed for many decades before the 1858 Darwin and Wallace essays on species transformation. The Scala Naturae doctrine accepted by the church of static heirarchy from the simple to complex, plant to mammal had been transformed to a dynamic ladder by several writers speculating on common descent and progression of life forms through the ages. (see Images 1&2) Over a century

EARLY STIMULANTS TO EVOLUTIONARY THINKING

Image 1

TIME & PLACE	SUBJECT AREA	SPECIFIC CONCEPT	AUTHOR, PUBLICATION
17th C Germany	Philosophy	Nature as self-organizing system displaying continuity and gradualism in the inner drive to perfection.	Gottfried Leibniz (1646-1717) <u>Protogaea</u>, 1693
18th C France	Literary Fantasy	Earth very old, extinction of species, origin of species by chance, external change.	Benoit de Maillet (1656-1738) <u>Telliamed</u>, 1748
18th C France	Natural History	Raised question of degeneration of animals from common origin, spontaneous generation, extinction of species, struggle for existence, uniform change over long period of time.	Georges Louis Buffon (1707-1788) <u>Histoire Naturelle</u>, 1749-1804
18th C France	Natural Philosophy	Development inherent in all matter, transmutation and diversification of species from 'errors,' some of which make an organism more adaptable.	Pierre de Maupertuis (1698-1759) <u>Vénus Physique</u>, 1745 Système de la Nature, 1751
18th C France	Natural Philosophy	Through variation species evolved over long period of time from primitive states to increasing complexity and specialization.	Denis Diderot (1713-1784) <u>Pensées sur l'interprétation de la Nature</u>, 1753
18th C Britain	Geology	Earth in perpetual state of cyclical change with "no vestige of a beginning--no prospect of an end."	James Hutton (1726-1797) <u>Theory of the Earth</u>, 1785
18th C Britain	Zoology	All species in state of perpetual improvement.	Erasmus Darwin (1731-1802) <u>Zoonomia</u>, 1794-1796
18th C France	Philosophy	Progressive development of masses from natural and human domination to emancipation.	Marie-Jean Antoine - Nicholas de Condorcet (1743-1794) "Sketch for a Historical Picture of the Progress of the Human Mind," 1795
18th C Britain	Political Economy	Population, unchecked, increases geometrically, food resources arithmetically yielding continuous struggle for existence.	Thomas Malthus (1766-1834) <u>Essay on the Principles of Population as it Affects the Future Improvement of Society</u>, 1798

CONTRIBUTIONS TO THE DEVELOPMENT OF A THEORY OF EVOLUTIONARY CHANGE

Image 2

TIME & PLACE	SUBJECT AREA	SPECIFIC CONCEPT	AUTHOR, PUBLICATION
19th C France	Natural History	Transformation - a non random advance of living things from lower to higher types via environmentally-induced behavioral changes and inheritance of acquired characteristics.	Jean Lamark (1744-1829) Philosophie Zoologique 1809
19th C France	Zoology	Comparative anatomy and principle of correlation of parts, geological epochs. Nonlinear, divergent classification of genera and species.	Georges Cuvier (1769-1832) Lessons in Comparative Anatomy 1802 Essay on the Theory of the Earth 1815
19th C Britain	Geology	Geological strata cannot be classified by mineral composition but only by the alteration of fauna fossilized within them.	William Smith (1769-1839) The Stratigraphical Systems of Organized Fossils 1817
19th C	Zoology	Uninterrupted succession in animal kingdom generated by environmentally-induced change.	Etienne Geoffroy St. Hilaire (1772-1844) Principles de Philosophie Zoologique 1833
19th C Britain	Geology	Uniform, gradual inorganic and organic change over very long time span, struggle for existence.	Charles Lyell (1797-1875) Principles of Geology 1830
19th C Britain	Natural History	Gradual, organic change towards increasing complexity, constant variation within species, spontaneous generation.	Robert Chambers (1802-1871) Vestiges of the Natural History of Creation 1844
19th C Britain	Natural History	Progression from original type by minute steps in various directions. Continuous variation within species, struggle for existence.	Alfred Russell Wallace (1823-1913) "On the Tendency of Varieties to Depart Indefinitely from the Original Type" 1858
19th C Britain	Natural History	Gradual transformation of species through process of natural selection on random variation within species.	Charles Darwin (1809-1882) "On the Variation of Organic Beings in a State Nature" 1858 On the Origin of Species by Means of Natural Selection or the Preservation of Favored Races in the Struggle for Life 1859

before the Origin Of Species was published Pierre de Maupertuis wrote

> couldn't we explain in this way how the multiplication of the most dissimilar species might have resulted from two single individuals? Their origin would be owing only to a few accidental productions in the embryo, in which the elementary particles would not have retained the arrangement which they had in the father and mother animals: each degree of error would have created a new species; and by dint of repeated divergences there would have come about the infinite diversity of species which we see today, which will perhaps increase with time, but to which the passing of the centuries perhaps brings only imperceptible additions." (Maupertius in Systeme de la Nature, 1751, quoted in Crocker 1954 p. 127)

The evolutionary theory of Darwin and Wallace was still, however, quite revolutionary. The fact that Darwin waited over two decades to publish is an indicator of the chasm between their views and orthodox explanations of creation and change. Darwin's theory was that the organic worlds experienced continual, gradual, non-teleological change. His emphasis was on the evolution of a population - a species. At any one point in time there was variation - diversity among the individuals of the species. This variation was usually slight and continuous across the whole population. Unchecked a species would reproduce at a considerable rate - but nature ensured checks. Competition for resources existed between species, but also competition for food, space, and sexual mates existed among individual of a species. Those that were able to survive and to reproduce the next generation were those whose slightly different traits allowed them to adapt most readily to their environment and to attract mates.

Variable traits could be inherited so eventually members of a species would look and act quite different from their predecessors millions of years ago.

For Darwin, although natural selection was creative as well as destructive, evolution was not progressive. There was no immanent potential being realized, nor a development from simple to complex, imperfect to perfect. Final causes, special creation, teleological determination had no place in Darwin's scheme. The idea of accidental or continuous random variation across a population confronted with natural selection yielded a stochastic process. "The Darwinian process of continued interplay of a random and a selective process is not intermediate between pure chance and pure determination, but in its consequences qualitatively utterly different from either" (Servall Wright quoted in Mayr 1982 p. 520).

A theory of organic evolution is obviously a theory of change over time and several writers have questioned why it was so late in developing given the acceptance of cosmic change and of Newtonian mechanics decades before Darwin's publication of the Origin of the Species. The uniform change postulated by Hutton and Lyell did draw on Newtonian mechanics and the idea of continuity of action. However, the dynamics of evolution are quite different as Geogescu-Roegen has pointed out: "Mechanics knows only locomotion, and locomotion is both reversible and quality-less" (Georgescu-Roegen 1971 p. 1)

Evolution is irreversable, qualitative change. It is not without direction, given the role of natural selection, but it does not have the goal-orientation suggested by earlier theories of progression toward sophisticated perfection. The static scala naturae doctrine which ideologically suited the centuries of fixed social hierachy under feudalism had been replaced by a hierarchy that incorporated mobility. The evolution of Darwin, however, knew no ultimate goal. Likewise development of organisms had been the subject of intense observation. Darwin did not dwell on this ontogeny, however, his focus was phylogeny. The subject was a population not an individual organism.

The characteristics of Darwin's and to some extent Wallace's theories that made them historically unique:

study of populations
assumptions of random logical variation
an investigation of stochastic processes

also rendered them ideally suited for the application of statistical analysis. Darwin did not apply such analysis, that was to come several years later with the work of Galton, Weldon, Pearson and other biometricians. Darwin was not comfortable with mathmatical or statistical analysis and admitted being 'muzzy' on the subjects of proportion and chance. He was, however, aware of the importance of measurement in verifying his ideas:

> The chief point which I am, and have been for years, very curious about, is to ascertain

> whether the young of our domestic breeds differ as much from each other as do their parents, and I have no faith in anything short of actual measurement and the Rule of Three. (Weldon 1901 p. 4)

The Rule of Three (calculating the 4th value of a proportional relationship from the other three) was an arithmetic rule. It was to be replaced by the biometricians with estimation of correlation and regression coefficients. A basis for the latter, least squares, had been established with the work of Gauss and others and applied to errors in measurement. The biometrician's investigation, however, was that of a stochas tic process of change over time not errors in mesurement of a fixed geometric relationship. The former required population thinking inherent in Darwin's analysis. Until his work much biological theory and application of probability analysis had assumed unvarying Platonic essence:

> The statistics of the essentialist are quite different from those of the populationist. When we measure a physical constant - for instance the speed of light - we know that under equivalent circumstances it is constant and that any variation in observational results is due to inaccuracy of measurement, the statistics simply indicating the degree of reliability of our results. The early statistics from Petty and Graunt to Quetelet was essentialistic statistics, attempting to arrive at true values in order to overcome the confusing effects of variation. Quetelet, a follower of Laplace, was interested in deterministic laws. He hoped by his method to be able to calculate the characteristic of the 'average man' that is to discover the 'essence' of man. Variation was nothing but 'errors' around the mean values. (Mayr 1982 p.47)

A major step had been taken in the seventeenth and eighteenth centuries in the development of a population concept. At the social level, the near-caste system of feudalism, a social scala naturae, had given way to a recognition of equalities that allowed the articulation of concepts such as a national populace. For example, in Karl Marx's analysis the development of capitalism was also the development of abstract labour. The quality, specifiticity and diversity of labour no longer mattered in the dynamics of value determination; only quantity of labour time was important.

Darwin's theory of evolution took the population concept one step further. A species was a population of similar, but not identical individuals. Diversity, within limits was recognized and such omnipresent variation was the source of possibilities for change of the entire population over time. The variation across the population was the result of random, accidental causes, but the change that resulted from this variation was not.

At first Darwin argued that favorable variation was rare but Wallace encouraged him to consider it frequent:

> Such expressions have given your opponents the advantage of assuming that favorable variations are rare accidents, or may even for long periods never occur at all and thus the argument would appear to many to have great force. I think it would be better to do away with all such qualifying expressions, and constantly maintain (what I certainly believe to be the fact) that variations of every kind are always occuring in every part of every species and therefore that favorable variations are always ready when wanted. I would put the burden of proof on my opponents to show that any one organ, structure

> or faculty does not vary, even during one generation, among all the individuals of a species. (Marchant, 1916 p. 142-143)

Darwin or Wallace reached their theories on population change over time by noticing simultaneous geographical variation. Climatic change could possibly explain why different fauna inhabited a region at different times. Likewise, different physical environments could explain why different varieties and species inhabited different geographical areas. Darwin and Wallace however, in their world travels were confronted with variation in the same time period and physical environment with small bodies of water being the only isolating barriers. In the Galapagos islands and even on opposite banks of the same river they were forced to explain the existence of remarkably different races of the same species under isolation but thriving in identical, close, physical environments. The only explanation was continuous random variation in isolated sub-populations of the same species.

The variation across the population was the result of random accidental causes (eg. in modern terminology genetic mutation or recombination in the linking of female and male chromosones). The change in species that results from this variation was not, however, random. Natural selection ensured a probable success to a favorable few. In the words of Darwin:

> "But let_the external conditions of a country alter... Now, can it be doubted from the

struggle each individual has to obtain subsistence, that any minute variation in structure, habits or instincts, adapting that individual better to the new conditions would tell upon its vigour and health?" (Darwin 1858 p 102)

Darwin's theory and the many supporting examples he presented in The Origin of Species focused on diversity within a population. Temporal continuity of some of the diversity ensured the transformation of a species and the multiplication of species. This temporal continuity came from inheritance. Darwin argued for a theory of 'soft' inheritance. Characteristics aquired within an organism's lifetime could influence the germ cells and be passed on. Darwin did not except a true theory of blended inheritance; for Darwin fertilization lead to a mixture and not a true fusion of two distinct individuals. His theory of pangenes, however lacked the arithmetic simplicity and determinacy of modern genetics.

> To sum up on the origin of our domestic races of animals and plants. Changed conditions of life are of the highest importance in causing variability, both by acting directly on the organisation, and indirectly by affecting the reproductive system. It is not probable that variability is an inherent and necessary contingent, under all circumstances. The greater or less force of inheritance and reversion, determine whether variations shall endure. Variability is governed by many unknown laws, of which correlated growth is probably the most important. Something, but how much we do not know, may be attributed to the definite action of the conditions of life. Some, perhaps a great, effect may be attributed to the increased use or disuse of parts. The final result is thus rendered infinitely complex. (Darwin (1858) 1928 p. 48)

Darwin's notion of the forces of inheritance left much to be desired in explaining the persistence of variable traits. As Karl Pearson noted in his Grammar of Science

> The reader can hardly fail to have been impressed in his past reading and experience with the great burden of explanation which is thrown on that unfortunate metaphysical conception force... He may perhaps have concluded, with the present writer, that the word is not infrequently a fetish which symbolises more or less mental obscurity. But the reason for the repeated occurance of the word is really is not far to seek. Whenever motion, change or growth were postulated, there in the old metaphysics force as the cause of change in motion was to be found. The frequent use of the word force was due to the almost invariable association of motion with our perceptions, or in more accurate language, to the analysis of nearly all our sense impressions by aid of conceptual notions" (Pearson (1892) 1969 p. 132)

Darwin's failure to articulate a modern theory of inheritence left him vulnerable to criticisms that minute variations were not sustainable from one generation to the next since they could be easily canceled out in sexual reproduction with individuals not possessing the same variation of characteristics.

One of Darwin's critics Fleeming Jenkins, argued in an 1867 article in the North British Review that the variation Darwin described could not be perpetuated. How could fortuitous variation explain evolution if a new characteristic possessed by just one or a few individuals could be easily eliminated through reproduction and inheritance? Jenkins argued that the variation, to be sustained, would have to have occured simultaneously to the majority of the species.

Darwin recognized the threat Jenkin's argument posed to fortuitous variation, but he was unable to answer the threat. An answer had already been written but acceptance of Gregory Mendel's ideas on genetic inheritance was not to come for several more decades.

The most obvious weakness in Darwin's argument was his scheme of inheritance. His attention was focused so much on diversity and change that he did not ask questions as to the stability of the species. In the words of Loren Eisely, Mendel "had intuitively grasped what seemingly no one else of his generation understood; namely that until we had some idea of the mechanisms which controlled organic persistence we would be ill equiped to understand what it was that produced evolutionary change." (Eisely 1958 p208)

It is one of the ironies of the history of science that the hereditary flaw in Darwin's argument lead to considerable advance in statistical methods that would not have taken place in the time and the form it did had Mendel's genetic theory of particulate inheritence been widely known and accepted. It was in pursuit of replacing the ambiguous "force of inheritence" with mathmatically precise "laws of inheritence" that the concepts of correlation and regression analysis were conceived and applied. These concepts were used to quantify change through blending inheritance. The latter was eventually seen as an obsolete, incorrect theory of inheritance by 20th century biologists, but the techniques developed to test such a theory remain today as essential components of statistical analysis.

<u>Inheritance, Correlation and Regression</u>

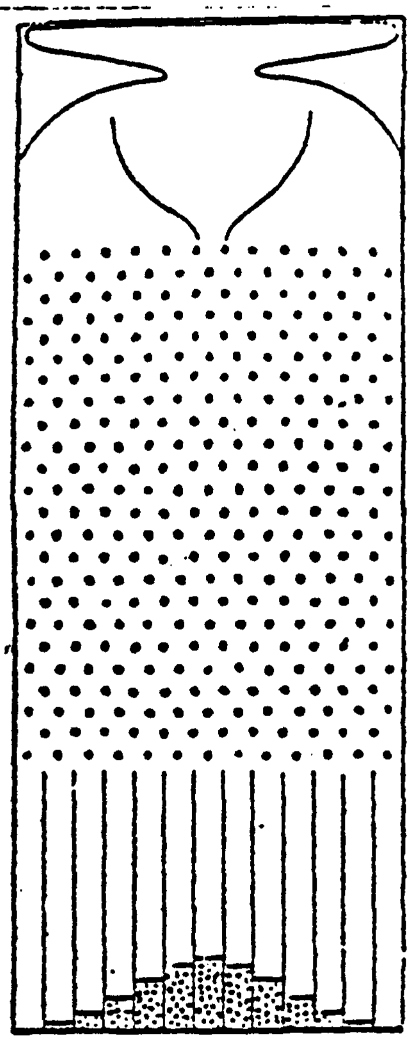
Considerable advances in statistical inferences were made in the late 1800's in the investigation of the mechanics of inheritance and evolution through natural selection. Biometricians, such as Francis Galton, Raphael Weldon, and Karl Pearson applied the 'law of error' to organic populations. In attempting to answer the questions posed by Darwin's theory, they formulated new frequency distributions, more refined statistical parameters of dispersion and coefficients of correlation and regression. Their goal was to quantify change over time and their use of concepts and techniques of logical variation to do so was unique.

Francis Galton, who is credited with first applying the correlation coefficient and regression analysis, stated as his goal "to place beyond doubt the existence of a simple and far-reaching law that governs the heriditary transmission of, I believe, every one of those simple qualities which all posses, though in unequal degrees" (Galton 1886 p. 246). To this goal Galton collected measurements of various parts of the human body and of other organisms. Galton was struck by the pattern that repeatedly appeared when he plotted the frequency distributions of these various attributes. They were all near identical to the distributions of measurements subject to observational errors and of observations derived by Quetelet from social data. Galton argued that this was further proof of order in apparent chaos:

> I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the "Law of Frequency of Error". The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshalled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along. The tops of the marshalled row form a flowing curve of invariable proportions; and each element, as it is sorted into place, finds as it were, a pre-ordained niche accurately adapted to fit in. (Galton 1889 p. 66)

To illustrate the reasons for the universality of this law, Galton constructed a mechanical demonstration (see Image 3) consisting of a funnel at the top and a succession of rows and pins arranged in quincunx fashion so that every descending object strikes a pin in each successive row. When the frame was held upright a charge of small shot passed through the funnel and cascaded down the structure. Invariably the shape formed in the bottom compartments by the shot approximated to the frequency distribution typical of errors in measurement; the normal curve as Galton called it.

Image 3



The principle on which the action of the apparatus depended was:

> "that a number of small and independent accidents befall each shot in its career. In rare cases a long run of luck continues to favour the course of a particular shot towards either outside place, but in the large majority of instances the number of accidents that cause Deviation to the right, balance in a greater or less degree those that cause Deviations to the left. Therefore most of the shot finds its way into the compartments that are situated near to the perpendicular line drawn from the outlet of the funnel, and the Frequency with which shots stray to different distances to the right or left of that line diminishes in a much faster ratio than those distances increase. This illustrates and explains the reason why mediocrity is so common." (Galton 1889 p.64-65)

Galton's understanding of the mean as the typical can be seen in the composite photographs made by him and Henry Bowditch. Both shot numerous portraits of individual members of "homogenous" groups . They blended these to construct a composite portrait that would represent the typical of that group. In a popular article in McClures Magazine in 1894, Bowditch attempted not only to capture the typical portrait of Boston Physicians but also to demonstrate how the typical Boston Physician changed over five years (Image 4). Taken to extremes the search for typical and the demonstration of the persistance of the typical despite random variation created the sophisticated horrors of eugenics when applied to racial and social subgroups of the human species.

The tendency toward mediocrity was to be a recurrent theme of Galton's work on inheritance. In these investigations Galton collected data on parents and

PLATE I. TWELVE BOSTON PHYSICIANS AND THEIR COMPOSITE PORTRAIT. THE COMPOSITE IN THE CENTRE.

PLATE II. THE SAME FACES SHOWN IN PLATE I., PHOTOGRAPHED FIVE YEARS LATER, WITH THEIR COMPOSITE—THE COMPOSITE IN THE CENTRE.

offspring of different species. His interest was human inheritence, but initially he was forced to concentrate on other organisms, such as the sweet pea. With the latter he noted that the variations of the diameter of the pods of the parent and the offspring were normally distributed and that the variance of both groups was equal.

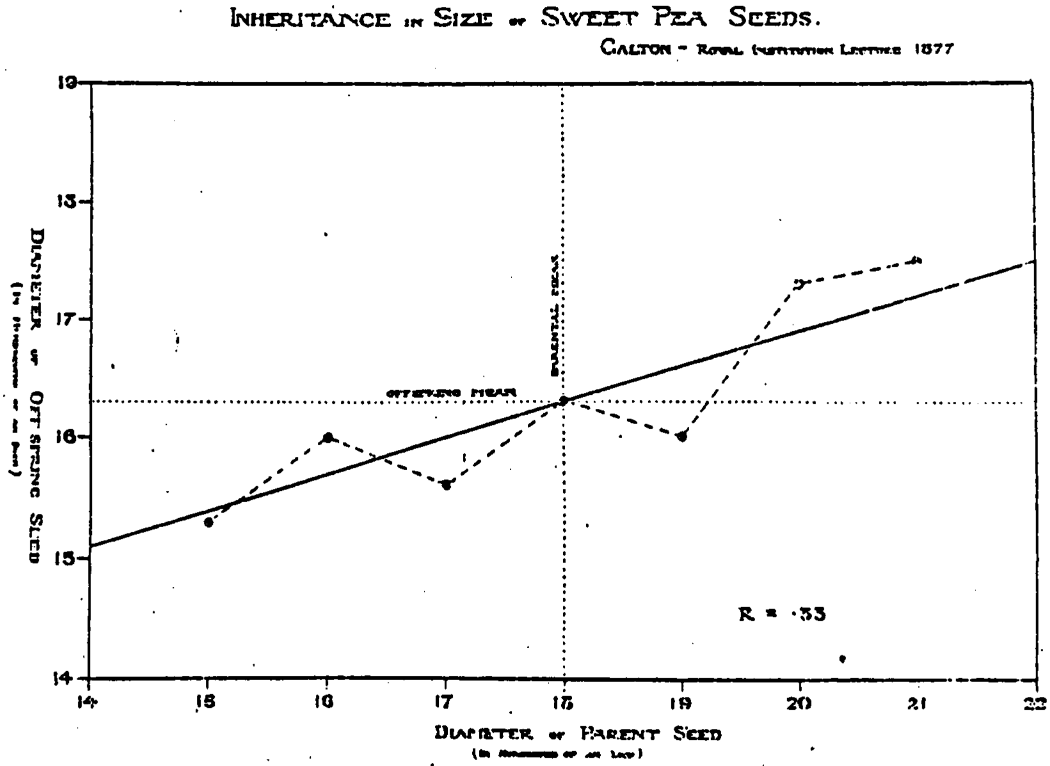
Galton chose sweet peas in an endeavor to find "a population possessed of some measurable characteristics that was suitable for investigating the courses of statistical simularity between successive generations of a people" (Galton 1889 p. 180). When Galton compared the diameter of the offspring pods with those of the parent he noticed that the coordinates did not fall around a line with a slope equal to one (see Image 5). This was, according to Galton, evidence of reversion; "the tendency of the ideal mean filial type to depart from the parental type reverted to what may be roughly and perhaps fairly described as the average ancestral type" (Galton quoted in Pearson 1920 p. 33). Galton later labeled this phenomena regression towards mediocrity. His 1877 experiments on sweet pea pods demonstrated:

> that the offspring did not tend to resemble their parent seeds in size, but to be always more mediocre than they - to be smaller than the parents if the parents were large; to be larger than the parents if the parents were very small...
>
> that the mean filial regression towards mediocrity was directly proportional to the parental deviation from it. (Galton 1886 p. 246)

In Galton's lecture on the sweet pea experiments,

Image 5
from Pearson, 1920

Table
from Galton, 1886



Diameter of Parent Seed.	Diameters of Filial Seeds.								Total.	Mean Diameter of Filial Seeds.	
	Under 15.	15–	16–	17–	18–	19–	20–	Above 21–		Observed	Smoothed
21	22	8	10	18	21	13	6	2	100	17·5	17·3
20	23	10	12	17	20	13	3	2	100	17·3	17·0
19	35	16	12	13	11	10	2	1	100	16·0	16·6
18	34	12	13	17	16	6	2	0	100	16·3	16·3
17	37	16	13	16	13	4	1	0	100	15·6	16·0
16	34	15	18	16	13	3	1	0	100	16·0	15·7
15	46	14	9	11	14	4	2	0	100	15·3	15·4

Febuary 9 1877, the correlation coefficient R was first articulated. His estimate of R for parent and offspring pod diameters was 1/3. It is interesting to note that Galton interpolated a line of regression from a polygon formed by connecting observed means of filial seeds corresponding to each value of the diameter of parent seeds. He chose parent seeds that had one of the seven diameters listed in the table in Image 5. From a frequency table of the sizes of the respective filial seeds. Galton calculated seven observed means. He then smoothed the polygon ensuring that the interpolated line passed through both parental and filial means.

In essence, Galton broke the filial seeds into separate samples determined by parental size. For each sample he constructed a frequency distribution and a mean. It was an ingenious application of analytical tools of logical variation to examine evolutionary variation. It is unlikely that he would have pursued this method had he not discovered a bi-variate normal surface in the joint frequency distribution. His construction of the regression polygon, however, is closely linked conceptually with the method of least squares and with the assumptions of error in equations. What remains unique about Galton's and the biometrician's work is the joint normal surface and the capacity for the subject of analysis for determining qualitative frontiers in the stochastic process, i.e. generation gaps. There is little room for question as to the appropiate intervals between the elements of the sequence.

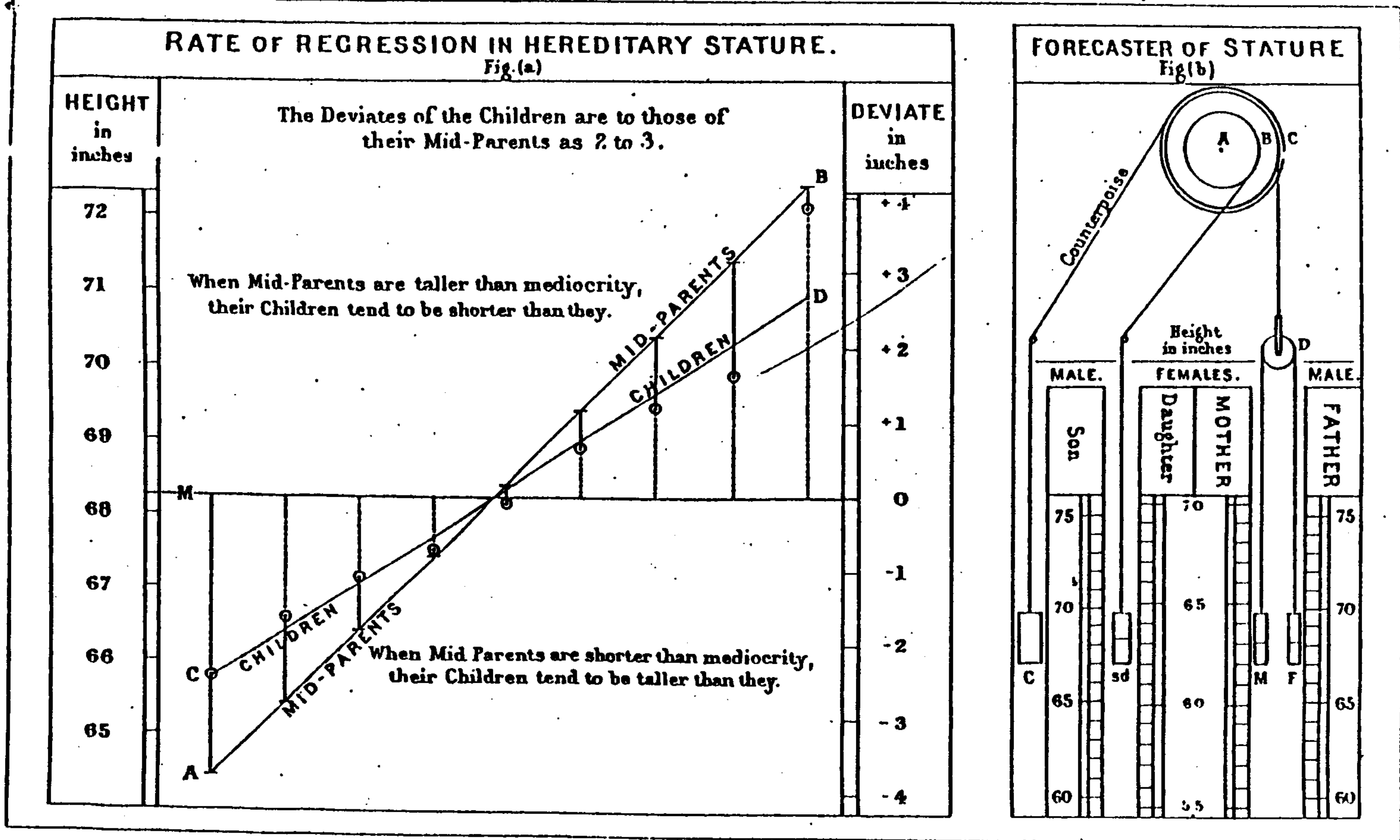
After resorting to offers of prizes for family records, Galton was finally able to collect enough data on humans to examine correlation of heights from one generation to the next. The data on 205 parents and 930 adult children yielded "the numerical value of the regression towards mediocrity in the case of human stature, as from 1 to 2/3 with coherence and precision" (see Image 6) (Galton 1886 p. 247). With the anthrometric data Galton worked out joint (bivariate) frequency distributions. Galton 'smoothed' the entries by summing at each intersection of a horizontal and vertical class the entries in the four adjacent squares (see Image 7). Galton in essence constructed a bivariate normal surface:

> I then noticed that lines drawn through entries of the same value formed a series of concentric and similar ellipses ....The points where each ellipse in succession was touched by a horizontal tangent, lay in a straight line inclined to the vertical in the ratio of 2/3; those where they were touched by a vertical target lay in a straight line inclined to the horizontal in the ratio of 1/3. Those ratios confirm the values of average regression already obtained by a different method, of 2/3 from the mid-parent to the offspring, and of 1/3 from offspring to mid-parent. (Galton 1886 p. 255)

Once again the law of error appeared, though this time in three-dimensional form. Galton thus concluded that "the characteristics of any population that is in harmony with its environment, may remain statistically identical during successive generations" and that the applications of the law of error "were found eminently suitable for expressing the processes of heridity (Galton

Image 6
Galton 1886

Plate IX.

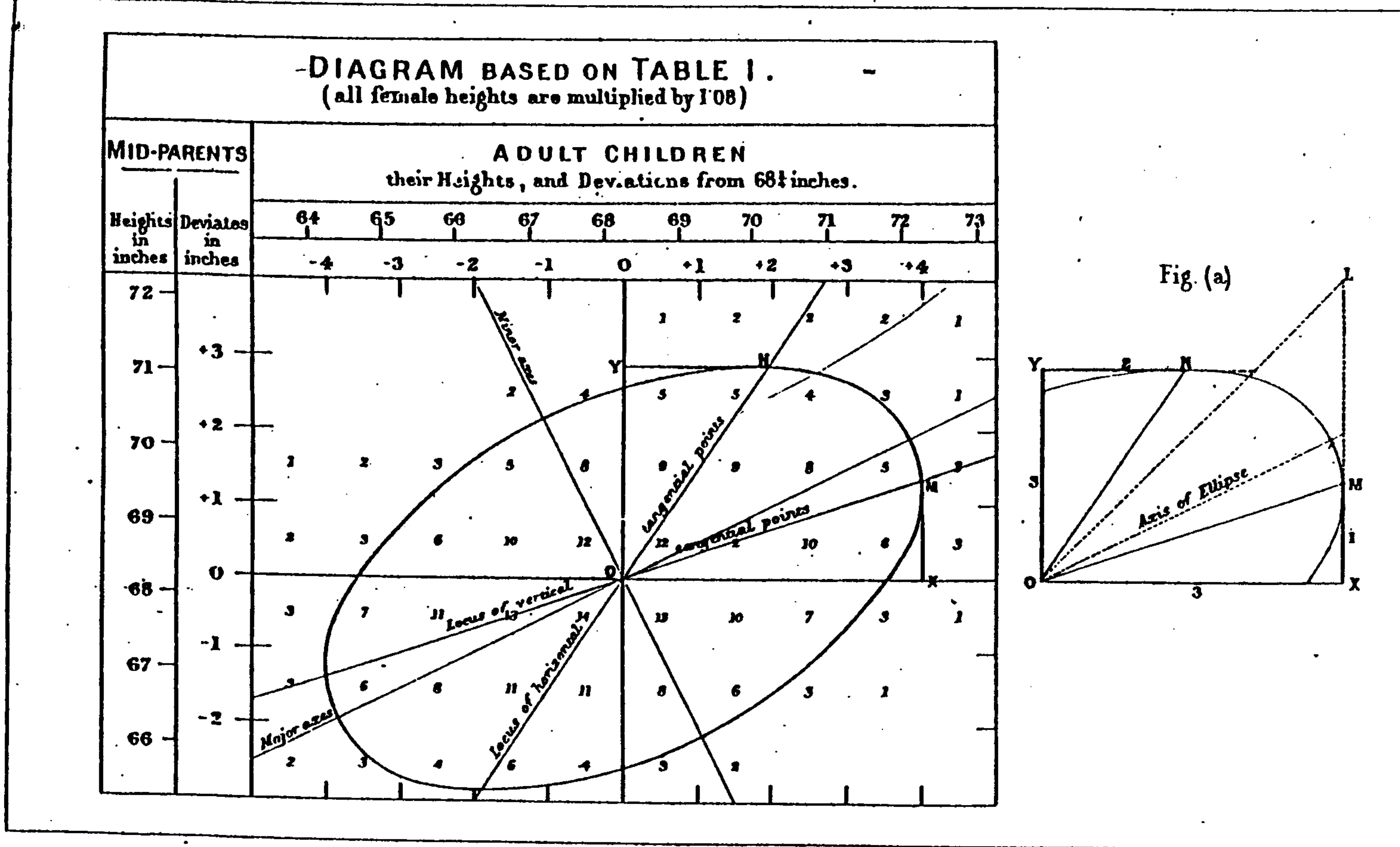


J.P. & W.R. Emslie, lith.

Journ. Anthropolog. Inst., Vol. XV, Pl. IX

Image 7
Galton 1886

Plate X.



J.P. & W.R. Emslie, lith.

Journ. Anthrop. Inst. Vol. XV, Pl. X

1889, p. 192).

Galton's description of the phenomena he observed as "regression towards mediocrity" implied a reduction in dispersion of the population from one generation to the next. This 'regression fallacy' is obviously not borne out by the data. Galtons work on the contrary illustrates the remarkable stability of species. His work on inheritance did not directly clarify evolutionary change. He was however, a consultant to those who did work on such possible change and he did not consider it impossible: "the limits of deviation beyond which there is no regression but a new condition of equilibrium is entered into, and a new type comes into existence, have still to be explained" Galton 1889 p. 258).

In applying quantitative analysis to inheritance Galton was faced with the problem of both parents and of many ancestors determining the characteristics of the offspring. Pearson labled the latter idea as Galton's law of Ancestral Heridity. Pearson's attempt to test the adequacy of this law led to the development of multiple regression. Each ancestor back to a certain point could be an explanatory variable.

Biometric Analysis of Evolution

Galton's work in heritance was a stimulus to Raphael Weldon and Karl Pearson, who eventually worked together to develop many concepts and techniques use in applied statistics. In an memorial tribute to Weldon, Pearson described him as one who "realised to the full that the great scheme of Darwin was only a working hypothesis, and that it was left to his disciples to complete the proofs, of which the master had only sketched the outline" (Pearson 1906 p. 282).

Weldon first searched for proofs with morphological and embryological studies. These methods, however, emphasized homogeneity and Weldon soon turned toward the study of variation and distributions that Galton used in his work on inheritance. Weldon went beyond using a frequency distribution to describing a population to asking how natural selection affected a distribution. In his work on shrimp and crabs (Weldon 1890, 1892, 1893) Weldon discovered:

> variations in size of the organs measured occur with the frequency indicated by law or error
>
> the "probable error" of the measurements of the same organ is different in different races of the same species
>
> the degree of correlation between a given pair of organs is approximately the same in local races of the same species
>
> in one species of shore crabs the frequency distribution for measurements of one organ was bi-modal.

Weldon hoped his findings on correlation could be expanded to many species to yield

> a new kind of knowledge of the physiological connexion between the various organs of animals; while a study of those relations which remain constant through large groups of species would give an idea, attainable at present in no other

way, of the functional correlations between various organs which have led to the establishment of the great sub-divisions of the animal kingdom. (Weldon 1892 p. 11)

The observation of a bi-modal distribution gave Weldon hope that he was gaining a glimpse of speciation in action: two populations that would eventually become two separate species. It also stimulated Pearson and others to examine non-normal frequency distributions. This examination eventually led to the analysis of skewed, chi-square, F and t distributions.

In response to a request from Weldon on breaking up a frequency distribution into two normal components, Pearson published in 1893 the first of a series of papers entitled "Mathmatical Contributions to the Theory of Evolution". In these papers Pearson developed:

the method of moments as a means to fitting a theoretical curve to observational data

a comprehensive system of frequency curves linked together by their derivation from a single differential equation

the chi-square test for goodness of fit

a method for correlating attributes

solutions to problems in multiple correlation

numerous statistical tables that are still in use today for test of significance and the fitting of probability distributions

The stimulus to these developments of statistical analysis was the study of evolution. In a paper that, accoding to Pearson "biometricians will always regard as a classic of their subject", Weldon stated

It cannot be too strongly urged that the problem of animal evolution is essentially a statistical problem: that before we can properly estimate the changes at present going on in a race or species we must know accurately (a) the percentage of animals which exhibit a given amount of abnormality with regard to a particular character; (b) the degree of abnormality of other organs which accompanies a given abnormality of one; (c) the difference between the death rate percent in animals of different degrees of abnormality with respect to any organ; (d) the abnormality of offspring in terms of the abnormality of parents and vice versa. These are all questions of arithmetic; and when we know the numerical answers to these questions for a number of species we shall know the deviation and the rate of change in the species at the present day--a knowledge which is the only legimate basis for speculations as to their past history, and future fate" (Weldon 1893 p.329).

The fertile bond between theory and practice, abstract concept and tools for analysis is vividly illustrated in the development of correlations and regressions analysis. The development of a theory of evolutionary change through static variation stimulated the development of statistical techniques which in turn further stimulated the development of theory in natural and social sciences. One method now used to fit regression lines to observations had been suggested nearly a century before the biometricians published their findings. The method of least squares, however, was first applied in a very different context.

Karl Pearson, in several of his early references on the history of statistics, gave credit to Pierre Simon LaPlace, Carl Friedrich Gauss and August Bravais for first developing the foundation for correlations and regression.

In his classic "Notes on the History of Correlation" in 1920 he apologized for such misleading statements and the injustice done to Galton and others. One of the themes of Pearson's 1920 article was that Laplace, Gauss and Bravias were attempting to deal with errors in measurement in investigating perfect geometric relationships; the biometricians were analysing imperfect organic relationships. It was only with the latter that the concept of correlation came into its own.

Gauss developed the criteria of least squares in the context of problems of observation arising in astronomy and surveying. In these situations there is an exact (perfectly correlated) relationships between variables, but actual observations of these variables render the measurements erroneous and thus the observed relationship inexact. Gauss's task was to reduce the influence of errors in measurement as much as possible:

The method of least squares that Gauss developed was thus used to accomodate errors in measurement and not errors in relationships or in equations. It could not in itself therefore lead directly to the concept of correlation. In Pearson's words:

> The point is this: that the Gaussian treatment leads (i) to a noncorrelated surface for the directly observed variates, (ii) to a correlation surface for the indirectly observed variates. This occurence of product terms arises from the geometrical relations between the two classes of variates, and not from an organic relation between the indirectly observed variates apppearing on our direct measurement of

them....

> It will be seen that Gauss' treatment is almost the inverse of our modern conception of corrleation. For him the observed variables are independent, for us the observed variables are associated or correlated. For him the non-observed variables are correlated owing to their known geometrical relations with observed variables; for us the unobservable variables may be supposed to be uncorrelated causes, and to be connected by unknown functional relations with the correlated variables....
>
> There is nothing in the memoirs of Gauss or Bravais that really antedates his (Galton's) discoveries. They were dealing with the relatively narrow problem of determining the probable errors of indirectly observed quantities. The product-terms that arise in their investigations were expressed in terms of differential coefficients; they were not treated as a means of determining organic relationships between directly measured variates. Galton, starting from the orgainic relationship between parent and offspring, passed to the idea of a coefficient measuring the correlation of all pairs of organs, and thence to the 'organic' relationship of all sorts of factors." (Pearson 1920 pp. 27,40)

The relationships analysed by the biometricians were stochastic. Two variables were co-related (either in the same organism or in parent and offspring) because their variation was partly due to common causes. The focus of the biometricians investigation was a process of change: inheritance and/or evolution. Examining samples of populations at one moment in time revealed the differences that were preconditions for change.

Variation, in the vocabulary of Galton, Weldon and Pearson was logical variation and it only took on systematic form in the context of a population at one moment in time. The idea that difference leads to change was taken to the extreme in Pearson's paper "On the Principle of Homotyposis and Its Relation to Heredity" (Pearson 1901A). In this paper Pearson suggested that resemblance between parent and offspring could be seen as a special case of resemblance between undifferentiated like organs in the parent. The correlation between two siblings is possibly an expression of the same phenomena as the correlation between two leaves on the same tree or any two measurements of similar characteristics of one organism:

> Now the reader will perceive at once that if we can throw back the resemblance of offspring of the same parents upon the resemblance between the undifferentiated like organs of the individual, we have largely simplified the whole problem of inheritance....
>
> If this view be correct, variability is not a peculiarity of sexual reproduction, it is something peculiar to the production of undifferentiated like organs in the individual and the problem must largely turn on how the resemblance between such organs is modified, if modified at all, by the conditions of nurture, growth and environment generally." (Pearson 1901A pp. 287, 288)

In an article typical of the acrimony between the biometricians and the Mendelians, William Bateson (Bateson 1901) was very critical of Pearson's suggestions. In that article and in Pearson's reply (Pearson 1901B) it becomes clear that one of the chief sources of conflict are differences in vocabulary; particularly different meanings attached to the word variation. Bateson argued:

> By the word <u>variation</u> we are attempting to express a great diversity of phenomena in their essence distinct though merging insensibly with each other. The attempt to treat or study them as similar is leading to utter confusion in the

study of evolution. (Bateson 1901 p. 204)

Pearson quotes Bateson's definition of variation from an earlier work and juxtaposes his own understanding of the word:

> Mr. Bateson's definition of variation: "For though on the whole the offspring is like the parent of parents, its form is perhaps never identical with theirs, but generally differs from it perceptibly and sometimes materially. To this phenomenon, namely the occurence of differences between the structure, the instincts or other elements which compose the mechanism of the offspring, and those which were proper to the parent, the name <u>Variation</u> has been given....
>
> Mr. Bateson's conception of variation is not that of a measure of the deviations of a population from its mean. To the biometrician variation is a quantity determined by the class or group without reference to its ancestry. To Mr. Bateson it is a measure of the deviation of the offspring from the parent." (Pearson 1901B p. 325)

The subject of the biometrician's study was that of a temporal process. There is discontinuity in the process from one generation to the next. Correlation, a measurement of degree of likeness or associaiton was developed to study this discontinous process. The data, however, was not in time series form. Cross-sectional observations were used for each generation. The statistical population was at one moment in time; variation measured difference not change:

> The starting point of Darwin's theory of evolution is precisely the existence of those differences between individual members of a race or species which morphologists for the most part rightly neglect. The first condition necessary, in order that any process of Natural Selection may begin among a race, or species, is the existence of difference among its members...

(Weldon 1901 p. 1)

> The Primary object of Biometry is to afford material that shall be exact enough for the discovery of incipient changes in evolution which are too small to be otherwise apparent. The distribution of any given attribute, within any given species, at any given time, has to be determined, together with its relations of external influences.... The orgainc world as a whole is a perpetual flux of changing types. It is the business of Biometry to catch partial and momentary glimpses of it, whether in a living or in a fossil condition, and to record what it sees in an enduring manner. (Galton 1901 p. 10)

Conclusion

The ideas of change, motion and progess took on a unique and very specific form in the Darwinian theory of evolution. At any moment in time there were small differences in the attributes of members of one species. Those members possessing the characteristics most suited to adapting to their environment thrived, reproduced and in some cases passed on their variant characteristics. Random logical variation, along with natural selection and inheritance, was the germ of evolutionary variation.

This concept of change differed from Lyell's view of gradual, cyclical change and also from the popular notions of progress from simple to complex, imperfect to perfect or chaos to order. The evolutionary process hypothesized by Darwin was irrevocable, non-teleological and stochastic. Perfection would never be achieved, differences would not diminish and the state of future generations could never be exactly determined. The specification of this process corresponded to statistical concepts. Again, however, the approach was a novel one. Darwin and the biometricians were populationists not essentialists. Their focus was on deviation rather than the mean. Under their direction, the mean changed from the manifestation of truth, divine order and perfection to a description of the momentary typical constituent of a population.

The curve of errors was a good fit for the frequency distributions of many observations on organisms. The Law of Errors became the "normal" distribution. Given their interest in questions of inheritance and evolution, however, the biometricians were not content to dwell on distributions from a cross-section of a species. Another dimension was needed. In his analysis of different generations, Galton discovered that the normal surface was a good fit to joint frequency distributions. He articulated the techniques of correlation and regression in explaining the stochastic process of inheritance. In their calculation of the coefficient of regression, the biometricians formed regression polygons. Their smoothed regression line connected the means of samples from separate statistical populations. In so doing they used the tools of logical variation to analyse temporal variation. The distinction between these two, however, was often unclear.

Confusion over several possible meanings of the term variation, as illustrated in the Pearson-Bateson articles and even in the quote at the beginning of this chapter, remain with us today. The attempt by the biologists to apply statistical analysis to understanding a process of change over time was carried on in studies of meteorology and economics. The biometry practiced by Galton, Weldon and Pearson soon gave way in biology to the individualistic, deterministic approach of the geneticists in the study of inheritance and to experimental methods such as the development of analysis of variance by Ronald Fisher. Ironically, current theories of evolution include elements of catastrophism, recessive mutation, and rapid change. It was the investigators of social policy and economic theory that carried on the techniques of correlation and regression. In these studies time presented far more serious dilemas and variation took on even more interpretations.

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CHAPTER 4

FROM THE SCIENCE OF MEANS TO THE LAWS OF MOTION

The Interaction of Statistics and Political Economy from 1860-1927

"Heraclitus: 'All things flow', contrasted with the modern view: 'All things are in motion'." (Pearson 1891)

"The Mecca of economics is economic biology rather then economic dynamics." (Marshall 1898)

"Mechanics knows only locomotion, and locomotion is both reversible and quality-less." (Georgesqu-Roegen 1971 p1)

Darwin's theory of the origin of species established coherent definitions of population and variation that closely corresponded to those developed in the probability analysis of games of chance and errors in measurement. Economic theory of the nineteenth century yielded neither a strong concept of population nor measurable units of analysis that could be marshalled into a distinct pattern of logical variation. Empirical political economy, however, did offer several tricks of the trade abstracted from commercial practices that became tools for fitting regression analysis to time series data.

The biometricians developed correlation and regression to determine change over time: inheritance and evolution. Their data, however, were cross-section observations. The statistical populations they worked with were measurements of organisms of one species at one moment in time and at identical stages of development. The cross-sectional approach was possible because they were examining the discontinuous change proceding from one generation to the next.

It was with the earliest applications of correlation and regression to economic phenomena that problems associated with time series were raised and that statistical models were used to study causal temporal relationships. As Alfred Marshall noted, time was the the chief difficulty of economic theory. Time was also to be the chief difficulty of econometric analysis

The frequency distributions of most of the cross-section samples collected by the biometricians displayed the characteristics of those of measurements of one object subject to observational errors. The observations were symmetrically distributed about a centre with most of the data grouped near the mean of the distribution.

There is a major conceptual distinction, however, between the mean of different observations of one physical entity and the mean of measurements from similar, but not identical organisms. Francis Edgeworth called the former an objective mean and the latter a subjective mean:

> The mean of observations is a cause, as it were, the source from which diverging errors emanate. The mean of statistics is a description, a representative quantity put for a whole group, the best representatives of the group, that quantity which, if we must in practice put one quantity for many, minimises the error unavoidably attending such practice... Observations are different copies of one original; statistics are different originals affording one 'generic portrait'. (Edgeworth 1887, p139-140)

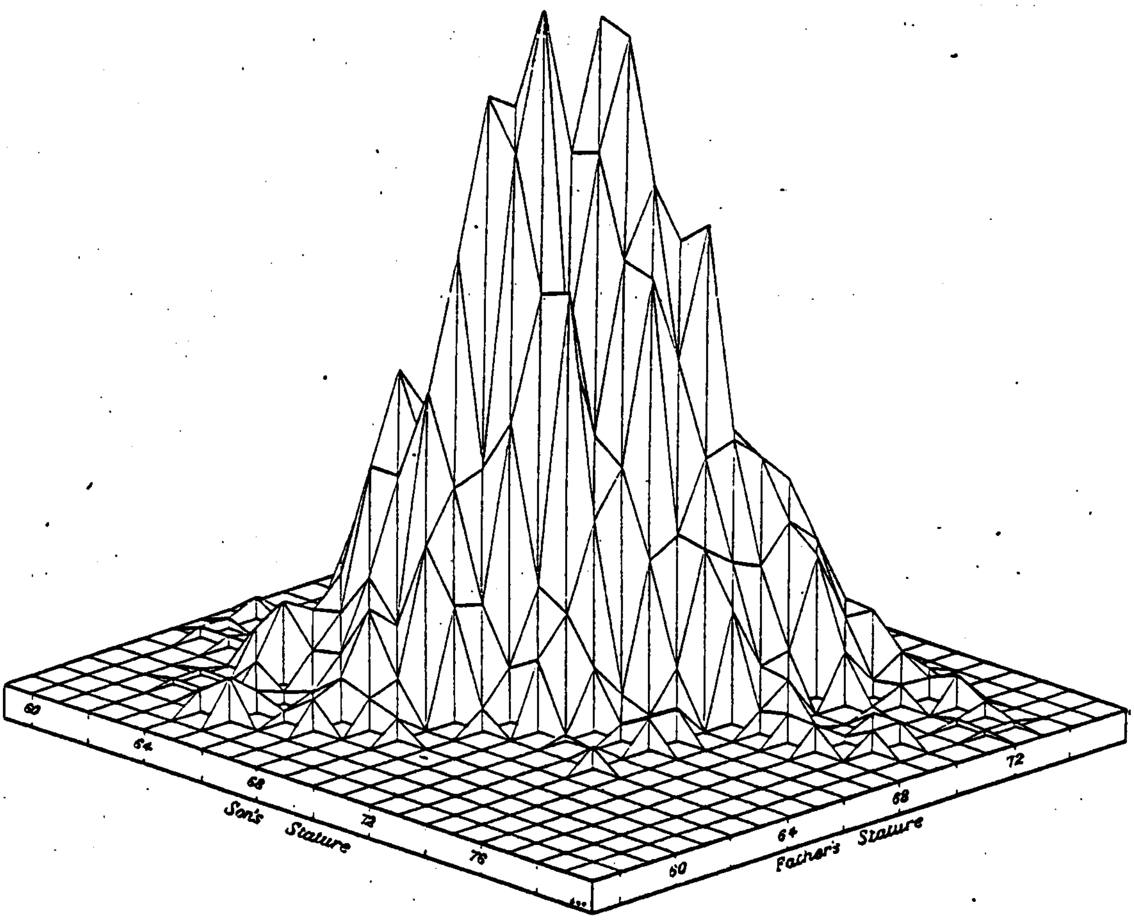
The subjective mean of the biometician's sample was not the true value of the astronomer's stellar position nor the bull's eye around which errant shots were distributed. It was a summary parameter of a whole population. Deviation from the mean in the biometric samples was not indicative of error or imperfection but the seed of creative change.

Within this new concept of population and variation, the assumptions of random sampling and the calculus of probability were very appropriate. The difference between objective and subjective means is conceptually related to the difference between descriptive statistics and statistical inference. In the examples of the biometricians, however, the relation of the average to the entire species yielded a law of error. Only with laws of error were there analytical images in which the mean was the focal point and deviation from the mean was regimented. In a summary article in the Encyclopedia Britannica, Edgeworth argued that the normal law was one type of law of error and that the law of error was one type of law of frequency.

> Laws of frequency, so far as they belong to the domain of probabilities, relate much to the same sort of grouped statistics as laws of error, but do not, like them, connote an explicit reference to an average... Every law of error is a law of frequency, but the converse is not true" (Edgeworth 1911 p309)

A leap in the history of statistical method was made when Galton noticed a three-dimensionsal display of the normal law in his bi-variate study of the diameter of sweet-pea seeds and in his studies on human heredity (Image 1). His use of the normal surface to quantify co-relationships between one generation and the next was analogous to the subjective mean. Friedrich Gauss and Auguste Bravais decades before had laid a mathematical foundation for the application of least squares to regression analysis. The relationships they estimated, however, were fixed; the only variability was caused by errors in measurement. Within the context of Edgeworth's classification, their work was analogous to the concept of a real, objective mean. Karl Pearson, using a similar distinction, argued that the credit for the development of correlation analysis lay totally with Francis Galton and that Gauss and Bravais contributed nothing of real importance to the problem.

> There is not a word in their innumerable treatises that what is really being sought are the mutual correlations of a system of correlated variables....It will be seen that Gauss' treatment is almost the inverse of our modern conceptions of correlation. For him the



..—Frequency Surface for Stature of Father and Stature of Son (data of Table III.): the surface is approximately of the ideal symmetrical or "normal" form.

Image 1

Yule 1911

> observed variables are independent, for us the observed variables are associated or correlated. For him the non-observed variables are correlated owing to their known geometrical relations with observed variables; for us the unobservable variables may be supposed to be uncorrelated causes, and to be connected by unknown functional relation with the correlated variables. In short there is no trace of Gauss' work of observed physical variables being-apart from equations of condition-associated organically which is the fundamental conception of correlation. (Pearson 1920 p. 27)

Darwin's focus on variation within a species and the biometrician's means of investigation of that variation yielded a similar analytical image to that of games of chance and errors in measurements. It was not, however, in essence identical. The variable population, the subjective mean and the organic co-relationship rendered statistical method a more flexible, applicable tool, yet one still grounded in the metaphysical calculus of probability.

Within a few years of Galton's investigations on anthrometry and inheritance correlation coefficients proliferated in studies of meteorology, psychology, social policy, and political economy. Each new subject area to which correlation and regression were applied presented unique problems and subsequent new parameters or techniques. For example, the relationships studied in psychology were often measured by incomparable units.

C. Spearman posed the problem:

> Suppose that we wanted to measure the correlation between the skin's spatial sense and its sensitivity to pain; we should measure both senses at a great many different places all over the body...But then arises the obvious question,

> how far are the variations of distance between the two points of the aeshesimet legitimately comparable with the variations of pressure of the agometer? (Spearman, 1901, p. 201)

Spearman circumvented the problem with the suggestion that measurements of the two different attributes be converted into ranked data then tested for correlation.

At the same time and in the same culture that the biometricians were honing their tools, some political economists were delving into quantitative analysis. For a few years in the late 1800's, the methodology of political economy seemed split between mathematical, reasoning and historical observation. By 1890 with the publication of Alfred Marshall's Principles of Economics and John Neville Keyne's, Scope and Method of Political Economy, the value and interdependence of both approaches was accepted.

Economic inquiry seemed suited for statistical measurement and verification. The subject matter was mass phenomena, a society with its individual base of the "economic man". Economic laws were treated as tendencies not the rigid certainties of physical laws. Equilibrium in value determination was revealed in the long run. Also there was an increasing interest in inductive, empirical approaches and in the quantification of relationships.

Economic theory and data, however, differed from astronomy and anthrometry in several respects relevant to statistical applications. As Warren Persons pointed out, "economics commonly, but not exclusively, is concerned with data in which the 'scatter' is great." (Persons 1925 p179). The observations were usually of aggregates or means of categorical variables rather than unmanipulated observations or repeated measurements of a single object. Also economic hypotheses took the form of multivariate relationships.

The most remarkable, unique feature of economic inquiry, however, was the importance of time. Several theories and descriptions of empirical phenomena dealt with time and/or change over time: long run tendencies, trade cycles, seasonal variation. Economic data usually comprised measurements taken at equally spaced intervals in time. The incorporation of time led to the development of new descriptive techniques, e.g. index numbers and moving averages, but it also raised questions about the applicability of probability analysis and technique based on assumptions of random sampling to time series data. Correlation and regression were readily applied to time series observations in economic inquiries. The perceptions of the nature of problems this raised varied considerably as did the forms of data manipulation and the techniques developed to eliminate temporal complications.

<u>The Method of Political Economy</u>

In the 1870's and 1880's considerable scholarship in political economy was devoted to questions of methodology. For a few decades there was a distinct chasm between a deductive, abstract approach and an inductive, historical approach. At times this separation was embodied in separate people and even in separate national schools. The work of Ricardo was held up as the thesis, the standard by which English classical economists aspired and the foundation of a mathematical approach. The attempt by William Roscher and others in Germany, to grasp the laws of the evolution of economies through historical and statistical observation was the antithesis.

The thesis was the method of abstraction and isolation; deduction through ceteris paribus. Heinrich von Thunen eloquently defended this method in the Preface to the second edition of The Isolated State.

> I hope that the reader who is willing to spend some time and attention on my work will not take exception to the imaginary assumptions I make at the beginning because they do not correspond to conditions in reality and that he will not reject these assumptions as arbitrary and pointless. They are a necessary part of my argument, allowing me to establish the operation of a certain factor, a factor whose operation we see but dimly in reality where it is in incessant conflict with others of its kind.
>
> This method of analysis has illuminated and solved so many problems in my life, and appears to me to be capable of such widespread application, that I regard it as the most important matter contained in all my work. (von Thunen 1966, pp.3-4).

The antithesis came several years after, at the time when ideas of variation and population had crystallized in the Darwinian paradigm of evolution. The subject of the economic inquiry became a population related through nationality. The object of study of the thesis had been the 'economic man', an individual, separated from time, place and others, acting under the singular influence of self interest. The antithesis was aggregation: of individuals, of motives, of causes.

The thesis had searched for universal laws of human nature. The focus was on essence not population. The antithesis assumed differences relative to place and time. The German Historical School turned to biological analogies of human growth, development, variation and evolution to guide their investigation. William Roscher called the 'Historical Method' the 'Anatomy and Physiology of Public Economy', and saw his task as capturing evolutionary themes of economic development.

Economics as well as economies evolved. Those advocating the historical method saw it as a natural maturing of the science of political economy:

> The abstraction according to which all men are by nature the same... as Ricardo and von Thunen have shown, must pass as an indispensable stage in the preparatory labors of political economy. It would be especially well, when, an economic fact is produced by the cooperation of many different factors, for the investigator to mentally isolate the factor of which for the time being, he considered as not operating and as unchangeable, and then the question asked, what would be the effect of a change in the factor to be examined, whether the change be occasioned by enlarging or diminishing it? But it never should be lost sight of, that such a one is only an abstraction after all, for which,

> not only in the transition to practice, but even in finished theory, we must turn to the infinite variety of real life (Roscher 1878, p. 105)

Likewise, Alfred Marshall argued that the assumptions of static equilibrium or of steady motion that enabled deduction through mere addition of forces or through mathematical formula were characteristic of the earlier stages of economic reasoning :

> There is a fairly close analogy between the earlier stages of economic reasoning and the devices of physical statics. But is there an equally serviceable analogy between the later stages of economic reasoning and methods of physical dynamics? I think not. I think that in the later stages of economics better analogies are to be got from biology then from physics; and consequently, that economic reasoning should start on methods analogous to those of physical statics, and should gradually become more biological in time... economic problems are not mechanical, but concerned with organic life and growth. (Marshall 1898, pp. 39, 44)

The development of economic reasoning according to Marshall had its parallel in the development of science:

> At the beginning of the nineteenth century the mathematics- physical group of sciences was in ascendent. These sciences, widely as they differ from one another, have this point in common, that their subject matter is constant and unchanged in all countries and in all ages... At last the speculations of biology made a great stride forwards: Its discoveries fascinated the attention of all men as those of physics had done in earlier years. The moral and historical sciences of the day have in consequence changed their tone, and economics has shared in the general movement. (Marshall 1885 p. 154)

The revelation of the vitality of economies was, according to the historical approach, to come from observation and measurement:

> These, then are the two methods: on the one hand, deduction from psychological motives—first and foremost, deduction from the motive of individual advantage, then from other motives; on the other hand, induction from history, from statistics, and from the less exact and less certain, yet indispensable, process of common observation and experience. (Wagner 1886 p.124)

The previous preoccupation of the classical economists with unidimensional 'economic man' was to give way to a science of measurable motives:

> The outward form of economic theory has been shaped by its connection with material wealth. But it is becoming clear that the true philosophical raison d'etre of the theory is that it supplies a machinery to aid us in reasoning about those motives of human action which are measurable. (Marshall 1885 p.158)

The Theory of Political Economy

By the 1890's it seemed a good start had been made toward answering Stanley Jevon's plea: "The deductive science of economics must be verified and rendered useful by the purely empirical science of Statistics. Theory must be invested with the reality and life of fact." (Jevons 1970). The synthesis, however, was not a comfortable one. For the majority of theorists, such as Jevons and Marshall, empirical work and theoretical work remained separate.

If the Mecca of economists was biology, Marshall and mainstream economic science were never to complete the pilgrimage. Marshall is remembered for his complete systematic mechanical analysis, not for his biological analogies. (see John Maynard Keynes (1925) 1966, Scott Gordon 1972.) Marshall had argued that equilibria should be seen as the balance of the forces of growth and decay rather than static points where the sum of mechanical forces cancels out. (Marshall 1898) The equilibrium analysis of The Principles of Economics, however, enabled Marshall to build up a "copernican system" of balance. This system of points at rest or in steady motion was strengthened with the logical, mechanistic concepts of margin and substitution.

In most of Marshall's analysis, equilibrium is at the end of an economic process, a point achieved in the 'long run'. In this sense it is closely connected to his use of the adjective 'normal':

> "The normal or 'natural' value of a commodity is that which economic forces tend to bring about in the long run. It is the average value which economic forces would bring about if the general conditions of life were stationary for a run of time long enough to enable them to work out their full effect." (Marshall (1920) vol 1 p347)

Marshall's concept of normal value went through a process of change that mirrored the transition from objective mean to subjective mean and abstract method to historical method. In <u>The Economics of Industry</u> first published in 1879, Marshall's concept of normal value is remarkably similar to that of Adam Smith's concept of natural value. It is the value toward which market prices would go in a climate of competition without the distortions of monopoly and government protection. Even with these obstacles the essence of human nature persists:

> Normal results are those which would be brought about by competition if it acted freely, and always had time to cause those effects which it had a tendency to cause...
>
> A man competes freely when he is pursuing a course, which without entering into any combination with others, he has deliberately selected as that which is likely to be of greatest material advantage...the normal action of economic forces is hindered, or even overidden, but never destroyed by friction, by combination or by those passing events which exercise a restless influence on market values. (Marshall 1881 p148, vi)

By 1890 when the <u>Principles</u> was published, Marshall's concept of normal no longer referred to the natural essence stressed by classical economics:

> Normal does not mean competitve. Market prices and Normal prices are alike brought about about by a multitude of influences, of which some rest on a moral basis and some on a physical; of which some are competitive and some are not. It is to the persistence of the influences considered, and the time allowed for them to work out their effects that we refer when contrasting Market and Normal price. (Marshall (1920) 1961 pp347-348)

With a similie included in the first edition of <u>Principles</u> but omitted in the second, Marshall explained the ever-illusive nature of 'normal' and 'equilibrium':

> "The economic conditions of the country are constantly changing, and the point of adjustment of normal demand to normal supply is constantly shifting its position. There are, indeed, constant tendencies towards that point as surely as, to use an old simile, there is a constant tendency of the surface of the sea towards a position of rest; but the moon and the sun are always shifting their places, always therefore changing the conditions by which the equiilibrium of the sea is governed; and meanwhile there are ceaseless currents of the raging winds; the surface is always tending towards a position of normal equilibrium, but never attains it."

The vocabulary of Marshall superficially corresponds to that of the biometricians. In Darwin's paradigm of evolution the tendency is toward adaptation but the goal is never achieved. Species are always changing because at any one point in time there is variation of attributes within a species and from one point in time to another there are changes in environmental conditions. Likewise 'normal' and 'long-run' were terms used in the late 1800's by statisticians. Marshall attempted to relate diverse applications of the former:

> ... every use of the term normal implies the predominance of certain tendencies which appear likely to be more or less steadfast and persistent in their action over those which are relatively exceptional and intermittent. Illness is an abnormal condition of man but a life passed without any illness is abnormal. (Marshall (1920) vol 1, p34)

The mean of a normal frequency distribution is the typical value. Quetelet's average man (l'homme moyen) is the centre of social gravity. Although one such person may not exist observation of the masses clearly points to this summary parameter. Marshall's normal value, unlike l'homme moyen, is not the mean of many observed prices. It is the end point of a process of change that had been isolated from other processes. The normal value is neither observed nor representative of observed values.

The long run of the statistician is, for example, the number of repeated trials that yield an acceptable level of stability in statistical ratios. The long run of the economist is the time period that will allow one to observe the ultimate effect of a steady force in isolation from all other causes and to draw conclusions about historical variation. The former requires mere patience as one records throw after throw of the dye or measurements of the same attribute; the latter requires a feat of the imagination: time without change.

Karl Pearson's method of moments did offer potential for reconciliation. The time it took for a body to come to rest was linked to a statistical measurement of dispersion. In Pearson's analogy, if a bar hung with weights be set:

> rotating on the given rough pivot at a certain speed, friction will bring it to rest in a given time.
>
> Now the greater the concentration of

> weights about the pivot, the sooner the bar comes to rest; the further out from the pivot the weights are, the longer it takes to come to rest. In other words, the time the bar takes to come to rest is a measure of the concentration or scattering of the weights along the range.
>
> Now physicists tell us that this time is proportional to the square of a certain quantity termed the spin or swing radius, and which I will denote by the Greek Letter (σ). (Pearson (1892) 1969 p386)

Marshall's long run, however, was a condition based on the rare simultaneity of time and ceteris parabus. The two usually converge only in the realm of abstraction or controlled experimentation. Marshall recognized that time was the centre of chief difficulty of almost every economic problem:

> It is true however that the condition that time must be allowed for causes to produce their effects is a source of great difficulty in economics. For meanwhile, the material on which they work, and perhaps even the causes themselves, may have changed; and the tendencies which are being described will not have a sufficiently 'long run' in which to work themselves out fully. (Marshall (1920) 1961 p36)

Economic theory of the late nineteenth century had little interaction with statistical method. It did not provide a clear definition of population and the variation hypothesized was unobservable. The concepts that were shared between the two--tendencies, normal, long-run--had very different contextual meanings.

Edgeworth stands out as a rarity in philosophically trying to mate the theories of statistics and economics. He published extensively in both fields and explored the possible analogies between the two. For example, in an article explaining methods of calculating the modulus and comparing two means Edgeworth stated:

> It is useful to have the ideal of proof before our eyes, even when we cannot realize it in practice. This function of the Calculus of Probabilities--to present an unattainable ideal--resembles that which the mathematical theory of Political Economy performs. (Edgeworth 1885 p. 194)

The strongest analogy Edgeworth drew was that between probability in statistical theory and utility in economic theory. Elements that both had in common, according to Edgeworth, included:

results often in non-numerical mathematical statements

assumptions of

equal frequency of chances

equal distributions among consumsers

maximisation of

probability used to determine frequency constants

utility used to determine equilibrium

subjective basis

probability-credibility

utility-personal satisfaction

Edgeworth's approach to economics and statistics was a philosophical one,. From that perspective he saw a very close relationship between utility and probability:

> This similarity of complexion is symptomatic of a deeper affinity between the two studies: the peculiarity that they are both in part metaphysical, unlike the recognised mathematical sciences in that they deal with subjective states of consciousness, belief and satisfaction. Jevons, in his powerful plea for the measurement of utility has dwelt on this analogy: 'previous to the time of Pascal who would have thought of measuring doubt and belief?' The successors of Pascal had already before the time of Jevons thought of measuring utility. Bernoulli had initiated the mathematical treatment of advantage, under the head of 'moral expectation'. Bernoulli , in effect, formulated the law of diminishing utility which plays so large a part in the mathematical theory of Political Economy....
>
> The cognate studies of probability and utility do well to keep together and support each other, for they have not yet won a recognised position among the arts and sciences. They occupy an insecure place between philosophical literature and mathematical physics, frowned on as pedantically precise by the one, and by the other as suspiciously inexact. (quoted in Bowley 1928 p119,120)

This philosophical link was, however, not reproduced in a common conceptual structure that would have enabled statistical verification of economic hypotheses. Edgeworth eloquently described why the path of applied economics was so slippery:

> We have laws almost as simple and majestic as that of gravitation, in particular those relating to value and distribution; but these laws do no afford middle axioms, such as the proposition that planets move in ellipses deduced from the law of gravitation. So dense is the resisting medium which obstructs the free movement of the market; and not only in general dense, but also variable from case to case. (Edgeworth 1925 p8)

In contrast to the interaction of biological theory and statistical method in the late nineteenth century, the application of statistical tools to political economy was not to test theories. The orientation was more in the realm of commercial practice than abstract theory. Statistics was primarily used to formulate motion of, for example, financial flows or to empirically examine policy issues.

Early Time Series Analysis

By 1890 the methodological debate seemed resolved and observation and measurement were important tools of economic analysis. Applied work, however, spanned a spectrum from anecdotal empiricism to statistical inference. Alfred Marshall was nearer to the former end of the spectrum. He was on the one hand very critical of the application of frequency distributions, least squares and regression analysis to economics, but on the other very enthusiastic on the necessity of observation. Observation through what Marshall called 'field work' took the form of numerous conversations with employers and unionists, diary entries of correlated events, and visits to factories and commercial ports. A good example of his method used to investigate Giffen's idea of inferior goods was related to in a letter to Edgeworth in 1909.

> Ever since I saw Giffen's hint on the subject I have set myself to compare the amounts of bread (and cake, wheaten biscuits, and puddings) eaten at first class dinners in private houses expensive hotels, with the

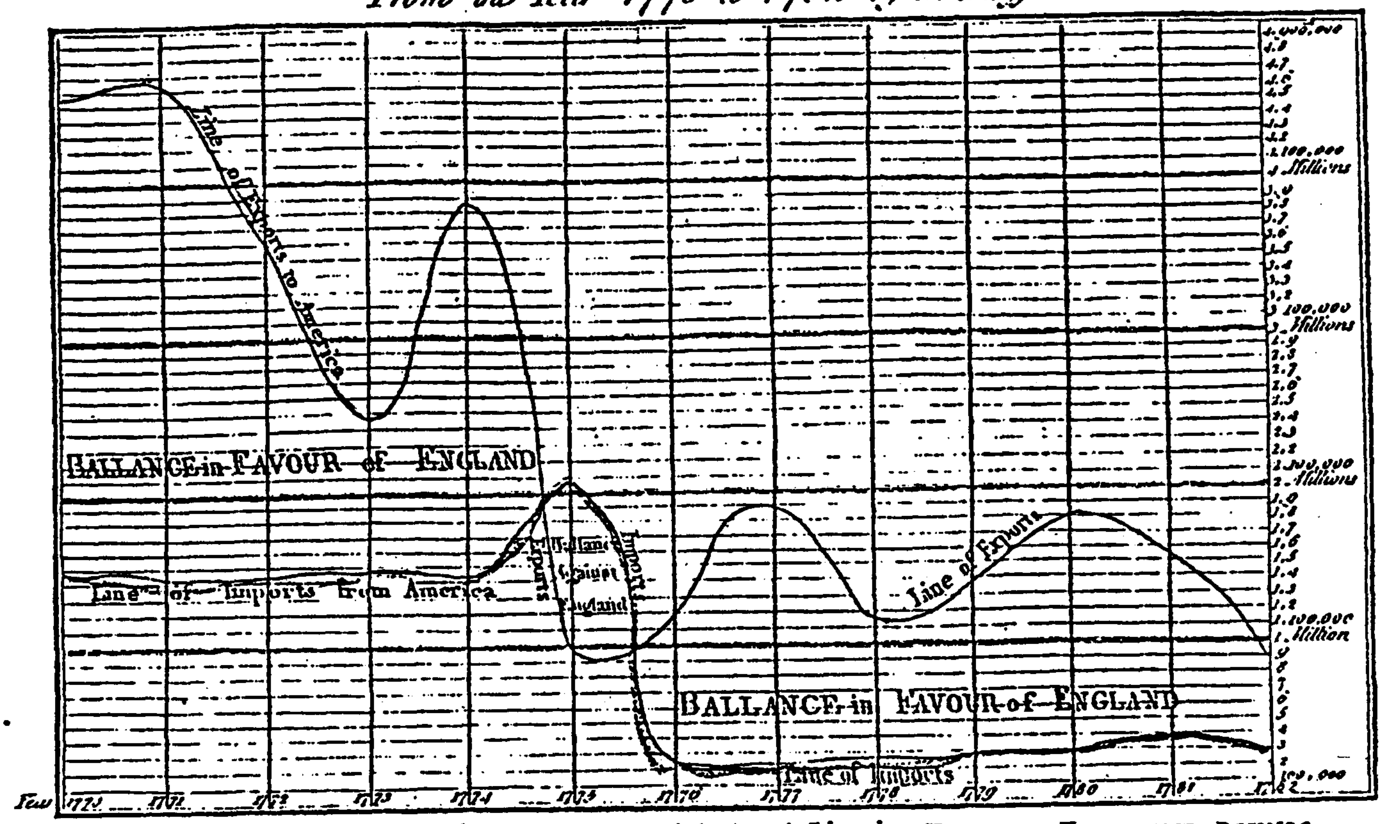
consumption in middle class houses and second-rate hotels; and again with the consumption in cheap inns, including a low grade London hotel: and I have watched the baker's supplies to cottagers. And I am convinced that the very rich eat less than half as much bread as the poorer classes; the middle class coming midway. This proves nothing: but it is a fair basis, I think, for a surmise as to a probability. (Marshall (1925) 1966)

Marshall's often-stated quest was for "the One in the Many, the Many in the One". His selective observation and armchair "data manipulation" enabled him to achieve empirical ceteris paribus. What was lacking in this approach compared to that of statistical inference, was numerical precision and measurement of the reliability of conclusions. Marshall's approach was a quaint marriage of deduction and induction, but it was not the stuff from which the dynasty of econometrics descended.

The dynamic interaction between political economy and statistical method was almost entirely through numerical measurement and quantitative description of historical variation: the relationship of variables to and through time. In trying to capture and abstract change, political economy, like the natural sciences, developed from descriptions of flows to analysis of motions.

This development is vividly illustrated in a comparison of the time series graphs of William Playfair in 1786 and Stanley Jevons in 1862 (Images 2 & 3). A first visual impression is that what is significant in the former is shape and area; in the latter points and lines.

CHART of IMPORTS and EXPORTS of ENGLAND to and from all NORTH AMERICA From the Year 1770 to 1782 by W. Playfair

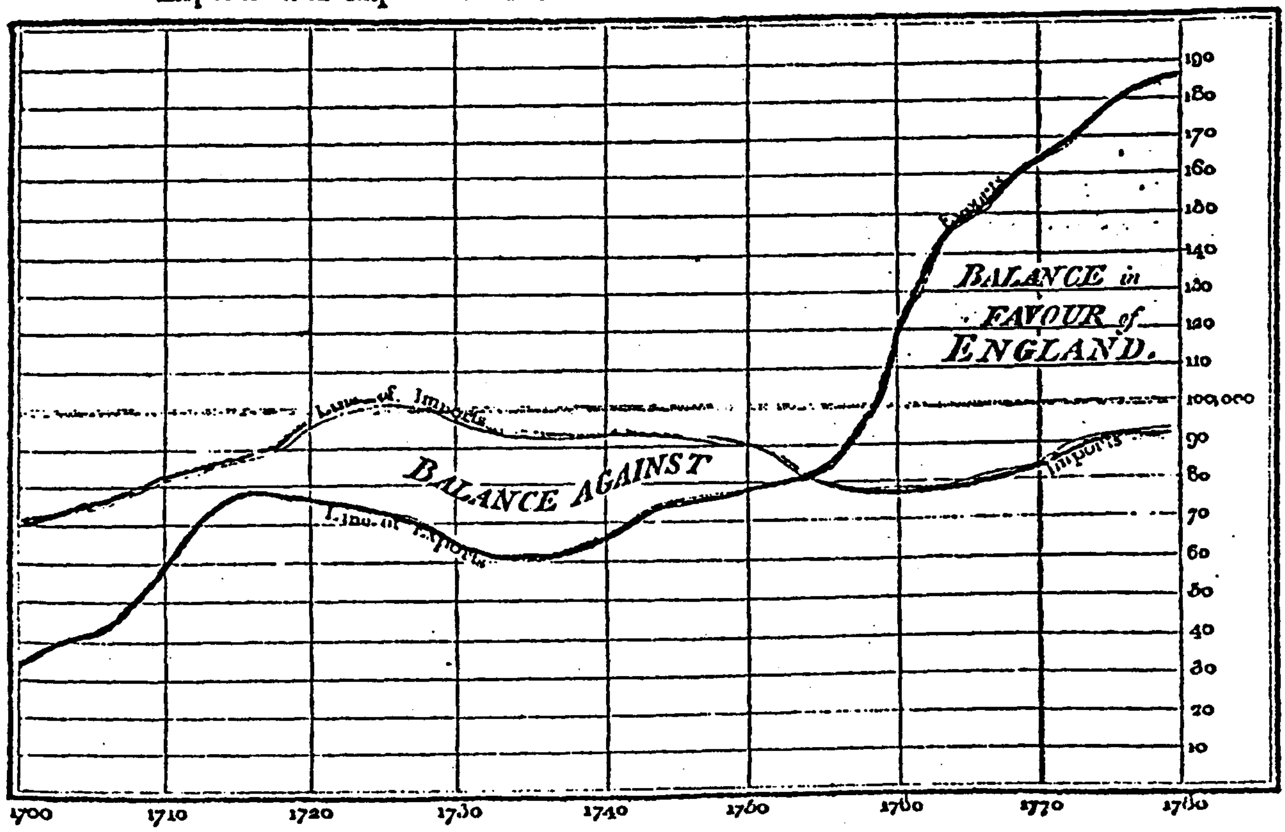


The Bottom Line is divided into Years the right hand Line into HUNDRED THOUSAND POUNDS

J. Ainslie Sculp.

Publish'd as the Act directs 20th Aug. 1785.

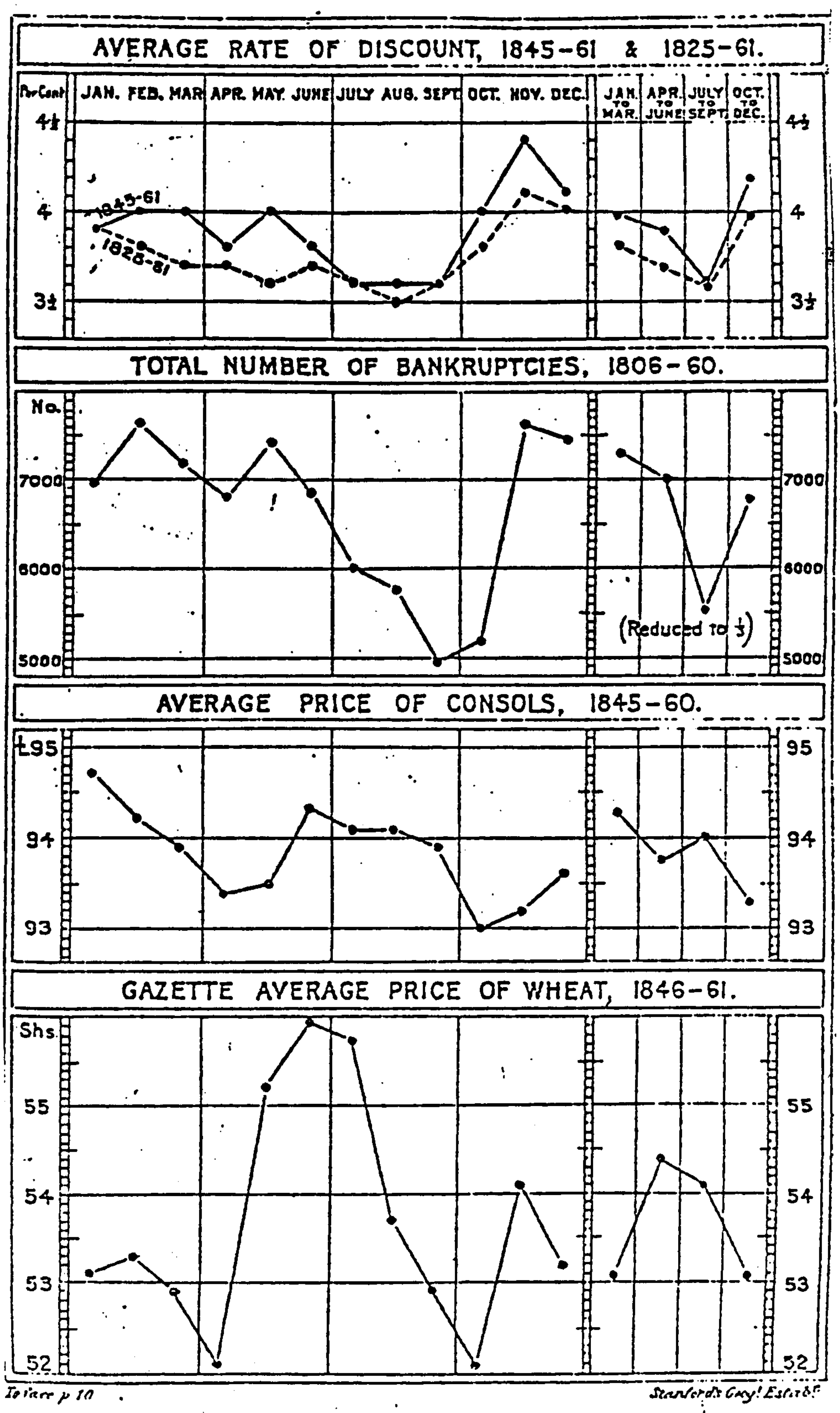
Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780



The Bottom line is divided into Years, the Right hand line into L10,000 each.

Published as the Act directs, 1st May 1786 by Wm Playfair

Playfair coloured defecits red, surpluses blue. Playfair 1786 Image 2



Jevons (1862) 1884
Image 3

As one glances from left to right Playfair's graph gives a sensation of organic flow, Jevon's a sensation of mechanism. The unity and wholeness of the former contrasts with the cusped vectors of the latter.

The earliest uses of the word "flow" are to describe movement or current in a liquid, and of "motion" to describe mechanical change of place. Adam Smith in his <u>Enquiry into the Nature and Causes of the Wealth of Nation</u> in 1776 explains the relative advantage of cities being determined by their proximity to rivers or seas which were the major mediums of commerce. John Hicks (1979) points out that in Smith's time Geneva was the only prosperous European city not located in a navigable river. Its prosperity was derived from the manufacture of watches, which compared to other goods had low transportation costs relative to final price. By the 1860's wind and current of water ways had been replaced by manufactured steam on the railways as the prime mover of exchange value. Motion became more useful than flow.

The essence of Playfair's graph (the top one) is the shifts in the Balance of Trade from 1770 to 1782. Although both axes are numerical, there are no highlighted coordinate data points. Turning points are smoothed. The story conveyed is a historical one: the dramatic changes in commercial relations between colony and colonizer before and during a war of independence.

Playfair was one of the first to use cartesian co-ordinates with one axis representing time and the first to apply principles of geometry to matters of finance. He called himself the inventor of linear arithmetic. The Commercial and Political Atlas published in 1786 contains the first time series plot of economic data and all but one of the 44 charts in the atlas were time series.

There are virtually no examples of economic time series plots between Playfair's work and that of Jevons in the late nineteenth century. Some political economists such as Marx and Roscher used statistics to investigate and illustrate their arguments, but these took the form of tables or isolated numerical insertions. Between Playfair and Jevons there was a visual vacuum. During that time, however, the law of error was being mapped to quantified biological and social phenomena. Jevon's visual images derive more from that development than from Playfair's diagrams of descriptive statistics.

In his study of periodic commercial fluctuations Jevons compared centres of gravity to draw conclusions about typcial, temporal maximums and minimums. The story conveyed in the his graphs is not an historical one; it is repeated year after year, if one ignores the noise. It is a change that is never identically observed but yet typical of experience.

In the example of the solid line in the first monthly graph, the steps in Jevon's construction were to:

1. Calculate the arithmetic mean of the rate of discount observed each week in Januray 1845, calculate the same from February 1845, etc until December 1861.

2. Group the data into monthly populations and calculate the arithmetic mean of the rates for each of the 12 months.

3. Plot the means against the corresponding time framework.

4. Connect the points with straight lines drawing attention to local minimas and maximas.

Jevons described his purpose:

> It seems necessary, then, that all commercial flucutations should be investigated according to the same scientific methods with which we are familar in other complicated sciences, such especially as meteorology and terrestrial magnetism. Every kind of periodic fluctuation, whether daily, weekly, monthly, quarterly, or yearly. must be detected and exhibited, not only as a subject of study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or non-periodic, and probably of more interest and importance.
> (Jevons (1862) 1884 p4)

His method of eliminating periodic variations to discover irregular ones was ironically to describe and model periodic variation by first eliminating the fortuitous logical variation displayed by the monthly populations:

> Taking the weekly accounts of the seventeen complete years, 1845 to 1861 inclusive, I have simply ranged them under each other in their

> numerical order within the year, and drawn the averages with all suitable precautions against error. All non-periodic variations seem to be nearly eliminated and the seasonal variations remain....
>
> The chief use of the table of quarterly variations, however, is that we may by it use eliminate these variations from the whole variations of the year by simple subrtraction. We thus ascertain the nature of the yearly variation which is due to natural causes as distinguished from the artificial distinctions of months and quarters. (Jevons (1862) 1884 p7,8)

Jevon's lines connect the subjective means of separate, fortuitous populations. The observations for all the Januarys could probably be considered as independent of each other and as random selections. The quarterly observations yield samples from different populations. The monthly and quarterly patterns plotted are similar but by no means identical. The motion is different and the stories used to explain such motions are different. They serve as excellent examples of complementary time-framework analyis.

Both Playfair and Jevons were interested in capturing change in their two-dimensional images. They were quantifying the relationship of commercial variables to time. Playfair in one picture vividly describes a moment in history of qualitative, revolutionary change in a multivariate relationship (exports-imports). Time in this picture is important in an absolute sense and the analysis focuses on the bi-variate relationship. Jevons describes a univariate pattern where time is important only in a relative sense. The image is of no historical signifigance, unless there is considerable structural change (note the dotted line in his first graph constructed from observations from 1825-1861). The usefulness of this quantified temporal relationship is in its universality. It is an analytical image of a "law" of seasonal variation. Its foundation was the law of error, and the averages and deviations pertained not to a sample of consecutive observations but to samples from homogeneous populations.

Jevons mixing of political economy and the law of error was an attempt to analyse motion. Aggregate commercial phenomena was such that fluctuations had long been a focus of inquiry. James Wilson is his tract on _Fluctuations of Currency, Commerce and Manufactures Referrable to the Corn Laws_ published in 1840 addresses the "frequent recurrence of periods of excitement and depression in the monetarial and commercial interests of the country." As Wilson points out numerous inquiries had been made on "the rapid and extensive fluctuations which for some years have continually been taking place" with most attention directed to the operation of the currency and banking system of the country.

Although what we now call trade or business cycles were in the 17th and 18th centuries called commercial crises, their periodicity was recognized early on, as demonstrated by an article by William Herschel in

Philosophical Transactions in 1801. Also Jevons pointed out that merchants and manufacturers were well aware of monthly and quarterly fluctuations in their own branch of industry.

> By the skill and rule-of-thumb knowledge which each one acquires in his own pursuits, they make allowances for such variations, and thus very rude comparisons of prices, stocks and sales enable them to detect irregular changes in their own markets, which is all they require. But this unwritten knowledge of commercail fluctuations is not available for scientific purposes, and is always of a very limited extent. (Jevons (1862) 1909 p3,4)

The task of the empirical political economists was to go beyond individual markets and look at aggregate phenomena and to translate rule-of-thumb practices into scientific method. For example, Jevons in his 1862 article notes that in market reports comparisons were made between numbers refering to the week, month or other parts of the year and those of corresponding parts of a previous year in order to avoid any variation due to time of year. Jevons took this a step further by reorganizing data from each month as a sample from a population for which an average and deviation from the average had summary significance and by connecting these averages into an analytical, time-series image.

John Norton (1902) in a later study of financial data also refered to the need for economists to refine commercial practices and even merchants' intuition into scientific analysis. Norton, like Jevons, studied the annual period of banking reserves using weekly data. The data Norton worked with were "not eccentric and solely a satisfaction of a scientific curiousity".

> They held "for over forty years a well recognized place of importance in the financial world. These statements have been again and again for weeks at a time the absorbing 'feature' of the security market." (Norton 1902 p22)

Likewise, Norton with sophisticated statistical technique was confirming the subjective experience of the banker: "The annual period of reserve deviations long known to bankers by experience but hitherto unmeasured, is clearly revealed." (Norton 1902 p102)

In subsequent papers, Jevons investigated the variety of 'tides' that revealed themselves in financial data. October 1865 was a time of remarkable withdrawl of coin from the Bank of England followed by a decrease in loanable capital and an increase in discount rates. The commercial press was in 'considerable perplexity' and Jevons applied his statistical and graphical tools to shed light on the dramatic increase in the demand for money.

Using data from the Bank of England from 1845-1881, Jevons studied monthly, quarterly, and annual variations. Jevons considered the first two artificial variations, due to social practice, He considered the "annual tide" in the accounts a natural phenomena and of more interest.

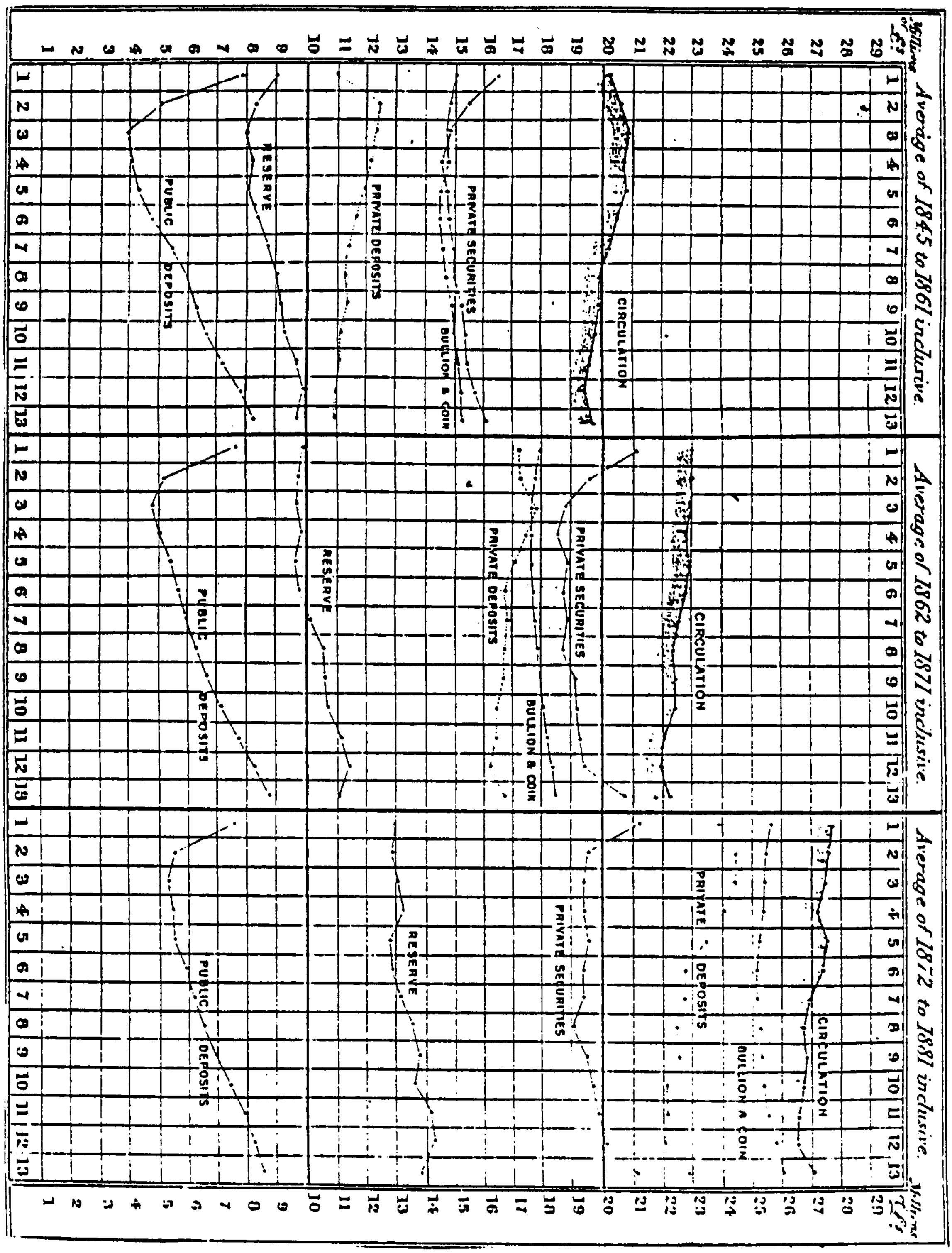
The different tides coincided in their effect in the Autumn:

> To sum up, then, the October drain is due, like many other economic disturbances, to a concurrence of causes... quite sufficient in certain states of the money market to engender a panic, unless, indeed, its normal and temporary nature be well understood. (Jevons (1866) 1884 p172)

In this examination of the special pressure of 1865, Jevon compared weekly figures for notes in circulation, bullion, and reserves of 1865 with those of corresponding averages for each week, from the period 1845-61. This indicated that 1865 was indeed unusual. Jevons addressed exceptional circumstances affecting trade in that year, but also the possibility of a "general and gradual development of our industry."

> Even without taking into account exceptional circumstances, the unbounded prosperity of the last few years seems sufficient to explain why the autumnal drain has of late manifested itself with far more that the normal severuty of the years 1845-61. We must bear in mind that we are moving onwards, and rapid progress such as ours, however desirable in itself, must beget some difficulties. (Jevons 1866 p177)

The trend element Jevons makes reference to is evident from a chart he appended to the paper in a later publication (see Image 4). In this chart the quarter is disected into weeks. The pattern of variation within the quarter remains similar in the three periods averaged, but the position of the lines for notes in circulation,



Jevons 1884
Image 4

private deposits and reserves indicates a trend.

The method of sampling from a population defined by sequence within a time framework was used by Jevons to study temporal variation from week to week within a quarter, month to month within a year, quarter to quarter within a year, and year to year within a trade cycle. The latter was applied in his papers investigating the relationship between commercial crises and sunspot activities. (Jevons (1875) and (1878) 1884). In all these attempts to clarify a law of temporal variation, Jevons averaged over values that varied according to a law of error.

In almost every empirical work of late nineteenth century political economy measurements taken at different points in time were used. In most studies, with the exception of Jevons, averages and even measures of dispersion were calculated from samples formed by consecutive measurements. Wilhelm Lexis was one of the few political economists to recognize that statistical ratios from such time series were often estranged from probability foundations and potentially unstable. His attempts to measure and test for stability were based on the question of singularity of population. He was one of a few statisticians to approach probability theory from induction rather that deduction.

Lexis argued that statistical method, not just statistics, was an essential tool of political economy for the study of "Massenerscheinurgen" (mass phenomena) and that empirical work was an economical means of determining laws.

> I conceive that political economy is an empirical science. The economic dealings of the conscious individual are its fundamental facts. From these, economic phenomena as they appear in the mass are to be explained. There being a large number of individual actions, the effects of some may serve to offset the effects of others. But other effects, again, are intensified, and bring about general phenomena, which, for the very reason that they made up a large number of individual actions, are subject to no rapid changes, and so possess a good degree of stability. Thus they become in a way independent of the will of the individual: they may even appear as forces controlling the action of the individual. In the flow of time they show such constancy that we may speak of "economic laws." This expression, while in strictness only figurative, is not open to objection, provided that it be not forgotten that the observed uniformities do not rest, like the law of gravitation, on some external force controlling the individual phenomena, but are simply the results of a number of individual acts, which, while doubtless reacting one on the other, yet are each independent....Thus we may reach, by the most trivial empirical means, and without any reference to immanent ideas or laws of evolution, the same conclusion as Marx's law of surplus value. (Lexis 1895 p32-33)

A theme throughout Lexis' work was the stability of social statistics. Those statistics directly related to natural conditions (eg. population statistics) were very stable but moral statistics less so. Time series data could reveal a variety of principal forms of fluctuation:

evolutionary- displaying historical development

undulatory- of which periodic is one type

oscillatory- of which a special class is the typical

The "typical" series was a group with the unique characteristics that individual values were in essence approximators of a constant underlying value differing from it only by random deviation. In all series average deviation could be calculated but if the series had an evolutionary or undulatory character one could only empirically characterize their variability, inference could not be drawn. Only in the 'typical' series was the relationship between individual values and the mean of a probability character.

A common theme in Lexis' studies was the stability of a series over time. Lexis derived measurements of stability that made comparisons between series possible. These measurements lead to the qualitative distinction of "super-normal", "normal" and "sub-normal" dispersion. The latter indicated that the individual events of the mass phenomena were interrelated and subject to regulations or norms. Super-normal dispersion was an indication that random fluctuations were combined with "physical fluctuations of the underlying probability". Only in the case of normal (or "random-normal") dispersion was there demonstration of a "constant probability underlying the observed numerical relation showing the same uncertainty as would be expected in a proper game of chance." (Lexis 1903 p7)

The game of chance analogy was applied by Lexis to statistics in the form of ratios. For example, the ratio of male to female births was like the ratio of white to black balls in a urn. The law of error was the proper analogy for series of absolute values, but Lexis concentrated on ratios. Super-normal dispersion within a series indicated that the probability structure had changed, that the balls were drawn from different urns, that the samples came from different populations.

With time series data, probability analysis and analogies could only be applied if values formed "typical" series and were not interrelated or not subject to an enforced law. Lexis did speculate on formulating curves of changing underlying probabilities in the case of evolutionary or undulatory series as Jevons had done. Lexis refused, however, to accord it a status of law:

> The curve of the underlying probabilities can be formulated only hypothetically. If one finds then that the average deviation between the calculated and the observed values is not appreciably larger or smaller than would be expected according to the theory of probability, the hypothesis is justified. One may assume then that the hypothetical curve reproduces approximately the true variations of the underlying probability in the given period of time. Such a curve, however, cannot be extrapolated since it did not represent a law in the past. It is nothing else but geometrical representation of what has taken place. The exact determination of such curves is usually not worth the effort. (Lexis 1903 p12)

Lexis did not see all periodic series as interrelated series, but he did argue that each phase of the cycle would have to be treated as a separate population to apply statistical analysis:

> The periodicity of the death rate is still more blurred by the seasons of the year. Series of this kind do not belong to interrelated series as all since the periodicity here is not determined by a definite rule but depends upon the periodical variation of external factors, the effect of which is not strictly uniform, varying greatly with many other conditions. In any case the periodicity of such a series cannot be neglected and each phase of the fluctuations must be treatd separately. (Lexis 1903 p6)

Although Lexis took an empirical approach to political economy and statistics, he searched for probability foundations in his work. His aim was not to capture laws of motion so much as to determine the distortions of temporal change on statistical series. In that determination he made the assumption that more than one population could be the source of observations taken over time on one variable. The observations had to be from a single population, the balls all from one urn to apply the analysis, whose beauty lay in the stability of the parameters summarizing a random world.

In their bridging of commercial practice and economic analysis, Jevons and others were in essence trying to determine the boundaries of populations. Statistics, as Edgeworth described it, was "the science of 'Means'... the term 'Means' of course implies the correlative conception: members of a class, or terms of a 'series'." (Edgeworth 1885 p182). The development of statistical method in political economy yielded such tools as index numbers, ratio charts, and moving averages. The common denominator of all these techniques was the focus on change. The use of rates of change in a variable rather than absolute levels enabled the establishment of statistical populations of very dissimilar objects, eg. price of wheat, price of iron, etc. Moving averages were recognized for their "smoothing" qualities in commerce long before they were applied by the economists to meet the challenge of population and temporal variation. These techniques were eventually used in regression analysis and sophisticated time series analysis.

Norton in his 1902 statistical study of the New York money market explained the importance to economists of adopting the merchant's focus on changes or deviations:

> It is with these changes that interesting problems in economics are connected, not with gross sums. Like the sailor or captain on the ocean, we are interested, not in the depth below of the ocean-bed,- for we are sure that the ocean-bed will remain during the little while that we are sailing over it,- but we do fear the height of the waves, not from the ocean-bed, but from the "ideal" surface. So with us,- we will measure the financial waves from an "ideal" standard, which for convenience we have called the growth axis.
>
> Nor are these waves unimportant. It was some chemist who said, "Never throw away your residues. Look in these for your results." This practice of studying the residues is perhaps most common in the business world. A glance by our leading journals will convince the reader how familiar this method is outside the text-books. The mill owner and the speculator are constantly watching the net changes, the differences, and these become the motives of their actions. The economists may profitably study these differences for the verification of

his laws. (Norton 1902 p34)

One year after his first publication on seasonal variation, Jevons published a pamphlet on the change in the value of gold. The journals of this time were well endowed with articles attempting to investigate the relationships of the standards of gold and silver to commodities and to each other. It was a classic, causal temporal enigma.

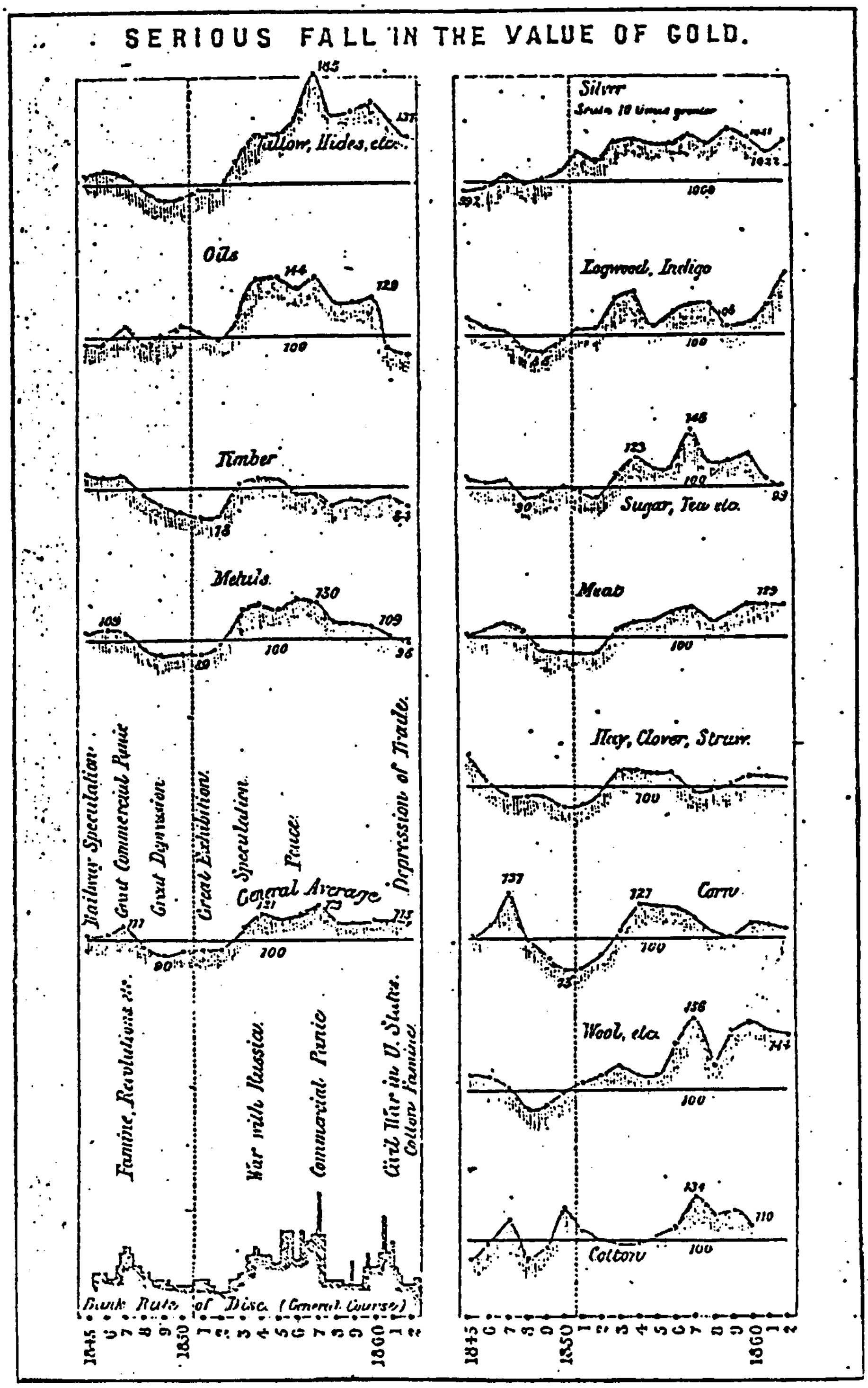
> In the following tract I commence by endeavoring to unfold the fundamental difficulties of the inquiry, afterwords discriminating the various causes of temporary fluctuations in prices, in order that we may the more surely recognize the effect of the permanent cause in question. I then introduce tables formed and reduced in accordance with the principles of the subject, so as to exhibit a depreciation of gold if any exists. (Jevons (1863) 1909 p16)

In this one study, Jevons introduced several analytical tools to unmask the essence of commercial motion: index numbers to focus on relative change rather than absolute level, ratio charts to compare proportional variation in prices and the geometric mean as an average of ratios.

Jevons established likeness in unlike things by concentrating on relative change. He explained that "there is no such thing as an average of prices at any one time... there is no such relation or similarity between a ton of iron and a quarter of corn as can warrant us in drawing an average between six pounds and three pounds." If one, however, measures the relative change in prices then "the ratios 100:150 and 100:120 are things of the same kind, but of different amounts, between which we can take an average." Thus Jevons created with rates of change a population of similar but variable (in the logical sense) observations for which a mean had significance.

No one variable, such as wheat, would suffice in answering the question of a possible depreciation in the value of gold. One could conclude that more gold was needed to buy the same amount of wheat then before, but one could not conclude from which side the change came from. Jevons argued that if several articles change in price the same way then it is more likely that the cause is due to a change in the value of gold. His use of an average of proportionate change of 39 articles enabled him to conclude that in fact the discoveries of gold in California and Australia had led to an alteration in the standard of value. Even during "comparative depressions of Trade" the general average (see Image 5) stayed above 100.

The focus on rates of change enabled Jevons and others to treat such changes in each year as a population and to compare movement in one series with that of another. This latter capacity was fully developed by Arthur Bowley in his famous study of wages (Bowley 1895).



Jevons (1863) 1884
Image 5

Working with a large variety of data on wages from different places, occupations and times, Bowley constructed a consistent index series from fragmented pieces. The links of the chain were rates of change.

The use of moving averages was practiced for many decades before being modified by political economists for statistical analysis. In his 1840 study on periodic commercial crises, James Wilson mentions the Bank of England's use of moving averages and their effect on the statistics.

> The fluctuations shown in this table, however great, are less than those which have actually taken place, owing to the mode in which the Bank's returns are made: being made once in each month, of the average of the three preceeding months.... Thus the smallest and largest amounts of bullion in possession of the Bank is never shown; and thus the fluctuations are even greater than displayed by the returns. (Wilson 1840 p3)

Calculating a moving average obviously meant calculating an arithmetic mean from consecutive observations, but it did not become a tool of the "science of means" until deviation from the moving average was also considered. J. Poynting in a paper read before the Royal Statistical Society in 1884 presented the 'process of averaging' in his investigation of a possible relationship between harvests of wheat, cotton and silk that would point to a common meteorological cause.

Poynting was aiming for a visual confirmation of a co-relationship between time series of three variables:

the price of wheat, British imports of cotton, imports of silk. Such a confirmation was not forthcoming from the plots of actual values. What was needed was a way to manipulate the statistics to show "the true fluctuations, whatever they may be, with the effects of war, increases of commerce, etc., as far as possible eliminated... fluctuations freed to some extent at least from accidental irregularities." (Poynting 1884 p35)

The method pursued to obtain "the true fluctuations" was to construct a standard of a ten-year moving average, or instantaneous average as Poynting called it, and to calculate deviations of a four-year moving average from the standard. The four-year average as a percent of the ten-year average was plotted as a time series. The result was synchronized curves; smoothed waves appeared to fluctuate over time in a similar manner.

Poynting had considered the possiblity that his method of averaging might have made the series appear to resemble each other even though there was no real physical connection. On mathematical advice from Balford Stewart, Poynting explained that some of the harmonics (infinite number of series of fixed periods composing a series fluctuating unequally with time) would be destroyed. He argued, however, that such a wide range of harmonics remained "that the fluctuations in the wheat and other curves do not arise merely from the process, but are fluctuations occuring in reality."

In the discussion that followed Poynting's presentation to the Statistical Society, Hyde Clark spoke of his frustrations over the past forty years of grasping at the perioicity of commercial phenomena. Clark warned that political economists must be "particularly careful in not stretching their knowledge too much, or supposing that they had too valuable a piece of economic machinery in this process, which had deservedly attracted the attention of so many eminent men of science." Another discussant, John Martin, was impressed that Poynting could "evolve order out of that which at first sight had the appearance of something very like chaos." Martin, however, wondered at the new paths the science of means was taking as time series were analysed.

> What was an average? Whether they took wheat, or silk, or cotton, or anything else, what was it that they were starting from... The system of averaging therefore for the purposes of economical practice was as far as he could see totally different from the system of averaging bases which commended itself to the mathematical mind.. Major Craigie hoped that one hundred years hence they would have a wider base and would get a better system of averaging, but if he (Mr. Martin) were still alive he was afraid he should still have to put the question 'What is an average?'." (Poynting 1884 p73)

Poynting had arbitrarily picked ten-year and four-year groupings to calculate deviations from an instantaneous mean. He made no assertion of cycles or periodicities. In later works (eg. Hooker 1901) it was recognized that the instantaneous average was only an appropriate method to investigate correlation in series that were periodic. The interval for averaging over was determined by the period of the cycle and the symmetry of this selection gave the instantaneous mean, or moving average, a link with the subjective mean of laws of error.

The concept of average developed in statistical analysis, whether it was an objective or subjective mean, was intimately linked to the concept of population. The individual elements of the population were of the same kind but not identical in measurement. The simularity was such that the numerical values of the elements showed a tendency to group about a center. Deviations from this center had analytical significance. Economic time series were observations of a single variable, but that was not a sufficient condition for assuming that the mean and deviations from the mean had analytical significance.

In the last few decades of the nineteenth century an often expressed concern was the small size of statistical samples. In the drive to realize the full potential of stable statistical ratios, political economists usually used every observation possible. Lexis and Jevons were unusual in testing for or using separate populations in their analysis of time series data. The problems that time presented defining population and thus applying probability analysis were particularly acute in investigations of co-relationships. The commercial tricks of trade were combined with regression and correlation to lay the foundation for explaining motion. With conflicting boundaries of population and manifestations of variation this explanation did not come easy.

Correlation Through Time

In the hands of the biometricians correlation and regression coefficients were applied to a variety of measurable characteristics of organisms. In his classic article on regression, Karl Pearson (1896) lists some of these earliest studies of correlation of sizes of organisms. There are several things they have in common that were not shared by most economic and time series investigations:

1. The co-relationship was between the same organ of different generations or different organs of the same organism. Thus the X and Y variables were measured in comparable units and although they were samples drawn from different populations, they were organically related.

2. The observations for each variable were taken in a cross-section of the population at one point in time. Although the goal was to study heredity and evolution, the observations comprised a sample from a static population. In a letter to Yule 1981, Pearson speculated on the analogy that "the further we go back the more we may find the laws of motion, mechanism, to be due to the structure of matter."

3. The observations for each variable displayed a 'normal' frequency distribution similar to the laws of error and the correlation table of paired variables yielded a normal surface.

Two decades separate Galton's first correlation of the diameter of sweet pea seeds and the first application of the technique to political economy and time series data. Political economy shared with meteorology temporal mass phenomena. These were the first areas after biometry that regression and correlation were applied. These two fields of inquiry had the following in common:

1. The data was usually in the form of time series.

2. The series usually displayed some periodicity. The temporal variation hinted at repetitious patterns.

3. The frequency distribution of each variable was usually skewed and the amount of variation was a considerable fraction of the mean.

4. There was spatial as well as temporal variation (eg. in prices or barometric heights at different locations)

Political economy and meteorology were different, however in the following respects:

1. The measurements of the X and Y variables in meterology were usually of the same "organ" eg. the barometer. Although the two samples being related were time series from two different stations, the assumption was often that the weather at one station would, in a few hours, be the weather at the other. The correlation of barometric heights at different stations was thus very similar to serial correlation.

2. The Meteorological observations used in the early studies rarely exhibited a perceptable secular change.

The problem of skewed frequency distributions was tackled by both Edgeworth and Yule. Edgeworth (1885) demonstrated a central limit theorem: even though samples may display skewed distributions, the distribution of the means of the samples from a population will be approximately normal. Yule (1897) used the condition of least squares to estimate a line of regression and deduce the formula for correlation and its properties without reference to the form of frequency distribution. The limiting criteria of "normal correlation" thus became insignificant, but other problems of time series still remained.

The obstacles confronting statistical investigations in meteorology and economics were demonstrated in the first attempt to apply correlation to economic data. In a series of articles Yule (1895, 1896 a&b, 1899) investigated Charles Booth's assertion that the proportion of relief given outdoors, as opposed to in the workhouse, had no general relation to the total percentage of pauperism in those districts that administered such an out-relief policy. Booth, a social reformer was concerned about the loss of liberty and the break-up of the family inherent in policies that were limited to indoor relief, i.e. the workhouses. He denied the arguement that outdoor relief would reduce the incentive to work.

Yule used data on over 500 unions in England that had been gathered in a survey in 1871 and again in 1891. Yule was first struck by the skewed frequency distribution displayed by measurements of the ratio of out-door to workhouse paupers and of the percentage of the population that was paupered. In his first article Yule calculates for both surveys a positive correlation coefficient, but warns that "as no theory of skew correlation has yet been published we cannot say what weight can be attached to the coefficient under these circumstances." (Yule 1895 p604) He concluded, however,

> Whether we deal with general pauperism at all ages, or with the case of males over sixty-five years of age, whether in the latter case we take an Urban or a Rural group of unions, whether we take the year's count or the day's count, we find the proportion of the population in receipt of relief to be positively correlated with the proportion of relief given out-of-doors, i.e. we find that a high pauperism corresponds on the average to a high proportion of out-relief. (Yule 1896 p618)

In the first few decades of the application of regression analysis the interpretation of regression was that it gave the average value of Y corresponding to a value of the X variable. This was consistent with the technique of fitting a line of regression to a regression polygon. In no publication of that time was a coefficient of regression translated as dY/dX. Yule, however, did make a table ranking districts from smallest to largest ratios of out-paupers to one in-pauper. For each class he gave the corresponding mean percentage of population in receipt of relief. He described this table as showing "that the rise in pauperism, as the out-relief ratio was increased was well marked and uniform" (Yule 1896, p613). A step had been taken in substituting logical time for historical time.

In his most thorough treatment of poor-law statistics, Yule (1899) investigated causes of <u>changes</u> in pauperism. Yule explained that:

> On the whole, it would seem better to correlate <u>changes</u> in pauperism with <u>changes</u> in various possible factors. If we say that a high rate of pauperism in some districts is due to lax administration, we presumably mean that as administration became lax, pauperism rose, or that if administration were more strict, pauperism would decrease; if we say that the high pauperism is due to depressed condition of industry, we mean that when industry recovers, pauperism will fall. When we say, in fact, that any one variable is a factor of pauperism, we mean that <u>changes</u> in that variable are accompanied by <u>changes</u> in the percentage of the

population in receipt of relief, either in the same or reverse direction. It will be better, therefore, to deal with changes in pauperism and possible factors. (Yule 1911 p192)

Rather than interpret the regression coefficient on time series data as the change in pauperism given a change in a factor, Yule correlated cross-section data of differences. The changes in pauperism, out-relief ratio, proportion of elderly, and population were all measured as percentage ratios eg. 100 x pauperism in 1881 divided by pauperism in 1871. Ironically, Yule did not use percentage change over the decade because of the problems of dealing with positive and negative signs.

Yule noted that this method of splitting an effect into portions due to several causes yielded estimates of the algebraic parts of the whole change not fractional parts. In some cases the change ascribed to a change in out-relief was greater than the whole change that had taken place, but against that there would be something to offset it due to another cause. Yule concluded that his method of multiple correlation of changes from 1871-1881 and 1881 to 1891 revealed that changes in pauperism were due to changes in administrative policy, not external causes such as growth of population or economic changes.

It is interesting to note that despite the theory advanced on non-normal distributions, Edgeworth in his discussion of Yule's presentation to the Royal Statistical

Society in 1899 worried about the lack of compliance with the laws of error: "Perhaps he went a little further than Mr. Yule in the importance which he attached to normality, but to him it appeared that if one diverged much from that rule, one was on an ocean without rudder or compass. This law of error was more universal perhaps than the law of gravity" (Yule 1899 p288). Edgeworth encouraged Yule to devote his attention to moral and social phenomena which fulfilled the law. Edgeworth's plea was made a few years <u>after</u> his own demonstration of the asymptotic normal distribution of means from non-normal samples and Yule's work on least-squares regression.

Yule attempted to explain change over time, with the use of cross-section data of the differences of one variable from one decade to the next. The first published application of correlation and regression to time series data was Karl Pearson's and Alice Lee's study on barometric pressure (Pearson 1897). Pearson and Lee noted that although barometric frequency curves were remarkably smooth when a large number of observations were dealt with, the distribution of frequency did not obey the normal law. Citing Yule's 1897 article, they argued that the coefficient of regression was still significant in the case of skewed correlation because it gave the slope of the line of closest fit to the locus of mean height of one meteorological station for successive heights at the other.

Pearson and Lee calculated coefficients of correlation and regression for pairs of stations that could be used to "predict the height of the barometer at one station from a knowledge of the contemporaneous height at a second" (p455). They were aware that there were several possible periodicities in the measurements taken daily at 9 o'clock in twenty stations in the British Isles over five to thirteen years. To ensure that mean and standard deviation were least effected by the periodicities they only included observations that covered an entire year. With regards to another possible cycle they wrote:

> It is hoped that the fraction of a monthly periodicity-if there be a lunar influence on the frequency- will not be very sensible, when the records deal with 65 to 170 lunar periods. Any long period, such as a 19-year period, would of course render less general a frequency distribution based on five to thirteen years of observation only. The possiblity of such a periodicity would render it very desirable to recalculate the frequency distributions for the same localities after, say, another ten years of observations. (Pearson and Lee 1897 p31)

The correlation coefficients calculated for various pairs of stations in the British Isles were high, between .75 and .98, but simultaneous correlation was not the most useful for forecasting. Cave-Browne-Cave a few years later lagged measurements at one station in each pair and used the criteria of greatest R squared to choose the most appropriate time interval between stations. Cave-Browne-Cave noted that the best interval was remarkably shorter in summer than in winter and split the data into equinox to equinox sets. She also correlated the daily rise and fall in barometric pressure at pairs of stations with different time intervals between the readings. In her article she asked for physical explanations of the time intervals that were determined to be the best, but she warned meteorologists that it could not necessarily be due to the average interval between the arrival of the same isobar at two stations since "correlation depends not upon the equality of pressure, but upon the proportionality of deviations from local means." (Cave-Browne-Cave 1904 p407)

The correlation coefficients calculated in the several meteorological studies at the turn of the century were taken as reliable values. The only problem usually perceived with applying correlation to this temporal phenomena were that of skewed frequency distributions and periodicities. Cave-Browne-Cave did notice a difference in the estimates of the parameters of the regression equations for two different decades. She had divided the measurements into these two separate samples because the observations were taken at different hours over the two decades. She doubted whether the diurnal variation could explain the whole difference in the estimates and speculated whether the remainder was due to variations of random sampling or indicative of a gradual change. With this one exception, however, progressive or secular change was not the problem in meteorological studies that it was in economic studies.

As can be seen in the table in Image 6 time presented to economists, as well as meteorologists, the question of intervals between cause and effect. Correlation with lagged values tackled this. For the economist, alone, however the applications of correlation begged the question of which of the several tides of time were to be correlated. Several of the earliest studies mention the different movements of a variable due to forces acting in different time frameworks. Among the movements described were:

seasonal movement within the year

rapid, irregular movements from year to year

oscillations of about ten-years corresponding to the "wave in trade"

slow, secular movement either non-periodic or periodic with a very long period

The earliest writers saw the time-correlation problem as isolating the different components and correlating only similar components of two or more variables. They soon recognized that the correlation coeffieients of unmanipulated observations only indicated a relationship between the secular changes. It was usually of greater interest to the economists to investigate correlation of short-term oscillations:

The Earliest Applications of Regression Analysis to Time Series Data

Image 6

Date:	Author:	Title:	Perceived Problem:	Solution:	Comments:
1897	K. Pearson A. Lee	"On the Distribution of Frequency (Variation and Correlation) of Barometric Heights at Divers Stations"	Skewed frequence distributions Possible irregularity due to periodicity	Yule (1897) had shown correlation still reliable. Selected observations to cover entire period	-X and Y variables mesaured same phenomana at same time at different locations -No trend
1899	G. U. Yule	"An Investigation into the causes of changes in Pauperism in England"	Skewed frequency distributions Causation of change	LS regression used Cross-section data of differences over a decade	-X causes, Y effect not measurements of same phenomena -Multiple regression
1901	R. H. Hooker	"Correlation of the Marriage Rate with Trade"	Compounding of several movements due to different causes Correlation only picks up trend not oscillations Marriage rate does not respond immediately to general prosperity	Deviations calculated from instantaneous average of all the observations in the period of which that moment is central point Determine lag of relationship by highest R^2	Population from which mean is calculated determined by one complete period
1902	J. P. Norton	<u>Statistical Studies in the New York Money Market</u>	Correlation of periodic movements not trends	Deviations calculated from "growth axis," an interpolated logarithmic curve	First correlation of financial date Used Regression Polygons
1904	F. E. Cave-Brown-Cave	"On the Influence of the Time Factor on the Correlation between Barometric Heights at Stations more than 1000 Miles Apart"	To forecast barometric pressure Correlation of changes in pressure Summer and winter different	Lag measurements from one station Use measurements of daily rise or fall (first differences) Calculate separate correlations	Dealt with structural change
1905	R. H. Hooker	"On the Correlation of Successive Observations Illustrated by Corn Prices"	To eliminate changing influences which affect the variables unequally	If oscillation periodic then caclulate deviations from instantaneous average If not periodic, correlate the differences between successive values	Explained that differences useful for correlating shorter rapid changes or where normal level of series not constant
1905	L. March	"Comparison Numérique de Courbes Statistiques"	To distinguish "des changement annuels, de changement polyannuel, des changements séculaires"	Correlate deviations from instantaneous average determined by graphical interpolation Correlate annual changes	Explained that all movements of greater or less rapidity due to different causes and generate different signs and magnitudes of correlation

> What we want is to separate the various movements so as to test which of them are correlated with any particular cause whose influence we may wish to ascertain. By the use of the higher mathematics it may often be possible to eliminate some of the movements of such variables, so as to test the correlation of others. What I wish to suggest here, however, is an elementary method of eliminating the general movement in the particular case- which is of frequent occurence- of phenomena exhibiting similar regular periodic movements, so as to enable us to correlate the oscillations. (Hooker 1901 p486)

Economists turned to statistics in search of "barometers" of trade cycles and correlations of short-term fluctuations. Unlike barometric pressure, many of the measurements of economic and commercial phenomena showed secular changes through "general movement" as well as random and periodic changes. Reginald Hooker's elementary method was to correlate deviations from an instantaneous average. Such an average was calculated as the average of all observations in an entire period (eg. nine years of the trade cycle) with each moment successively serving as a central point. It was the method suggested by Poynting, but Hooker went beyond visual confirmation to quantify correlation. Of more significance, he recognized that the foundation of this method of averaging was the periodic movement of a series.

The curve or line representing the successive instantaneous averages Hooker called a trend. The trend showed "the direction in which the variable is really moving when the oscillations are disregarded." (Hooker

1901 p486) John Norton also interested in correlating oscillations, suggested a similar method. The line from which he calculated deviations he labeled a growth axis.

When weekly observations of time series were connected on a graph the result was a complex polygon. Norton argued that the statistician's task was to "resolve the motion of the polygons into elements." An element was an "ideal influence which is at work in the curve in a certain motion correlate with time." The elements that Norton isolated were:

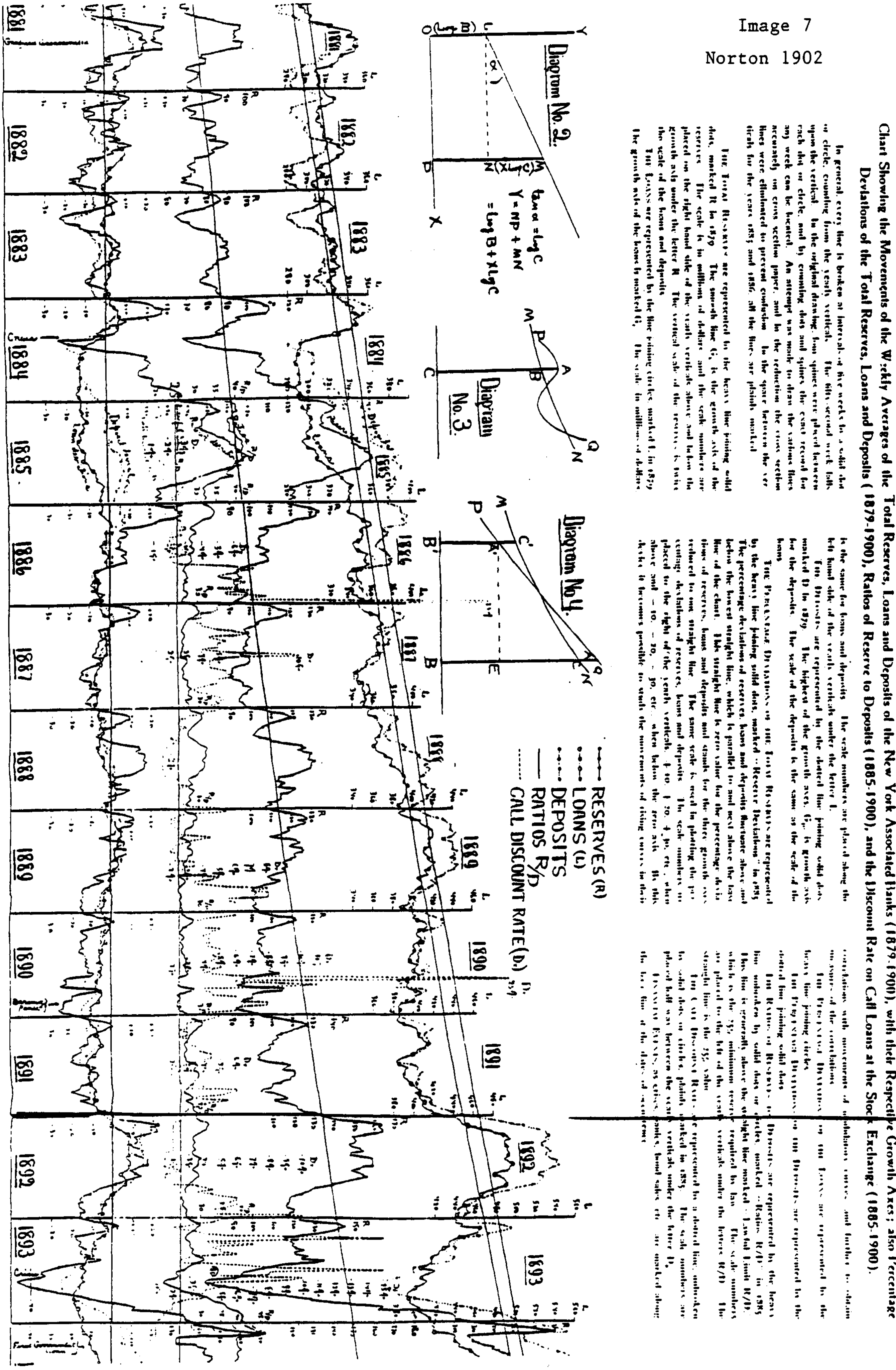
- The Growth Element due to a continuous passage of time
- Periodic Elements due to recurrent periods of time
- Dynamic Elements

To capture the growth element Norton graphically interpolated growth axes in each series (the ascending solid lines in Image 7). His primary interest, however, was in the percentage of deviations of reserves, loans and deposits from the respective growth axes and the correlation of these deviations:

> In this Chart are represented the really important movements in these financial statistics. The motion of the growth element is slow and gradual. Its effect is scarcely felt. But in the deviations are the movements which are forever puzzling financiers, and upon whose often apparently eccentric movements great fortunes are made or wrecked, panics are bred and crises precipitated. These deviations do not, it is hardly necessary to say, produce such serious calamities as crises; but they are the

Image 7

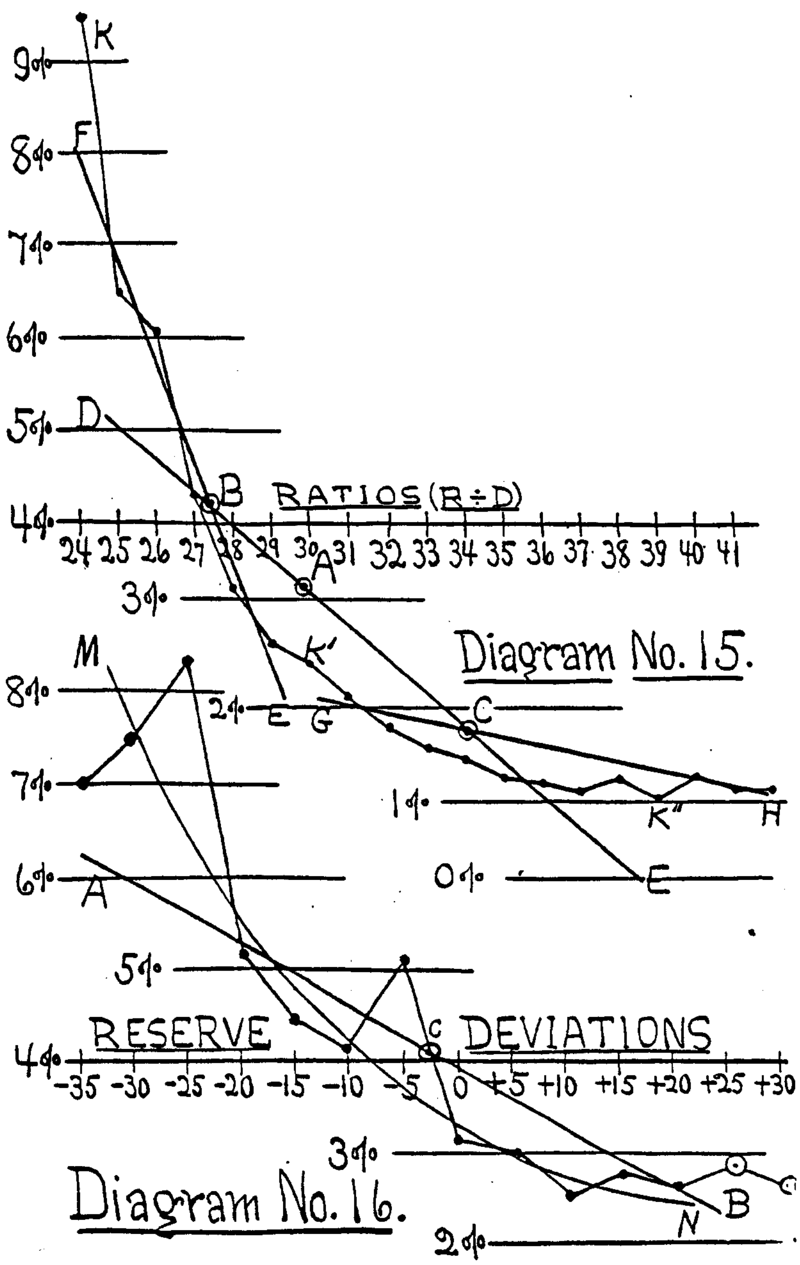
Norton 1902



> barometers of the state of that conglomeration of many tendencies in the societary circulation, working for good or for ill, that are in themselves prosperity or depression. Indeed, they may be made to form a measure of the severity of crises or of the affluence of periods of prosperity, as we shall later see. (Norton 1902 p36)

It is interesting to note that in fitting lines of regression to the deviations from the growth axis, Norton used the biometrician's method of regression polygons rather than Yule's method of least squares. This enabled Norton to easily determine when a unique line was not a good fit and to break his data into sets with narrower ranges and to fit separate lines for each set. Thus, an interpolated curve through the entire range could be approximated with lines of regression. This is illustrated in his Diagram no. 15 in Image 8 of the ratio of reserves to deposits. Through the several points forming the polygon, Norton fitted two separate regression lines and compared that to one line fitted to the entire data set.

Nikolai Kondratieff in 1925 (english version 1935) applied a combination of some of these techniques to test his theory of "long cycles" in the development of capitalism. He used least squares to fit trend lines to per capita data of several time series. He then smoothed deviations from the trend using a nine-year moving average. His purpose was to eliminate all movements other than long waves of approximate 50-year periods. Kondratieff treated these long cycles as empirical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phenomena with no direct causes but rather manifestations of capitalist development.

The method of correlating deviations from an instantaneous or moving average was recognized as only being applicable if there was a periodic movement in a variable. Lucien March (1905) and Reginald Hooker (1905) independently suggested that correlation of first differences could be used to capture the correlation of short-term changes in cases of non-periodic variables.

A few years later the correlation of differences was modified to the variate-difference method. Oscar Anderson and "Student" (William Gossen) in articles in the 1914 issue of Biometrika argued that correlating first differences was only valid when the connection between the variables and time was linear. They suggested that the variables be differenced until the correlation of the nth difference of X and Y was the same as the correlation of the (n+1) difference. The effect of this would be "to eliminate variability due to position in time or space and to determine whether there is any correlation between the residual variations." (Student 1914 p180)

In a review of the variate-difference method, Yule pointed out the large gap between Hooker, March, et al. and Anderson and Student. He argued that the view of the earlier writers was that the essential difficulty of correlation of time series was:

isolating for study different components in the total movement of each variable... They are wavelike movements, movements which can be readily represented with a fair degree of accuracy over a moderate number of years by a series of harmonic terms but which cannot be represented in the same way, for example, by a polynomial....the problem of time-correlation may be said to be the isolation, for separate study, of oscillations of differing durations. Most writers up to 1914- indeed all writers so far as I am aware- seem to be agreed on this. (Yule 1921 p501)

In contrast, the advocates of the variate-difference method saw the problem as "spurious correlation" due to variables being functions of time. Their solution was to eliminate all components which were functions of time and to correlate the serially independent residuals. Yule questioned the usefulness of isolating random residuals and demonstrated that in fact the variate-difference method tends to give correlations due to two-year oscillations.

Yule stands out as one of the few that philosophicaly struggled with the blending of the law of error and laws of motion. His first attempt to analyse change was through cause and effect. Changes in the proportion, of out-relief "probably" caused changes in pauperism. He first induced this from his correlation of measurements of ratios from different provinces on one day. Even in this context the frequency distributions were skewed.

Yule's second step was to correlate the "tendencies" by calculating the decennial change in each ratio in each province. Within a few years he was describing the solution of the time-correlation problem as one of decomposition. For Yule, however, it still remained a "problem" and at times a source of nonsensical results.

In a profound way Yule demonstrated the source of the nonsense in an article in 1926. He took the extreme tract, to demonstrate how we could get a correlation of -1 or +1 if there was zero correlation between two time series. His model was one of secular, but periodic change. The samples drawn were from only a fraction of the period of the series. Yule calculated correlation coefficients from various simultaneous segments of two variables that were simple harmonic functions of time of the same period, but differing by a quarter period in phase. He then constructed a frequency distribution of correlations from random samples of such segments (Image 9).

The frequency distributions of the correlation coefficients for all segments that were less than the entire period were u-shaped. If the correlation coefficient had been calculated over a whole period or any number of whole periods the value would be zero. Correlating segments less than a whole period yielded +1 and -1 as the most frequent calculations: "the distribution always remains u-shaped, and the values of the correlation as far as possible removed from the true

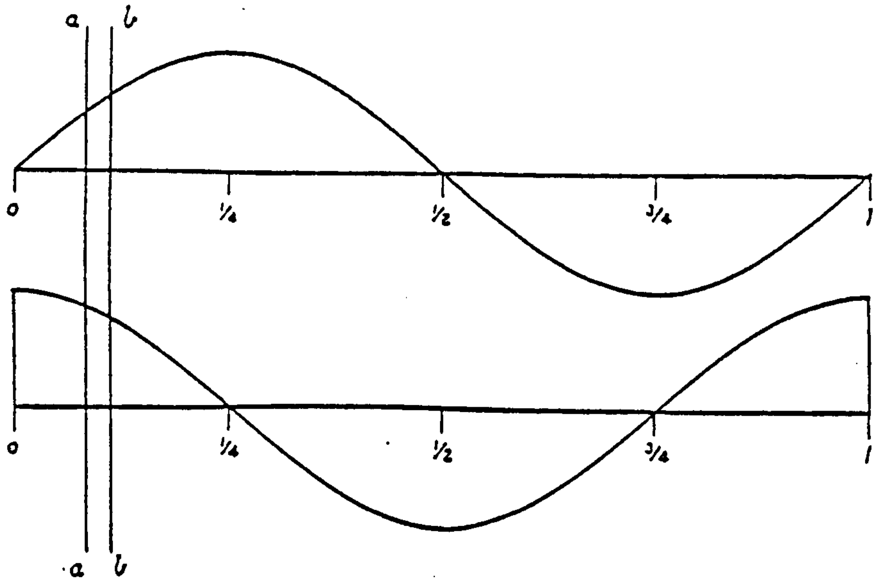


FIG. 2.—Two sine curves differing by a quarter-period in phase, and consequently uncorrelated when the correlation is taken over a whole period.

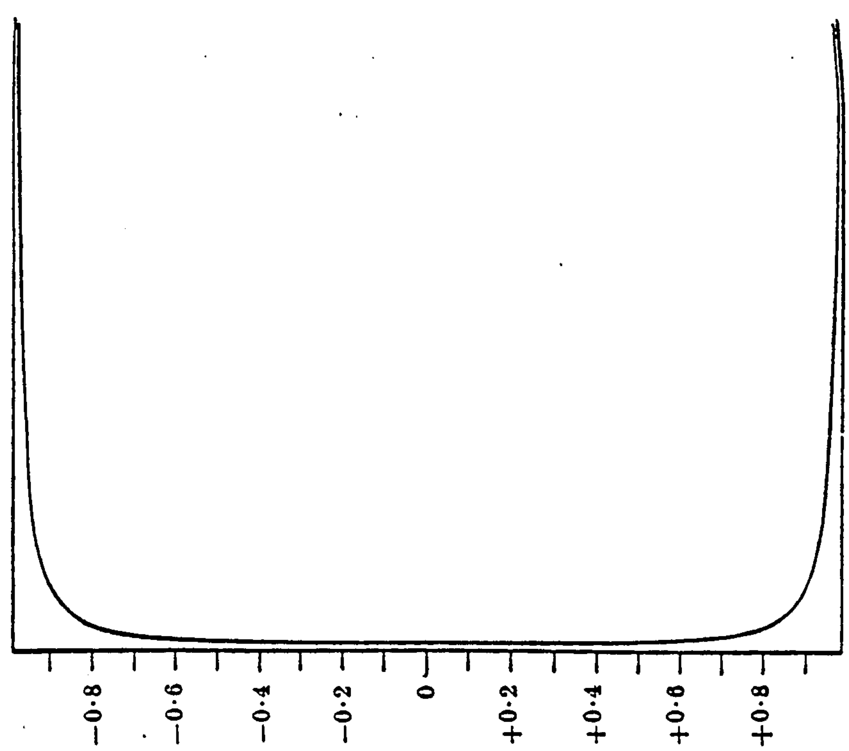


FIG. 5.—Frequency-distribution of correlations between simultaneous elements of the harmonic curves of Fig. 2, when the length of the element is 0·1 of the period. The following Figs. 6 to 9 show the change of form as the length of the element is increased from 0·1 to 0·9 by steps of 0·2.

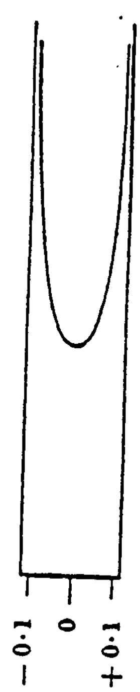


FIG. 9.—*Cf.* Fig. 5. Length of element, 0·9 of the period.

Yule 1926
Image 9

values (zero) always remain the most frequent" (Yule 1926 p10).

In searching for the source of this nonsense, Yule summarized the assumptions made in calculating statistical ratios such as standard error:

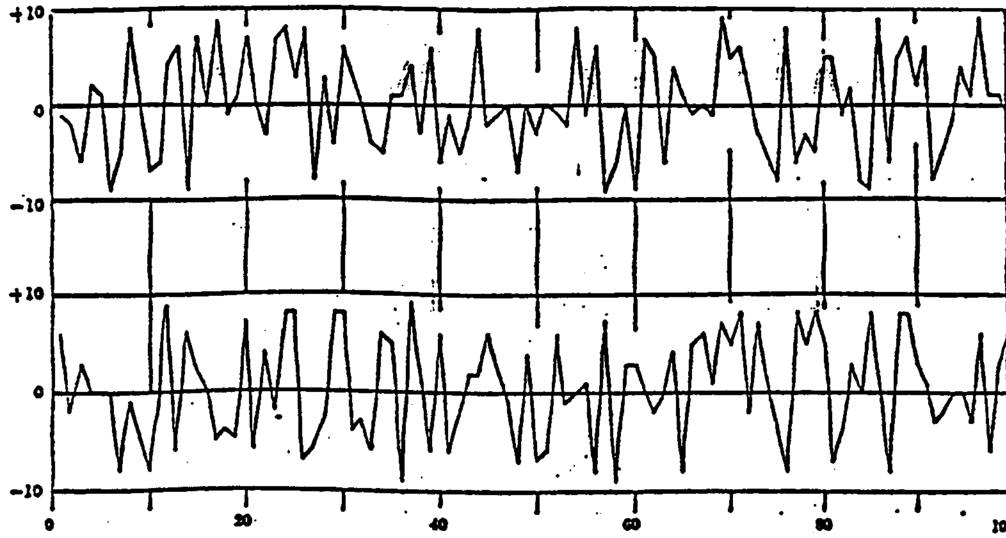
> (1) that we are drawing throughout from the same aggregate and not taking one sample from one aggregate, a second sample from another aggregate, and so on; (2) that every card in each sample is also drawn from the same aggregate, in such a way that the 1st, 2nd,...nth cards in any sample are each equally likely to be drawn from any part of the aggregate, not the first card from one batch, the second from another, and so on; (3) that the magnitude of x drawn on, say, the second card of the sample is quite independent of that on the first card, and so on for all other pairs in the sample; and similarly for y; there must be no tendency for a high value of x on the first card drawn to imply that the value of x on the second card will also probably be high; (4) in order to reduce the formula to the very simple form given, we have also to make certain assumptions as to the form of the frequency-distribution in the correlation table for the aggregate form which the samples are taken. (Yule 1926 p5)

For Yule time series posed the most serious threat to assumptions two and three. The problem was one of sampling. In his demonstration the most frequent segments that could be chosen were ones that would manifest a trend in the sample series, for example the segment between a & b in Image 9:

> We tend- it suggests- to get "nonsense-correlations" between time-series, in some cases, because some time-series are in some way analogous to the harmonic series that we have

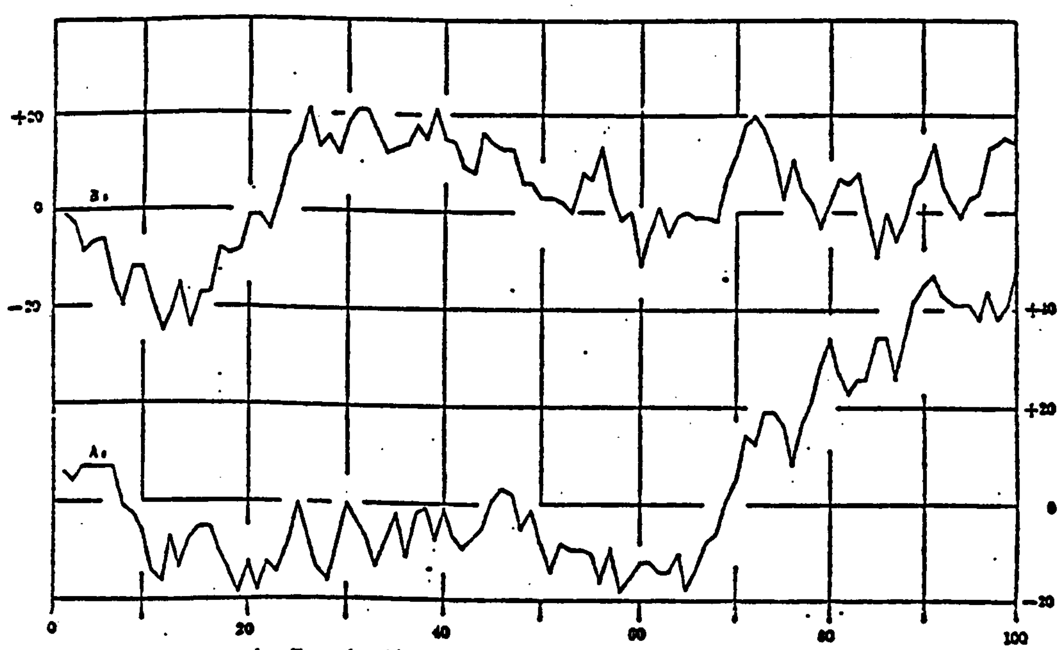
> taken as illustration, and our available samples must be regarded as very small samples, if not practically infinitesimal, when compared with the length required to give the true correlation...
>
> The phenomenon is clearly related to the fact that a small segment of a sine-curve, taken at random, tends to be either rising or falling, not more or less level, and consequently tends to give high correlations of either sign with other segments taken at random. (Yule 1926 p12, 14)

Yule went on to demonstrate that the most serious problems are with series for which all the serial correlations are positive and whose differences are also positively correlated. The tools of probability analysis applied to random series (Image 10a). Yule remarked that graphs constructed from experimental random series "are not, to the eye at least, very unlike graphs of some annual averages in meteorlogical data" (p26). In economics the most common series were either conjunct (serially correlated) series with random differences (Image 10b) or conjunct series, the differences of which are themselves conjunct series (Image 10c). The latter type that showed secular movement with correlated differences yielded the most nonsensical results when correlation analysis was applied. Samples from conjunct series with random differences gave a widely dispersed distribution of correlations; samples from positively correlated series with positively correlated differences gave a completely u-shaped distribution with over one-third of the correlations exceeding $\pm.9$.



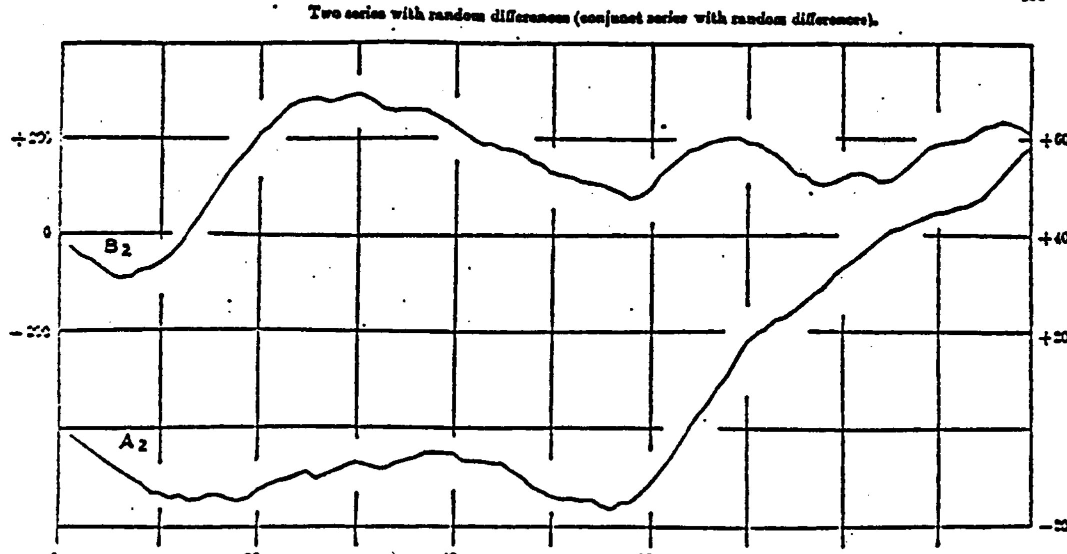
Two random series.

A



Two series with random differences (conjunct series with random differences).

B



Two series with positively correlated differences (conjunct series with conjunct differences).

C

Yule 1926
Image 10

The perspective of the advocates of the variate-difference method was that the time-correlation problem was that time itself was a causal factor; the variables were functions of time and thus correlated with time. Yule did not find this description intellectually satisfying; he did not regard time per se as a causal factor:

> We see, in fact, that conjunct series with conjunct differences are not necessarily correlated with the time, so the phrase criticized is at least inexact. But, successive differences being correlated with each other, there is a tendency for the curve to rise or fall consistently over more or less prolonged periods; there is a greater or less degree of continuity with the time over short samples. This is, I think, the only sense in which the "common influence of the time factor" can be held responsible. (Yule 1926 p40)

In this 1926 article Yule, for the first time introduced the concept of serial correlation. It was in the context of his experiments to demonstrate the conditions for nonsensical results. His conclusion was that serial correlation of the difference series were the most important factor. One year later Yule used serial correlation to analyse Wolfer's sunspot numbers. These measurements obviously displayed periodcity, but periodogram analysis did not yield satisfactory results.

Periodograms had been used from 1898 to analyse time series in economic and terrestial phenomena. This analysis was based on the criteria of least-squares and worked well on series that were simple harmonic functions of time even if they contained small random errors. Few economic time series, however, had periodcities masked soley by random "superposed fluctuations." Yule argued that the motion was affected not just by superposed fluctuations but by "true disturbances." The existance of disturbances ensured that unpredictability rapidly increased with time.

To estimate the parameters of motion, Yule suggested linear regression on $U_x = (2 \cos a) U_{x-1} - U_{x-2} + e$, where e was an error varying with the disturbance. In his experiment "the errors or disturbances e being given by dice throwing", Yule demonstrated the inadequacies of the periodogram in analysing disturbed periodic movement:

> And I would like in conclusion to suggest that many series which have been or might be subjected to periodogram analysis may be subject to "disturbance" in the sense in which the term is here used, and that this may possibly be the source of some rather odd results which have been reached. Disturbance will always rise if the value of the variable is affected by external circumstance and the oscillatory variation with time is wholly or partly self-determined, owing to the value of the variable at any one time being a function of the immediately preceding values. (Yule 1927 p417)

Yules regression equation of U_x on U_{x-1} and U_{x-2} for Wolfer's numbers did not yield totally satisfactory results, but it did perform much better than the periodogram. It was a landmark in statistical analysis for several reasons:

Yule's work became the foundation of modern time series analysis.

It was one of the first formulations, along with that in Eugen Slutsky's "The Summation of Random Causes as the Source of Cyclic Processes" (1927) of stochastic processes.

Yule focused on the random properties of the error term rather than of the series itself.

Serial correlation was reminiscent of the very first application of statistical analysis to investigate correlation of the same variable measured on successive generations.

These several accomplishments are hinted at in a quote from the 1926 article:

> In a mathematical series any term Us is some definite mathematical function of s, and has precise and definite mathematical relations to the terms that precede and the terms that follow. In a statistical series Us is no longer a definite mathematical function of s, and no longer has a precise and definite relations to the terms that precede and follow it. I have suggested replacing, as we usually have to do in statistics, the conception of mathematical functionality by the conception of correlation, and thus specifying the characteristics of the series by its serial correlations. Apart from its application to the theory of sampling in time-series, such a specification is of interest in itself as a method of analysis. (Yule 1926 p41)

With regards to contemporary mathematical vocabulary, I think the term sequence rather than series is the most appopriate. Yule essentially specified a form for a stochastic process: a stochastic sequence. The observations of the variable did not comprise a sample from a unique population, they were successive terms in a sequence. The disturbance term was the dye and an element of a statistical population.

Yules variables were not functions of time; they were functions of previous terms in the sequence. It was as if Yule had taken hundreds of generations of one family of sweet peas. He was correlating measurements from parent and offspring, but the observations were not across a species at one moment in time. The offspring of the one correlated triad was the parent in the successive group and the grandparent in the next. Variable population and the law of error were only manifest in the disturbance to the sequence. The imperfectability of the co-relationship was not due to the pull of the ancestral mean but to the influence of chance.

Conclusion

Yule's technique of serial correlation, and his replacement of a mathematical function with a correlation concept of sequence subject to random disturbances, had potential for resolving laws of error and laws of motion. Political economists from the mid nineteeth century had been searching for ways to analyse temporal variation. The mere assumption of change or flow was not sufficient.

The development of the analytical tools of logical variation held out promise for such analysis. Applications of the science of means to political economy and time series data, however, raised fundamental problems that are still with us.

Under the influence of the historical antithesis, economists set sights on the Mecca of biology. Ironically it was the tools of the biometricians not the organic models of growth, form and evolution that set the course. Regression and correlation analysis combined with refined commercial practices and an emphasis on differences as change and series as samples became the foundations for econometrics.

It is also ironic that the key problem in this econometric practice was the undirectional, evolutionary character of secular change. Time posed minor, surmountable problems if change were of a random or short-term periodic nature. In fact changes, in the form of first differences in magnitudes or in the form of a period of one cycle, became a new basis for using the science of means in index numbers and moving averages. Directional, qualitative change, however, begged the questions "what is an average?", "what is a statistical population?".

Within the context of a science of means, deviation from the mean has a special significance. If deviation is to be more than just an empirical calculation, i.e. it is to be used for inference, then the observations from which it is calculated should be related within a singular statistical population. As Lexis argued, there is often the possibility that observations in an economic time series come from more than one population.

Resolutions to this problem took several forms. Several social scientists manipulated the raw data to form series of differences or of deviations from a trend. This was consistent with their focus on short-term flucuations. Yule introduced the algorithm of least squares to fitting a regression line; this at least circumvented the problem of non-normal frequency distributions. It also brought attention to the role of residuals in stochastic relationships.

The most significant attempt to resolve the conflicting criteria of statistical inference and characteristics of economic time series, however, was the formulation of stochastic processes and the generation of errors in equations. These were introduced together in Yule's treatment of serial correlation. This had several points in common with that of Galton's and Pearson's specification of the process of inheritance. The biometricians estimated the relationship of an organ's dimension with that of previous generations. In essence they were regressing Yt on Yt-1, Yt-2,.... They used cross-section samples of a species in each generation as their data. This was consistent with the theory of natural selection that difference creates the conditions for change; logical variation is necessary for evolutionary variation. Yule, however, used one familial lineage as his data. In the economists' case the early applications of statistical method were not to test theories of processes of change but to address commercial and policy matters and to some extent to predict future courses by mapping motion.

Another key difference is that the intervals between Yt and Yt-1 are clearly determined in the biological case by the physical separateness of each organism and thus each life cycle. In economic data, however, intervals between observations of time series are not forgone conclusions. They are one of the most important choices made in empirical work. Although Jevons experimented with different intervals for the same variable, little attention was given to the link between the specification of intervals and the specification of models.

The importance of intervals became even more obscure with the change in focus from the statistical properties of the series to those of the residuals. In an extension of the technique of regression polygons, Yule applied the criteria of least squares to fitting lines of regression. Several years later he introduced the concept and notation of the error term to linear multivariate relationships. What these three have in common is the decomposition of the data into separate samples with separate distributions and separate means for each value of the explanatory variable. Thus the entire series of a variable loses its status as a sample from one population. This put to rest some of the questions of the applicality of statistical inference to economic time series data. It also, however, locked econometrics into an errors-in-equations mode. This was to be the format by which a science of means would be used to reveal the laws of social motion.

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Chapter 5

Conclusion

"We shall not cease our exploration and the end of all our exploring shall be to arrive where we started and to know the place for the first time." (T.S. Elliot "Little Gidding" Four Quartets)

Summary

Alfred Marshall's quest was to discover the One in the Many, the Many in the One. It is connected through two centuries to the goal of "enlightened" scholars to observe uniformity amid variety. The science of observation started from a recognition of population and mass phenomena bound by the equality of the individual members. With comparative measurements of populations came the discovery of the remarkable stability of ratios. A significant step was taken with the calculations of parameters that tended, as observations increased, toward a constant truth and a revealed order. A science of means developed with the acknowledgment that differences within a population were as analytically significant as the typical to which they were compared. The idea that difference was a necessary condition for change established the potential for statistical analysis of motion.

Political Arithmetic is an appropriate beginning to such a history. John Graunt and William Petty stated their goal as the foundation of a science based on measurement of social phenomena. Both made references to the stimulation from Francis Bacon. Petty was a founding member of the Royal Society and Graunt was invited to join after the publication of his observations on the Bills of Mortality. In keeping with the founding spirit of the Royal Society, both directed their work to the sovereign. Petty was employed by the State and most of his political arithmetic was on matters of fiscal or cameral policy. Graunt doubted that his observations would be of interest or use to those not concerned with governing.

Graunt and Petty took as their starting point the human populace of the kingdom. Birth and death were common to all, and the plagues reinforced the experience of mass phenomena. Graunt searched for patterns revealed in the numbers posted weekly to warn the wealthy of epidemics. Petty not only equated one person to another but even land and people through calculation of the monetary value of each. He considered the latter equation the most important consideration in "political oeconomies" because it was a condition for allocation of the proportionate and thus fair share of the tax burden. Both men drew many conclusions from their data, but one common aim was to estimate the population of the county or various regions.

In their estimation, however, they did not treat their data as a sample, nor did they explicitly search for summary parameters to measure a central tendency or dispersion of a distribution. Graunt's work, however. did lay a foundation for statistical analysis with his treatment of ratios. Out of the total number of deaths, he looked at those due to different causes. He even calculated a "chance" of death due to one cause as the 20-year ratio of deaths from that cause to total death. The most significant observation Graunt made was on the stability of some of the ratios over several years.

Graunt did not recognize that an increase in stability comes with an increase in sample size. His use of ratios, however and his acknowledgement of the regularity of death, inspired Edmund Halley in 1693 to make an "Estimate of the Degrees of Mortality of Mankind" and to utilize it for one of the first formulations of life insurance. From the beginning, state and money were linked to the quantitative concept of population. Theoretical political economy, however, had little interaction with statistical method until two centuries later when the impetus from the historical school focused study on national populations and state policy.

John Graunt's observations inspired Johann Peter Sussmilch's work on population. That in turn inspired Thomas Malthus and the latter was a source of inspiration to Charles Darwin. Parish registers had been kept for centuries, however, Graunt, according to Sussmilch, was the first to discover in them the Divine Order. The ratios that Graunt observed were for the most part relatively stable ones. Although his observations were over a considerable length of time, the instability that Lexis and others noted in "moral" and financial time series was not a problem.

Graunt saw order and stability in "Chronical Diseases" but dismissed the possibility in most cases of accidents. It was the remarkable pattern of order observed in the accidents of games of chance that provided a foundation of mathematical reasoning for a Science of Observation. Graunt calculated ratios and even made references to the "chance" of death by cause. His analysis, however, lacked the measurement of uncertainty that was to come from studies of games of chance. The dice or deck of cards served as an experiment that could be repeated many times. The equal chances of being drawn or thrown were not variable with the environments in which the experiments were made. The outcomes were discreet and easily measured by frequency ratios. Although each face of the dye has an equal chance of being thrown, the numerical combinations and permutations possible had different relative frequencies that were revealed by many plays and confirmed by mathematical reasoning. Experience, experiment, and deductive reasoning were in consensus in the analogy of the games of chance.

Subjective expectation and mathematical expectation were one, if the former was based on a large number of trials. That condition became the basis for attempts to measure probability a posteriori. Experiments with games of chance showed that the accuracy of observed ratios increased with the number of observed trials. The analogy of chance was used to analyse effects due to accidental or a multiplicity of causes. No functional relationship could appropriately link cause and effect in such cases nor could any analytical image fit sequential variation of the value under study. The remarkable feature of outcomes due to chance or accidental causes is the simple, archetypical analytical image revealed in a comparison of different frequencies of outcomes if a large number are observed.

A major step was taken in the development of this image by Abraham De Moivre: In 1730 he printed the first formulation of the probability integral that is the basis for the normal curve. With this he demonstrated that accuracy increases inversely as the square root of the number of experiences and he gave a means for determining the probable deviation of a statistical ratio.

The strongest link between calculations of play and scientific analysis, was the development of the analytical image of logical variation in studies of errors in measurement. Galileo and others in the seventeenth century had reasoned that erroneous measurements would be symmetrically grouped about the true value. The distribution of the measurements were as archer's marks around a bull's eye. The further away from the center, the less frequent the marks. The center itself was not necessarily one of the many actual measurements made, but it was the true value revealed by the relationship of all the measurements. Thomas Simpson in 1757 demonstrated, using the analogy of dice, that the mean measurement was preferable to any single observation made. Friedrich Gauss and Pierre Laplace in the early 1800's clearly defined the exponential curve of errors and developed the theories of least squares and the combination of observations.

Although Gauss' equation for the exponential curve of the frequency distribution of errors was the same as that used for the probability integral of De Moivre, it was the former whose name was appended to the curve. Two possible reasons for this are that the study of observational errors involved a posteriori reasoning and was thus more general. Also, from very simple axioms Gauss went directly to the equation for the normal curve without resorting to approximations of the probability integral. With mathematical reasoning, rather than probabilistic calculation, a law was revealed. From the relationship of observations a true measurement, the mean, was derived. Arithmetic and the Rule of Three were replaced with calculus and the Law of Error.

Adolphe Quetelet applied the law of errors in a grand manner to mass social phenomena. Frequency distributions of heights, crimes, suicides, births, deaths, etc. yielded a visual display of quantitative observations remarkably similar to that of the distribution of measurements of one object. For Quetelet, l'homme moyen was the social centre of qravity. If there was secular change in social history it would be revealed in the changing quantitative characteristics of l'homme moyen. Progress was manifested in the narrowing of the dispersion about the social, arithmetic mean.

The idea of statistical population reached its full maturity with the investigations of Charles Darwin's and Russell Wallace's theories of evolution. In the reasoning on the games of chance, errors in measurement, even in Quetelet's social physics, the mean was a true value. The search was for the essence. Deviation from the mean, although measured, was at best of no analytical merit or at worst indicative of error and imperfection. For De Moivre, Sussmilch, Quetelet and others of their time, the mean was the Original Design of the Deity. The stability of statistical ratios was evidence of this Design veiled by irregularities. The Darwinian theory of evolution of a species gave prominence to the variation within a population. The unit of analysis was the species- which for Darwin was a population of similar but non-identical individuals. The differences were of the nature of random variations. Environmental exigencies forced a natural selection of the most favorable characteristics. Natural selection, inheritance and isolation of sub-populations gradually led to the changing, extinction and origin of species.

The writings of Darwin and Wallace did not in themselves apply the analytical image of the law of errors to variation across a species. There is little treatment in their theories of a mean or typical individual. Variation across a population was the beauty of the system. Perfection, as near as possible, was achieved, not by a narrowing of the range, but by the persistence of differences. Evolution was a mass phenomena. Individuals could not evolve, but differences between them were a condition for the process of change. The focus on variation lent itself to quantitative analysis. Many characteristics of organisms could be measured continuously or classified into discrete attributes. The biometricians took on the task of fitting quantitative observations to theories of evolution and inheritance. In the studies of Francis Galton, Ralph Weldon, and Karl Pearson, the frequency distributions of continuously variable measurements on members of a species were often the same pattern as the law of errors. It occured so often that Pearson called the frequency distribution, the "normal" distribution. In the few cases of bimodal distributions, the biometricians considered themselves fortunate witnesses of a key moment in the evolution of species- the moment of bifurcation in phylogency- the origin of a species.

A frequency distribution taken at one moment in time was a static comparison of differences in a species. Within each generation there was variation displayed as a normal curve or law of errors. The dynamics of inheritance were searched for in a frequency surface. Galton first witnessed a normal surface in the spatial relations of the joint frequencies of diameters of parent and off-spring pea seeds. This pattern was repeated in many cases of the paired measurements of one organ from two generations of one family. The mean coordinates were paired to establish a center of gravity of the system. To the biometricians this was the ancestral mean to which the next generation regressed. Darwin had spoken of the "Greater or lesser forces of reversion and inheritance." Correlation was the mathematical measurement of heredity. A correlation coefficient of less than one indicated that inheritance was not perfect, there was some regression to the ancestral mean.

Within this context the mean and deviation from it took on new significance. The mean was not a true objective value amidst errors and imperfection. It was a typical value, one of several possible summary parameters. In the context of correlation of measurements of the same variable in two different generations, it represented the systematic pull of the species counteracting the inheritance of the family. If data could be collected over many generations significant changes in mean values and standard deviations would reveal secular natural selection.

The concept of evolutionary change and the statistical techniques used to investigate it captured the imagination of scholars in other natural and social sciences. Political economy was no exception and statistical method was applied to analyse sequential variation. The abstract method of political economy did not, however, lend itself to statistical analysis. There was some focus on mass phenomena. For example, from the interaction of individual self interest came the harmony of the whole metophorically described as the invisible hand. The classical method, however, took as its starting point universal human nature. It was much closer to the essentialist view rather than the populationist view.

The historical method was the antithetical reaction to the abstract method. The focus of the former on difference and change and on national populaces rather than human nature was a ripe environment for quantitative and statistical studies. The historical method emphasized induction, observation and measurement. The audience, as with Political Arithmetic, was often the state. The 1890 publications of Alfred Marshall and John Neville Keynes heralded a synthesis in method. For the classical economists, the natural price was the universal potential realized when the market was unfettered with monopoly and government protection. In Marshall's Principles, this was replaced with the concept of normal price realized in the long run when cause, with isolation and time, could determine effect. In this synthesis, political economy and statistical method finally shared a common vocabulary: normal, long run, variation, tendencies. There was imperfection, however, in the correspondence of these terms. The source of that imperfection was what Marshall saw as the chief source of difficulty in economics theory- time.

In the latter half of the nineteenth century statistical method was more readily applied to refine commercial practices than to test economic theories. The interest was in understanding laws of motion. In most cases the data took the form of time series. Bankers and merchants had developed rules of thumb to construct crude barometers from prices and measurements of production. The annual period or seasonal variation was grasped in comparing observations from the same months or weeks rather then with a sequential comparison. First differences or deviation from an average were often recognized as more useful than the absolute values. The moving average was used in quoting financial flows. Stanley Jevons, John Norton and others consciously strived to turn the tools of merchant's intuition into scientific practice. They grouped data into statistical populations and calculated not only means, but deviations from means. The new tools developed included the periodization of time series, index numbers and deviations calculated from moving averages.

By the turn of the century economists were combining these techniques with correlation and regression analysis. Meteorology and political economy were unique in that their data usually took the form of time series. From the earliest applications of correlation and regression to time series, problems were percieved. Skewed frequency distributions, non-random and successively dependent observations, time lags between cause and effect, and periodic and secular movement threatened the applicability of techniques grounded in logical variation and probability. Least squares regression replaced normal correlation and apparently solved the problem of skewness. Time lags were estimated with the use of non-simultaneous correlation. Short-term flucuations were correlated by creating new series of deviations from a trend or of first differences.

With the exception of the advocates of the variate-difference method, statisticians of the early 1900's saw the time-correlation problem as one of isolating the different temporal movements of a series. Analogies were made to the tides and even to the movement of the earth being both a revolution around the sun and a rotation on its axis. The primary interest was in correlating flucuations of short-periods. Correlation of most economic time-series, however, only captured relationships between secular changes or trends.

From various perspectives secular change was seen as the primary problem of correlation of time series. Udny Yule demonstrated that if the secular change is a periodic one, and if series do not cover the entire period, the results will usually be nonsense correlations. He suggested that it would be more useful to see observations of variables not as functions of time but as terms of mathematical sequence. The residuals are not deviations from a mathematical function of time but random disturbances in a correlated sequence. The most useful correlation is not between a variable and time but serial correlation.

Yule's introduction of an error term in the serial relationship became the population/probability window of econometrics. The elements of the series were neither random, independent nor from a normal distribution. The disturbance term was all of these. The analysis of errors in measurement was applied to errors in equations. Logical variation could be used to analyse sequential variation.

The End is Where We Start From

History can be a tool for rotating perspective and thus a source of conceptual resolution and development. This particular history has insights for econometrics that include alternative perspectives on:

Mean

Statistical Population

Analysis of Change

Mean

In the nineteenth century, statistical technique developed from a focus on a true, objective mean to a focus on deviation from the typical value. This development was analogous to the change in approach from that of the essentialists to populationists in Biology. In economic theory there was a corresponding development from the abstraction of universal nature to the assumption of tendencies amid irregular variation. Edgeworth called statistics the science of means and he argued it only became such when deviation from a mean had analytical significance. Edgeworth, Pearson, Yule and others recognized that the normal distribution or law of errors was not a necessary prerequisite to use of the science of means. Edgeworth, however, always stressed the difference between the mean as "moyen typique" and as a mere "indice". An arithmetic mean could be calculated among any numbers, but it was only a statistical mean if there was a relationship between those numbers such that they clustered about a mean.

In a 1925 article that reviewed the application of statistics to economic theory, Warren Persons explained the importance of Edgeworth's work on frequency distributions. He highlighted Edgeworth's conclusion in an 1885 article that the distribution of averages will be approximately normal even though the distributions of the original observations composing the averages are not from normal frequency distributions. Persons asserted:

> Although this theorem is not even mentioned in most current textbooks in economic statistics it provides the justification for applying the theory of probability to the problems of sampling and computation of means from economic data differing markedly from the normal type. (Persons 1925 p185)

Edgeworth, however, did not use his theorem to ignore the frequency distributions of variables. He always insisted that calculation of deviation from a mean assumed a "correlative" relationship among the observations that was manifested in a variation of density about the central tendency. Fourteen years after the 1885 article Persons referred to, Edgeworth cautioned Yule that "if one diverged much from that rule (the normal law), one was on an ocean without rudder or compass." (discussion on Yule 1899 p288)

There was a corresponding attempt to transcend the limitations of normal surfaces. The biometricians tools of correlation and regression could only be generalized to economic data if there was calculation suitable for non-normal multi-variate frequency surfaces. Yule took a major step in this direction with the application of least squares to the "swarm" of co-ordinates. Karl Pearson in his classic article on the history of correlation questioned whether Yule's generalization was the real line of future advance:

> Now while these methods are convenient or utile, we may gravely doubt whether they are more accurate theoretically than the assumption of a normal distribution. Are we not making a fetish of the method of least squares as others made a fetish of the normal distribution? For how shall we determine that we are getting a 'best fit' to our system by the method of least squares?...
>
> we can only assert that least square methods are theoretically accurate on the assumption that our observations of y for a given x obey the normal law of errors. That is the proof which Gauss gave of his method and I personally know no other...Hence in disregarding normal distributions and claiming great generality for our correlation by merely using the principle of least squares, we are really depriving that principle of the basis of its theoretical accuracy, and the apparent generalisation has been gained merely at the expense of theoretical validity. (Pearson 1920 p45)

I think we have been too hasty in accepting Edgeworth's and Yule's work as proof that we can ignore the frequency distribution of economic variables. Regression analysis is the king pin of econometrics because it focuses on the properties of the error term. Our series lack the prerequisites for applying a science of means, but the residuals can meet the criteria. The necessity to generate errors in equations has severely limited our use of other statistical techniques. Also, no matter what method of estimation we use in regression we are quantifying a relationship on the basis of deviation from the mean of each variable. The usual method of sampling time series renders deviation from a mean analytically insignificant because we ignore the importance of statistical population.

Statistical Population

Population is defined by both identity and difference. Political arithmetic and the calculus of probability were conditional upon the recognition of the similarities of individual people and of individual outcomes. The orders of equivalence and of chance laid the foundation for measurement and the stability of parameters constructed from measurements. A statistical science, however, also required the acknowledgement of variation within the limits of an order. The assumptions of true essence and natural value were not fertile ground for quantitative comparison. The generalization of the mean was only possible when significance was accorded to deviations from the mean.

What is an average? The question is rarely asked in our sophisticated statistical analyses of time series data. We usually seek as many observations as possible. The time intervals between observations are equal and our choice of interval duration is based on the exigencies of bookkeeping. The problems of time are perceived a posteriori: Do empirical results indicate we have multicollinearity, autocorrelation or structural change?

I suggest we take more into consideration in our a priori approach and define statistical populations before constructing samples. With time series data this requires acceptance of the principle of complementarity and of the necessity to choose interval duration. A series of observations united by a common name is not necessarily a sample from a singular statistical population. Also, the causes that influence the variation of a variable do not all act within the same time framework.

The principle of complementarity recognizes that for total comprehension of some phenomena, different explanations must be accepted even though analysis dictates their mutual exclusion. The usefulness of this principle in economic theory is its capacity for resolving contradictory time-bound hypotheses and paradigms. In econometrics this principle translates into model specification and the choice of observations or intervals determined by the time frame of reference. The goal of avoiding bias by including all significant variables is replaced by the attempt to achieve a correspondence of specificaitons of model, of statistical population and of duration between time series observations. Econometricians must be selective, not just of calendar time covered by the sample, but also of intervals between observations.

To accept the principle of complementarity is to accept the simultaneous existence of very different estimated models to explain variation in one variable. Jevons' studies of commercial fluctuations came close to this. He averaged not over an entire series, but over observations corresponding to one time period. The polygon connecting the twelve monthly averages was different from that connecting the quarterly averages. They called forth different stories of annual fluctuation of the same variable.

Regression polygons, as used by Galton, Pearson and Norton also hold out promise for complementary analysis. The construction of the polygons involves the separation of the observations of the dependent variable into samples corresponding to values of the explanatory variables. A visual examination of polygons for different interval specifications could serve as an exploratory tool. The contruction of the polygons is obviously based on logical variation, but it does not inevitably lead to the enforcement of logical time. That comes with the step of fitting a unique line of regression to the polygon and using the slope of that line to infer relationships of change over time.

The usual approach in using time series data is to average over the entire series at some point in the analysis. Wilhelm Lexis pointed out that this was satisfactory as an empirical description, but it lacked the foundation of probability in many time series. Lexis' concern was that the observations were from different populations. In the analogy of games of chance, the balls were drawn from different urns. Our a priori reasoning should distinguish the urns.

John Maynard Keynes (1921) in his attempt to go beyond the calculus of probability to explain its logic was critical of the necessity of assuming ignorance of the conditions of the material observed:

> Our state of knowledge about our material must be positive, not negative, before we can proceed to such definite conclusions as they purport to justify. To apply these methods to material, unanalysed in respect of the circumstances of its origin, and without reference to our general body of knowledge, merely on the basis of arithmetic and of those of the characteristics of our material with which the methods of descriptive statistics are competent to deal, can only lead to error and to delusion. (Keynes 1921 p384)

Our state of knowledge should include an historical, institutional and empirical comprehension of the possible populations into which our observations can be addressed. John Hicks (1979) argues that economics holds a key position because it is on the edge of the sciences and also on the edge of history. We can realize the full potential of this position by recognizing complementary statistical populations and complementary models in econometrics.

In practice, this means careful selection not just of observations, but also of intervals between observations.

These intervals may not be equal over the entire sample. For example, we may choose observations only at the turning points of a trade cycle or only when another variable surpasses a threshold. It will be necessary at some point to distinguish the procedures for forming complementary models or populations in the cases of flow variables measured over a period, stock variables, indicators such as price, unemployment rate, and variables differenced over a period.

There has been considerable debate over the problems of measurement without theory and theory without measurement. We should give attention to the problems of measurement without history. We do not have the clear-cut generation boundaries that the biometricians had to determine the intervals between Yt and Yt-1. Economists have a choice of intervals. Given our theoretical orientation to the concept of choice, we should not find it difficult to perceive this opportunity. In contrast to our assumptions in problems of resource allocation, however, acknowledgment of complementarity allows for the simultaneous explanatory powers of different histories.

Analysis of Change

The long-term and secular change had been in the scope of several classical economists, Karl Marx and the historical school. The early econometricians, however, were interested in the short-term irregular or periodic movements of economic variables. This may well be due to their attention to social and economic poilcy. As John Hicks (1979) has argued, economics is unique among the social sciences because it is specially concerned with the making of decisions and with the consequences that follow from decisions. As economics evolved into addressing the decisions of individuals, firms and states, the relevant time frame became shorter.

I think the focus on the short-term and its relevance to decision making has been confining. Secular change was recognized as the key problem in early econometric studies. We could, however, see secular change not as something we have to eliminate in our data but as something we should be searching for. With such an approach we may have to forsake probability analysis, statistical inference and forecasting. I do not, however, see secular change and measurement as mutually exclusive. Useful analytical images and generalizations can be made. Visual images and computer graphics would be particularly relevant to such analysis.

One conclusion of this history is that biology is a very appropriate Mecca for economics. Developmental variation over a life-cycle, evolutionary variation and even organic periodic variation are useful analogies to social change. With regards to periodic variation, econometricians have looked to the analogy of light rather than temperature. The soltices and equinoxes occur with fixed, predicative periodcity. The response of terrestial heat to light is not a perfect one. Variation in temperature is analogous to flow, and to Yule's concept of serial correlation. It is more useful to see temperature and many economic variables not as functions of time but as functions of previous values of the same.

The study of patterns in nature, of the relationship between growth and form, of evolutionary change, and of ecosystems serve as interesting analogies useful to the study of economics. The economy is a social and thus organic entity. To foresake laws of mechanical motion is not to relinquish hope of discovering order in change. Morphology is a stunning example of this.

A possible econometric project that could draw on various biological analogies is a comparison of the annual pattern revealed in the monthly or weekly data of Stanley Jevons ((1862) 1909), and John Norton (1902) with that revealed in present data. Norton and Jevons both saw the annual period as a natural one:

> Laws may change, theories may change, local changes of vast importance may come and pass, but the primary facts on which this period rests are not ruled by laws of men, nor modified by theories of reformers, nor escaped by local shifting. It rests on the facts of nature, on the temperature of hemispheres, on the very twistings of the earth upon its axis, and as long as summer follows winter and we obtain our foods from the soil of the earth by the growth processes of nature, somewhere ceaselessly responding will be found this banking rhythm. (Norton 1902 p59)

As economies have seemingly moved further away from a foundation of agriculture, how have the annual patterns of finance changed? What has been the evolution over more than a century of the response of the banking rhythm to the annual organic cycle?

There are several projects such as the above that are suggested by this history. Whatever the specifics of investigations, however, the pattern in the tapestry suggests some universal themes. An historical comprehension of statistical analysis yields tools for critical assessment of the logic of econometric techniques. Probability and population are conceptually bound, and to ignore the latter is to invalidate the former. The questions and the data of economics usually concern change and time, but our answers usually assume no change or at best locomotion. Logical time and logical variation are not synonomous. Logical variation is a very powerful analytical tool, but we must be aware of its historical rationale, the analogies on which it is based and thus its potential and its limitations for comprehending sequential variation.

Weaving and history of thought are processes of teleological construction. The venture into what has been is usually preceded by an uncomfortable fit with what is and a wondering of what ought to be. My hopes for a new methodology include: a liberation from the dominance of regression analysis and models in logical time, an analysis of change that bridges quantity and quality, an environment conducive to measurement with history. To change the way we understand change is a task that would surely require a near life cycle. In the hopes of quickening the wisdom that comes with age, I have turned to history.

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