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PROBES IN GASEOUS PLASMAS

by

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ABSTRACT

From the time of Langmuir measurements of the current-potential characteristic of a small electrode probe immersed in a partially ionised gas have been used for the quantitative study of plasmas. The plasma parameters of interest are electron and ion temperature, electron and ion concentration, and plasma potential.

The proposed probe theories may be divided into two parts depending on whether the ions and electrons come under the influence of the probe's potential in a collisionless plasma or a collision dominated plasma. In a collisionless plasma the motion of the charge carriers is described by a Free Fall theory while in a collision dominated plasma a Diffusion theory is needed. Various theories describing the behaviour of probes in these two regions are presented and their assumptions and their regions of validity discussed.

When a moving probe's velocity through a plasma is

comparable to the charge carriers' thermal motion modifications to the simple probe theory must be made. This is necessary for Space probes and the required modifications to Langmuir's theory for a collisionless plasma are made.

The theory of the Resonance probe is presented.

Perturbations to the current-potential characteristic occur if the probe disturbs the plasma or if instabilities are present in the plasma. A quantitative estimate of some of the disturbances is possible.

Static and dynamic measuring techniques for determining probe characteristics are discussed along with ideas on probe design and construction.

CONTENTS

CHAPTER 1

INTRODUCTION

In studying plasma kinetics one usually requires a knowledge of electron and ion temperatures, electron and ion concentrations, and plasma potential. These plasma parameters may be measured by a variety of techniques. Some of these techniques include optical and mass spectroscopy, microwave absorption, and electric probes.

This dissertation is concerned with the last of these techniques; the use of electric probes in partially ionized gases. The technique is based on the measurement of probe current as a function of probe potential.

The electric probe has the advantage of being able to measure local plasma parameters and is thus suitable for measuring the spatial distribution of the parameters. Providing certain precautions are taken information on the plasma may be gained using relatively simple measuring techniques.

Langmuir was the first to use the electric probe for the systematic study of low pressure plasmas. His analysis involved the idea of orbital motion of carriers in a space charge sheath surrounding the probe and it is applicable only if there are no collisions within this sheath. An electric probe used under these conditions is generally known as a Langmuir Probe. A difficulty in applying the theory is the necessity of knowing the thickness of the space charge sheath. Also, the theory does not satisfactorily explain the collection of positive ions.

More recent theories have overcome the failings in Langmuir's work and they have been extended to cover the case when carriers undergo numerous collisions once they have come under the influence of the probe's electric field.

Floating double probe techniques have extended the use of probes to the study of electrodeless plasmas. It can be shown that the single probe is just a special case of the double probe in which the area ratio of the two probes is very large.

A high frequency probe technique has been developed that enables carrier concentrations to be obtained form a measure of the increase in probe current when the plasma is in resonance with the probe circuit. The theoretical interpretation of high frequency probe characteristics is, at the moment, undergoing much scrutiny. It is now generally believed that the resonant increase in probe current does not occur at the plasma's electron resonant frequency.

On the experimental side, it is essential that a probe immersed in a plasma should produce as little disturbance to the plasma as possible. The effects of numerous types of disturbances have been analysed and one should always take them into account when assessing the accuracy of probe measurements. Techniques have been developed that enable complete probe characteristics to be determined in very short time intervals. This is useful when a knowledge of instantaneous plasma parameters is required in fluctuating plasmas.

Care must be taken in the interpretation of these instantaneous characteristics as they are only meaningful when the carriers have reached equilibrium with the probe's instantaneous electric field.

The following chapters are intended to give a systematic outline of the developments made in probe theory from the time of Langmuir up to the present date. The last chapter indicates a number of experimental techniques that are available for the practical measurement of probe characteristics.

CHAPTER 2

LANGMUIR PROBE THEORY.

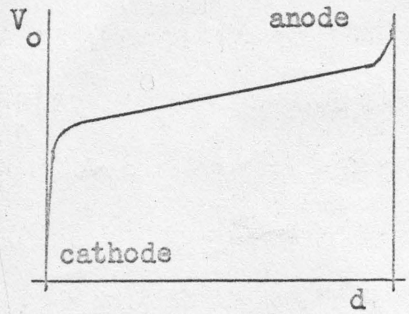
2.1. Introduction

2.1.1. The probe and plasma.

For the purposes of the following discussion a probe may be considered as a spherical, cylindrical or plane electrode whose supports and lead wires are completely insulated from the plasma in which it is immersed.

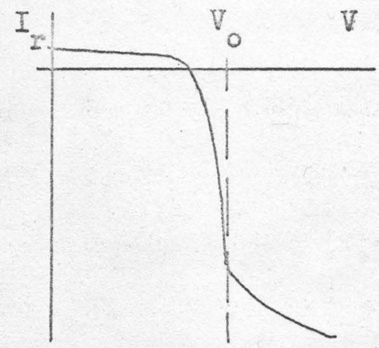
A plasma is taken to mean any partially ionized gas in which the concentration of negatively charged carriers is roughly equal to the concentration of positively charged carriers. Such a plasma is readily realizable in a hot or cold cathode discharge, an after-glow, and in ionized gas layers surrounding the Earth. The dimensions of the probe and its supports should be such as to produce negligible disturbing effects on the plasma being investigated.

In developing a qualitative theory of a probe immersed in a plasma it is convenient to consider one particular type of plasma in order that one may better visualize the physical problems involved. For this reason consider the behaviour of a probe when immersed in the positive column of a low pressure glow or arc discharge. Figure 1 shows the variation of plasma potential with position in such a discharge.



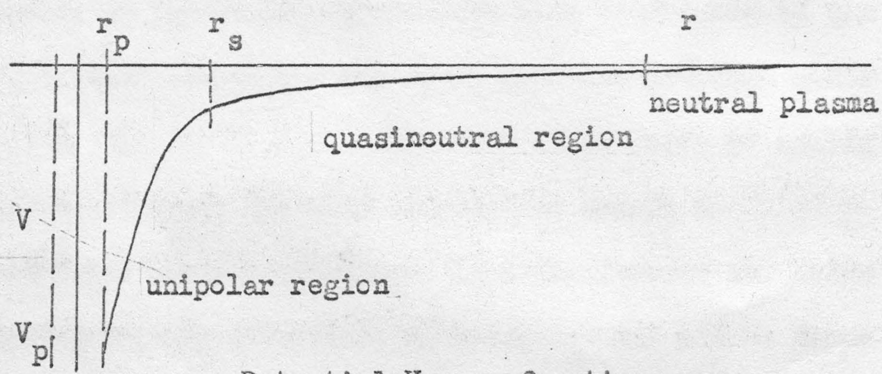
Plasma potential V_o as a function of distance d from the cathode of a glow discharge

Figure 1



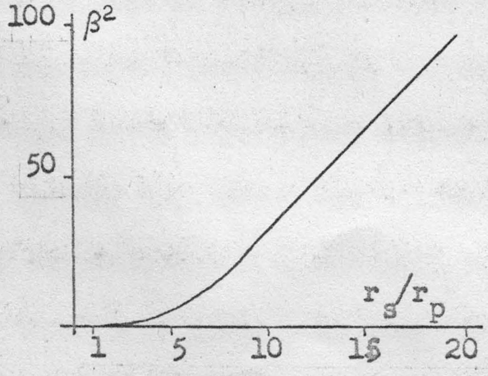
Probe current I_r as a function of probe potential V

Figure 2



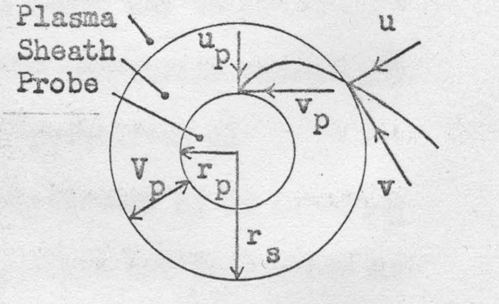
Potential V as a function of radial distance r from the centre of a cylindrical probe

Figure 3



β^2 as a function of r_s/r_p (4)

Figure 4



Carrier velocity components in sheath region of a cylindrical probe

Figure 5

2.1.2. The probe characteristic.

If a probe is placed in the positive column and connected externally to the cathode the potential of the probe will be V_o negative with respect to the plasma and a current will flow from the plasma to the probe. Under these conditions the probe will attract positive ions and repell electrons collecting only those electrons of energy greater than eV_o. If a battery is placed between the probe and cathode and the probe made less and less negative with respect to the plasma more and more electrons would reach the probe until a point is reached when the rate of arrival of electrons just equals the rate of arrival of positive ions. The probe potential at which this occurs is known as the Floating Potential V_f; it is usually a few volts below plasma potential and is the potential that an isolated probe would aquire if immersed in the plasma. When the probe is at a potential V_o positive with respect to the cathode the probe is at Plasma Potential and it neither attracts nor repells electrons or ions. The current flowing to the probe is just equal to the random current in the plasma. As the probe is made positive with respect to the plasma potential it attracts electrons and repells ions and the circuit current flows from the probe into the plasma. Figure 2 shows the variation of current with probe to cathode potential.

It is the aim of this dissertation to present a review of the theories that will exactly describe the characteristic shown in Figure 2 under various plasma conditions. It will then be possible to obtain information on the plasma from experimentally determined probe characteristics.

2.1.3. Sheath formation.

A negative probe (with respect to the plasma) will repel electrons and attract positive ions to form a positive ion space charge sheath. The positive charge in the sheath will just balance the negative charge on the probe and so the field surrounding the probe does not penetrate beyond the sheath edge.

The sheath surrounding a strongly negative probe may be divided roughly into two regions. The inner region next to the probe consists almost exclusively of positive ions with perhaps one or two very high energy electrons. Outside this region is an outer sheath in which ions and electrons are present in almost equal quantities but in which the normal conditions within the plasma have been modified due to the withdrawal of positive ions. The variation in potential with position from the surface of a probe is shown in Figure 3.

Most of the potential drop occurs across the inner sheath. In Langmuir's original probe theory no account was taken of the potential drop across the outer sheath. More recent theories have shown that this penetrating electric field is important when considering the collection of positive ions by a negative probe.

2.1.4. Basic assumptions.

The following assumptions were made by Langmuir (1) in developing his probe theory.

(a) Carrier densities are known at the sheath edge.

(b) Carrier velocity distributions are known at the sheath edge.

(c) All the probe potential is developed across the inner sheath.

(d) Gas pressures are sufficiently low for collisions within the sheath to be negligible.

(e) Carriers are neutralized on reaching the probe.

(f) Carriers are not reflected at the probe's surface.

(g) The edge effects of plane probes are neglected; cylindrical probes are assumed to be infinitely long compared with their diameter; the supports and lead wires to cylindrical and spherical probes are assumed to produce a negligible disturbance to the potential distribution in the surrounding plasma.

Assumption (d) implies that a carrier entering the sheath will be subject to the probe's field and will describe an orbit around the probe without undergoing collisions. The path of the orbit will depend on the potential drop across the sheath and on the velocity of the carrier on entering the sheath. There are three types of orbits: those that terminate at the probe's surface; closed elliptical orbits; hyperbolic orbits that pass close to the probe within the sheath. The closed elliptical orbits arise from collisions within the sheath. As the number of such collisions has been assumed to be negligibly small they will not be considered.

The current to the probe may be calculated from a consideration of the orbital motion of the carriers within the sheath.

Alternatively the current flowing from the sheath edge to the probe surface may be calculated by considering the sheath edge and the probe surface to be the two electrodes of a space charge current limited diode.

2.2. Diode equations

If a sharp boundary is assumed to exist between the sheath and the undisturbed plasma at a point r equals r_s in the case of a cylindrical and spherical probe, and at a point x equals x_s in the case of a plane probe, expressions for the probe current may be obtained in terms of the potential drop across the sheath. In deriving these expressions a knowledge of the carrier velocity distributions at the sheath edge is required.

2.2.1. Plane probe.

When the initial velocity of the electrons leaving the sheath edge is negligible in comparison with their final velocity at the positive probe the electron current density flowing to a plane positive probe is

$$j = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{1/2} \frac{V_p^{3/2}}{x_s^2} \qquad \ldots(2.1)$$

where e is the magnitude of the electronic charge, m is the electron mass, V_p is the probe to plasma potential, and x_s is the sheath thickness. Equation (2.1) was obtained by solving Poisson's equation and applying the boundary conditions that at

$x = x_s, \; V = 0$

$x = 0, \; V = V_p$

$x = x_s, \; \frac{dV}{dx} = 0$

If the velocity distribution of the electrons at the sheath edge corresponds to an electron temperature T_e °K the electron current density flowing to a plane positive probe is

$$j = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{1/2} \frac{(V_p - V_m)^{3/2}}{x_m^2}\left[1 + 2.66\left(\frac{kT_e}{e(V_p - V_m)}\right)^{1/2}\right]$$

where at $x = x_m, \; V = V_m$

$x = 0, \; V = V_p$

$x = x_m, \; \frac{dV}{dx} = 0$

The potential distribution inside the sheath region passes through a minimum value of $V_m < 0$ just inside the sheath edge at x equals x_m (2).

In deriving equations (2.1) and (2.2) it has been assumed that only electrons are present in the sheath and that all the positive ions have been repelled.

The principle use of these equations is in the estimation of the sheath thickness. Equation (2.2) should be used in preference to equation (2.1) as it is more reasonable to assume that the carriers are emitted from the sheath edge with a finite initial velocity rather than with zero initial velocity.

The assumptions and theory outlined above may be applied directly to the case of a negative probe and the collection of positive ions (3).

2.2.2. Cylindrical probe.

When the initial velocity of the electrons leaving the sheath edge is negligible in comparison with their final velocity at the positive probe the electron current density flowing to a cylindrical positive probe is

$$j = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{1/2} \frac{V_p^{3/2}}{r_p^2 \beta^2} \qquad \ldots(2.3)$$

where β is a function of (r_s/r_p) and has been tabulated by Langmuir and Blodgett (4) for values of (r_s/r_p) from one to infinity. r_s and r_p are the sheath and probe radius respectively. Figure 4 shows the variation of β^2 with (r_s/r_p) for values of (r_s/r_p) from 1 to 20.

The most probable energy of an electron may be expressed in electron volts defined by the relation

$$eV_e = kT_e \qquad \ldots(2.4)$$

where k is Boltzmann's Constant and T_e is the electron temperature in degrees Kelvin. Langmuir and Blodgett (4) have considered the effects of a tangential velocity component at the sheath edge. They show that if $eV_{e,t}$ is the electron's energy component tangential to the sheath edge and if V_p is the probe to plasma potential the electron cannot reach the probe if

$$V_{e,t} > V_p\left(\frac{r_p}{r_s}\right)^2 \qquad \ldots(2.5)$$

The application of equation (2.3) therefore requires the probe to plasma potential V_p to be much greater that V_e in order that the greater majority of the electrons emitted form the sheath edge should reach the probe.

The assumptions and theory presented above may be applied directly to the case of a negative probe and the collection of positive ions.

2.2.3. Spherical probe.

When the initial velocity of the electrons leaving the sheath edge is negligible in comparison with their final velocity at the positive probe the electron current density flowing to a spherical positive probe is

$$j = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{1/2}\frac{V_p^{3/2}}{r_p^2\alpha^2} \qquad \ldots(2.6)$$

where α is a function of (r_s/r_p) and has been tabulated by Blodgett and Langmuir (5).

A complete analysis of space charge limited currents to plane, cylindrical and spherical probes has been made by Langmuir and Compton (6). The analysis includes a study into the effects of the initial velocity of the carriers emitted from the sheath edge.

The diode equations enable the sheath radius and hence the current density at the sheath edge to be estimated (7). If the carrier temperature at the sheath edge is known the carrier concentration can then be found.

2.3. Probe characteristic equations.

The probe theory initially developed by Langmuir was first published in a series of articles in the General Electric Review during 1923 and 1924 (7 to 12). The theory presented in these articles is reviewed and developed in a paper by Mott-Smith and Langmuir in 1926 (1).

The theory is built up around the assumptions listed in sub-section 2.1.4.

2.3.1. Cylindrical probe - general distribution.

The orbital motion of carriers within the space charge layer surrounding the probe will now be examined. Motions perpendicular to the probe's axis need only be considered as motion parallel to the probe's axis does not contribute to the current.

The length of the cylindrical probe L is assumed to be very much greater than its radius r_p. V_p is the probe to plasma potential; the potential drop across the sheath region. If V_p is positive negatively charged carriers are attracted and if V_p is negative positively charged carriers are attracted by the probe. Let u and v be the radial and tangential carrier velocity components at the sheath edge and let u_p and v_p be the corresponding components at the probe surface. Let the generalized carrier mass be $\mathcal{M}$ where $\mathcal{M}$ = m for electrons and $\mathcal{M}$ = M for positive ions. Finally let the generalized carrier charge be e_o where $e_o = -e$ for electrons and $e_o = +e$ for positive ions. Figure 5 shows the carrier velocity components at the sheath edge and probe surface.

By the conservation of energy

$$\tfrac{1}{2}\mathcal{M}(u_p^2 + v_p^2) = \tfrac{1}{2}\mathcal{M}(u^2 + v^2) - e_oV_p \qquad \ldots(2.7)$$

By the conservation of momentum

$$\mathcal{M}v_pr_p = \mathcal{M}vr_s \qquad \ldots(2.8)$$

Equations (2.7) and (2.8) may be solved simultaneously for the tangential and radial velocity components at the probe surface,

$$v_p = v\left(\frac{r_s}{r_p}\right) \qquad \ldots(2.9)$$

$$u_p^2 = u^2 - v^2\left[\left(\frac{r_s}{r_p}\right)^2 - 1\right] - \frac{2e_oV_p}{\mathcal{M}} \qquad \ldots(2.10)$$

For a carrier to contribute to the probe current its radial velocity component at the probe surface must be greater than zero. It follows that a carrier, having a radial velocity component at the sheath edge equal to u, must have a tangential velocity component at the sheath edge v given by

$$v^2 \leqslant \left(\frac{r_p^2}{r_s^2 - r_p^2}\right)\left(u^2 - \frac{2e_oV_p}{\mathcal{M}}\right) \qquad \ldots(2.11)$$

for the carrier to reach the probe.

It is seen from equation (2.11) that the lowest possible value for u may be found by putting v equal to zero. The value of u is then

$$u^2 = \frac{2e_oV_p}{\mathcal{M}} \qquad \ldots(2.12)$$

The limiting values of v and u that a carrier can have in order that it may reach the probe are therefore

$$v_1 = -\left(\frac{r_p^2}{r_s^2 - r_p^2}\right)^{1/2}\left(u^2 - \frac{2e_oV_p}{\mathcal{M}}\right)^{1/2}$$
$$v_2 = +\left(\frac{r_p^2}{r_s^2 - r_p^2}\right)^{1/2}\left(u^2 - \frac{2e_oV_p}{\mathcal{M}}\right)^{1/2} \quad ...(2.13)$$

for all values of e_oV_p and

$$u_1 = \left(\frac{2e_oV_p}{\mathcal{M}}\right)^{1/2}$$
$$u_2 = \infty \quad ...(2.14)$$

for values of $e_oV_p > 0$ and

$$u_1 = 0$$
$$u_2 = \infty \quad ...(2.15)$$

for values of $e_oV_p < 0$.

The probe current I may now be expressed in terms of the carrier velocity distribution function at the sheath edge. Let f(u,v)dudv be the fraction of carriers at the sheath edge in the velocity range u to u + du and v to v + dv and let N be the total number of carriers per unit volume at the sheath edge. I is then

$$I = 2\pi r_s LNe_o \int_{u_1}^{u_2}\int_{v_1}^{v_2} uf(u,v)dudv \quad ...(2.16)$$

where the limits of integration have been defined by equations (2.13) to (2.15).

2.3.2. Spherical probe - general distribution.

By applying the laws of conservation of energy and momentum to the motion of carriers within the sheath region around a spherical probe the carrier current is given by

$$I = 4\pi r_s^2 N e_o \int_{u_1}^{u_2}\int_{q_1}^{q_2} qug(q,u)\,dq\,du \qquad \text{...(2.17)}$$

where u is the radial velocity component and q is the resultant of v and w, the other two rectangular velocity components. The limits of q and u are defined by

$$\left.\begin{aligned} q_1 &= 0 \\ q_2 &= \left(\frac{r_p^2}{r_s^2 - r_p^2}\right)^{1/2}\left(u^2 - \frac{2e_oV_p}{m}\right)^{1/2} \end{aligned}\right\} \qquad \text{...(2.18)}$$

for all values of e_oV_p and

$$\left.\begin{aligned} u_1 &= \left(\frac{2e_oV_p}{m}\right)^{1/2} \\ u_2 &= \infty \end{aligned}\right\} \qquad \text{...(2.19)}$$

for values of $e_oV_p > 0$ and

$$\left.\begin{aligned} u_1 &= 0 \\ u_2 &= \infty \end{aligned}\right\} \qquad \text{...(2.20)}$$

for values of $eV_p < 0$

2.3.3. Cylindrical probe - Maxwellian distribution.

If the carriers at the sheath edge are assumed to have a Maxwellian velocity distribution the distribution function in equation (2.16) is given by

$$f(u,v)\,du\,dv = \frac{m}{2\pi kT}\exp\left[-\frac{m}{2kT}(u^2+v^2)\right]du\,dv \qquad \text{...(2.21)}$$

where T is the carrier temperature in degrees Kelvin.

Substitute equation (2.21) into (2.16) and integrate between the appropriate limits. The carrier current is then given by

$$I = 2\pi r_p L N e_o \left(\frac{kT}{2\pi m}\right)^{1/2} \left\{ \frac{r_s}{r_p}\left[1 - \mathrm{erf}\left(\frac{-r_p^2\eta}{r_s^2 - r_p^2}\right)^{1/2}\right] + \exp(-\eta)\,\mathrm{erf}\left(\frac{-r_s^2\eta}{r_s^2 - r_p^2}\right)^{1/2} \right\} \quad \ldots(2.22)$$

for $\eta < 0$ and

$$I = 2\pi r_p L N e_o \left(\frac{kT}{2\pi m}\right)^{1/2} \exp(-\eta) \quad \ldots(2.23)$$

for $\eta > 0$ where η is defined by

$$\eta = \frac{e_o V_p}{kT} \quad \ldots(2.24)$$

and erf(x) is defined by

$$\mathrm{erf}(x) = \frac{2}{\pi^{1/2}} \int_x^\infty \exp(-y^2)\,dy \quad \ldots(2.25)$$

2.3.4. Spherical probe - Maxwellian distribution.

If the carriers at the sheath edge are assumed to have a Maxwellian velocity distribution the carrier current flowing to a spherical probe is given by

$$I = 4\pi r_p^2 N e_o \left(\frac{kT}{2\pi m}\right)^{1/2} \left\{ 1 - \left[1 - \frac{r_p^2}{r_s^2}\right] \exp\left[\frac{r_p^2 \eta}{r_s^2 - r_p^2}\right] \right\} \quad \ldots(2.26)$$

for $\eta < 0$ and

$$I = 4\pi r_p^2 N e_o \left(\frac{kT}{2\pi m}\right)^{1/2} \exp(-\eta) \quad \ldots(2.27)$$

for $\eta > 0$.

2.4. <u>Elimination of sheath radius from probe characteristic equations</u>

In general the sheath radius may be determined from the diode equations given in sub-section 2.2. The application of these equations requires that carriers of only one sign are present within the sheath and that the probe to plasma potential is sufficiently great to enable the probe to collect all the carriers entering into the sheath region. Assuming that these conditions are satisfied and r_s is determined its value may be used in conjunction with equations given in sub-sections 2.3.3. and 2.3.4.

Due to the inevitable vagueness of the space charge sheath radius it is, perhaps, more convenient to consider under what circumstances the probe current equations given in sub-sections 2.3.3 and 2.3.4 become independent of r_s.

2.4.1. Cylindrical probe.

When (r_s/r_p) tends to infinity equation (2.22) can be shown to be independent of the sheath radius (13) and the resulting expression for the carrier current is

$$I = 2\pi r_p L N e_o \left(\frac{kT}{2\pi m}\right)^{1/2}\left[\frac{2}{\pi^{1/2}}(-\eta)^{1/2} + \exp(-\eta)\,\mathrm{erf}(-\eta)^{1/2}\right] \quad \ldots(2.28)$$

for $\eta < 0$.

Equation (2.28) may be further simplified for $\eta < -2$ when the carrier current is

$$I = 2\pi r_p L N e_o \left(\frac{kT}{2\pi m}\right)^{1/2} \frac{2}{\pi^{1/2}}(1-\eta)^{1/2} \quad \ldots(2.29)$$

2.4.2. Spherical probe.

When (r_s/r_p) tends to infinity equation (2.26) can be shown to be independent of the sheath radius (13) and the resulting expression for the carrier current is

$$I = 4\pi r_p^2 N e_o \left(\frac{kT}{2\pi \mathcal{M}}\right)^{1/2}(1-\eta) \qquad \ldots(2.30)$$

for $\eta < 0$.

2.4.3. Plane probe.

The carrier current flowing to a plane probe is clearly independent of the dimensions of the sheath. Expressions for the probe current may be obtained from the spherical probe current expressions by putting $4\pi r_p^2$ equal to A, the area of the plane probe, and letting r_p tend to infinity. Equations (2.26) and (2.27) then become,

$$I = ANe_o\left(\frac{kT}{2\pi \mathcal{M}}\right)^{1/2} \qquad \ldots(2.31)$$

for $\eta < 0$ and

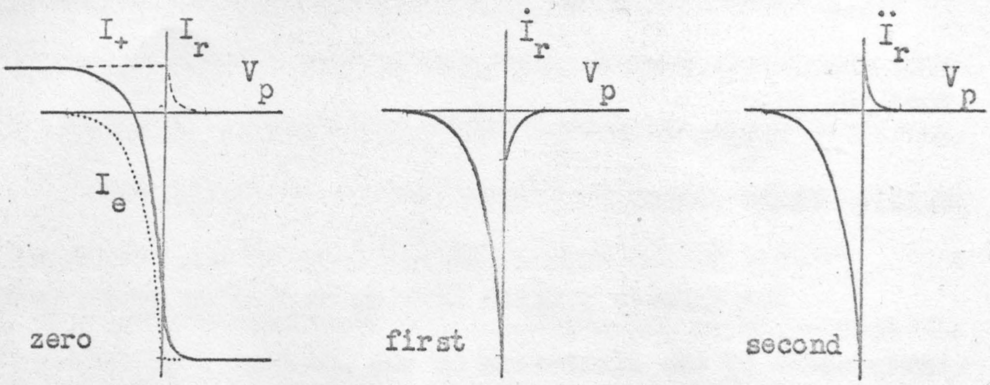
$$I = ANe_o\left(\frac{kT}{2\pi \mathcal{M}}\right)^{1/2}\exp(-\eta) \qquad \ldots(2.32)$$

for $\eta > 0$.

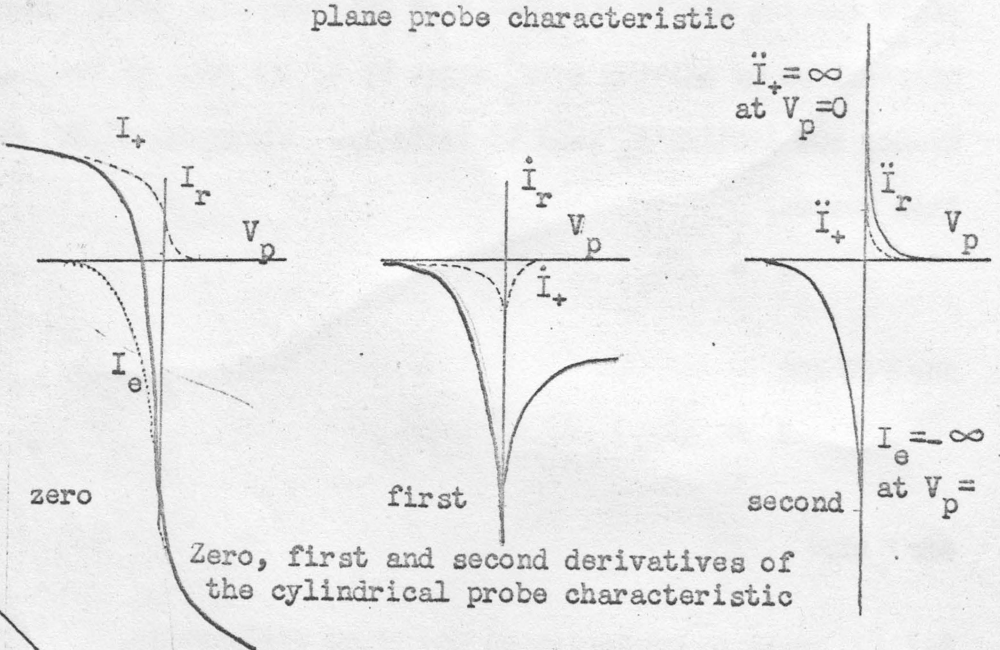
2.4.4. Probe characteristics and their derivatives.

The resultant carrier current flowing to a probe is given by the sum of the positive ion and electron currents. Expressions for these have been derived in the previous sections, where $\mathcal{M} = M$ and $e_o = +e$ for positive ions and where $\mathcal{M} = m$ and $e_o = -e$ for electrons. If I_r, I_+, and I_e are the resultant, positive ion, and electron currents respectively.

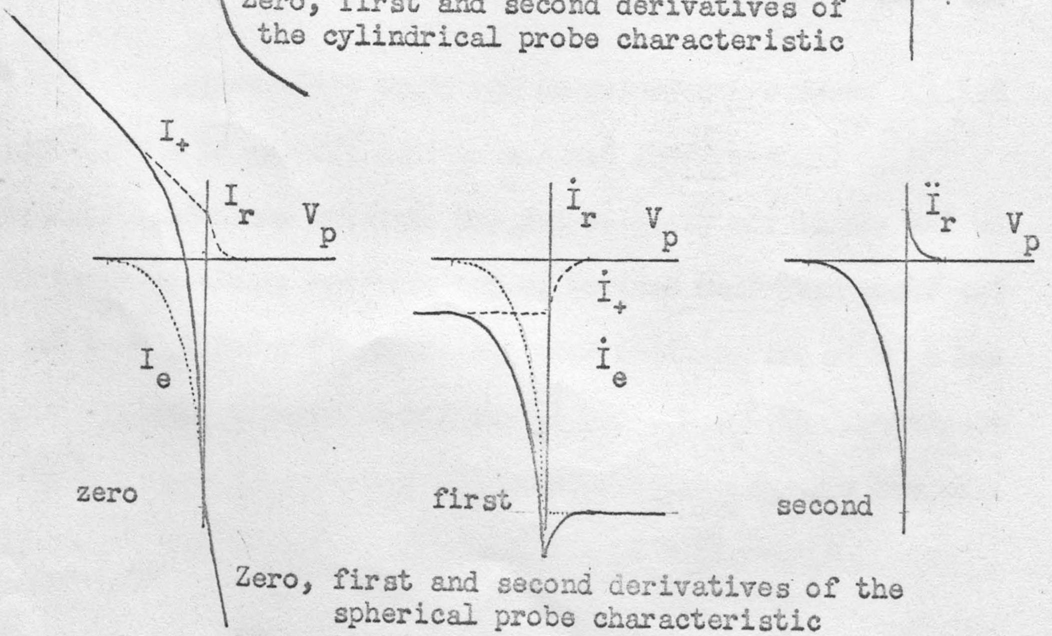
$$I_r = I_+ + I_e \qquad \ldots(2.33)$$



Zero, first and second derivatives of the plane probe characteristic



Zero, first and second derivatives of the cylindrical probe characteristic



Zero, first and second derivatives of the spherical probe characteristic

Figure 6

Equation (2.33), together with its first and second derivatives are plotted in Figure 6. These characteristics have been drawn assuming (r_s/r_p) tends to infinity. Equation (2.33) is only really true when either $I_+ \gg I_e$ or $I_e \gg I_+$ because in the derivation of the carrier current equations carriers of only one sign are assumed to be present in the sheath region. When carriers of both signs are present there will be a partial neutralization of the space charge. To compensate for this there will be an increase in the effective sheath thickness.

CHAPTER 3

DETERMINATION OF PLASMA PARAMETERS FROM PROBE CHARACTERISTICS

If the assumptions listed in sub-section 2.1.4. are applicable and if a Maxwellian distribution of carrier velocities can be assumed Langmuir's probe theory, as presented in Chapter 2 may be used for the determination of plasma parameters.

Experimentally it is not possible to apply a potential directly between the probe and the plasma; instead it is applied between the probe and a reference point. The potential difference between the undisturbed plasma, in the neighbourhood of the probe, and the reference point is known as the plasma potential. Providing that the plasma potential remains constant the potential applied between the probe and the reference point is equal to the probe to plasma potential plus a constant. In interpreting probe current characteristics as a function of probe to reference point potential the equations derived in Chapter 2 must be corrected to allow for the additional plasma potential.

3.1. Electron and positive ion temperatures.

Langmuir's theory assumes that the carriers have a Maxwellian distribution of velocities and they can, therefore, have a temperature associated with their mean energy.

The necessity of assuming a Maxwellian distribution is in the application of Boltzman's law in the case of a retarding probe. To apply this law equilibrium must exist between the energetic carriers and the retarding probe's electric field. This equilibrium state is discussed in detail by Langmuir in his paper 'The theory of collectors in gaseous discharges' (1) and in his review article 'Electrical discharges in gas' (6).

If a Maxwellian distribution of velocities exists and Boltzman's law is applied a straight line plot should be obtained if the logarithm of the probe current is plotted against the retarding probe potential. Langmuir and others have found that, in practice, this straight line plot may extend over three or four magnitudes of electron current. It has generally been concluded from this that the electrons possess a Maxwellian distribution of velocities. Langmuir could not explain satisfactorily how the electrons could aquire a Maxwellian distribution of velocities in a gas discharge and this delema is known as Langmuir's paradox (14).

In determining the electron temperature by observing the variation of electron current with retarding probe potential one, in practice, measures the resultant probe current and either assumes that the probe current is entirely an electron current or, if this assumption is not justified, corrects for the positive ion current.

For probes of any shape the electron current flowing to a negative probe is given by

$$I_e = I_{eo} \exp\left[\frac{eV_p}{kT_e}\right] \qquad \ldots(3.1)$$

where $V_p < 0$, e is the magnitude of the electron charge, and I_{eo} is the electron current flowing to the probe when at plasma potential ($V_p = 0$). I_{eo} is given by

$$I_{eo} = \frac{A}{4} N_e e \left(\frac{8kT_e}{\pi m}\right)^{1/2} \qquad \ldots(3.2)$$

In practice probe potentials are measured with respect to some reference point. If V_o is the plasma potential in the neighbourhood of the probe and V is the probe to reference point potential.

$$V = V_p + V_o \qquad \ldots(3.3)$$

Substituting $V_p = V - V_o$ into equation (3.1) and taking natural logarithms gives

$$\ln I_e = \ln I_{eo} - \frac{eV_o}{kT} + \frac{eV}{kT_e} \qquad \ldots(3.4)$$

A plot of $\ln I_e$ against V has a slope of e/kT_e or using equation (2.4) in conjunction with equation (3.4) the slope is $1/V_e$. The mean electron energy expressed in electron volts $e\bar{V}_e$ is related to eV_e by

$$e\bar{V}_e = \frac{3}{2} eV_e \qquad \ldots(3.5)$$

As the probe is biased more and more negative and positive ion contribution to the resultant probe current becomes significant and has to be corrected for. This can be done by extrapolating the positive ion current characteristic, for a strongly negative probe, back to plasma potential. In the case of a plane probe extrapolating the characteristic back to plasma potential is quite straight forward as ideally the positive ion current is constant and independent of probe potential. In practice, however the positive ion current shows a linear dependence on the probe potential. Langmuir (9) explained this linear dependence to be due to a change in the effective collecting area as a result of the edge effects of the sheath region. A linear extrapolation may also be used in the case of a spherical probe. For a cylindrical probe, the simplest method of extrapolation is to use equation (2.92). Equation (2.29) can only be used if the extrapolation need not be extended to within $2kT_+/e$ of plasma potential. These extrapolations assume that r_s/r_p tends to infinity and that the ions, at the sheath edge, may be assigned a temperature corresponding to their mean kinetic energy in the plasma.

Experimentally it is not easy to investigate whether or not the ions possess a Maxwellian distribution. Theoretically, on the other hand, it is difficult to see how, in a confined discharge, they can aquire a Maxwellian distribution as they are generated with roughly zero initial velocity (15). The ions gain their energy in the electric field created by the negative potential on the walls of the vessel containing the discharge. An exchange of energy will occur between the ions and the neutral gas atoms and it is thought that the mean ion energy in the bulk of the plasma is roughly equal to that of the neutral atoms. As the ions do not move randomly within the plasma but have a directed motion towards the walls of the vessel it seems unlikely that they possess a truely Maxwellian distribution of velocities.

Measurements of ion energies by Langmuir (16) using a perforated collector show that ions possess a roughly Maxwellian distribution of velocity corresponding to an ion temperature of approximately half the electron temperature (15, 17, 18). By using the idea of a penetrating electric field, Langmuir (17) was able to explain how the ion temperature at the sheath edge was very much greater than in the bulk of the plasma. In this theory a small proportion of the potential drop between the probe and plasma occurs across a region surrounding the sheath: this region is known as the extra sheath. The electric field in the extra sheath causes the ions to be accelerated towards the negative probe and gain a mean energy equivalent to approximately $T_e/2$ on arrival at the sheath edge. A more detailed discussion of this process is given in Chapter 6.

If the equations developed in Chapter 2 are to be applied to the collection of positive ions by a negative probe the ion temperature at the sheath edge must be put equal to $T_e/2$.

3.2. Electron and positive ion concentrations.

3.2.1. Electron concentrations by random current method.

At plasma potential no space charge sheath is formed around the probe and the resultant probe current is equal to the sum of the random electron and ion currents crossing a surface area equal to that of the probe. As the random electron current is generally at least one hundred times the random positive ion current the resultant probe current may be considered to be effectively due to electrons.

$$I_r \simeq I_e = \frac{A}{4} N_e e \bar{c}_e \qquad \ldots(3.6)$$

where $\bar{c}_e$ is the mean electron velocity and is given by

$$\bar{c}_e = \left(\frac{8kT_e}{\pi m}\right)^{1/2} \qquad \ldots(3.7)$$

Substituting (3.7) into (3.6) and solving for N_e gives

$$N_e = \frac{I_r}{Ae} \left(\frac{2\pi m}{kT_e}\right)^{1/2} \qquad \ldots(3.8)$$

Equation (3.8) alternatively could have been obtained from equations (2.23), (2.27) and (2.32) for cylindrical, spherical, and plane probes respectively by putting V_p equal to zero.

3.2.2. Electron concentrations when r_s/r_p tends to infinity.

In the electron accelerating region the resultant probe current is effectively only due to electrons and may be given by (6), (8),

$$I_r = AN_e e \left(\frac{kT_e}{2\pi m}\right)^{1/2} \left(1 + \frac{eV_p}{kT_e}\right) \qquad \ldots(3.9)$$

in the case of a spherical probe and by

$$I_r = AN_e e \left(\frac{kT_e}{2\pi m}\right)^{1/2} \frac{2}{\pi^{1/2}} \left(1 + \frac{eV_p}{kT_e}\right)^{1/2} \qquad \ldots(3.10)$$

in the case of a cylindrical probe providing $V_p > 2kT_e/e$. The error in equation (3.9) is less than 1% for $V_p > 5kT_e/e$ or less than 5% for $V_p > 3kTe/e$. The error in equation (3.10) is less than 1% for $V_p >$ $3kT_e/e$ or less than 5% for $V_p > 2kT_e/e$. N_e is found from the slope of the plot of I_r against V_p for a spherical probe, or I_r^2 against

V_p for a cylindrical probe. In practice it is not the variation of I_r with V_p that is measured but the variation of I_r with V where V is defined by equation (3.3). This does not, however, affect the determination of N_e.

3.2.3. Electron concentration when r_s/r_p is large.

When $1 \ll r_s/r_p \ll \infty$ the method given in sub-section 3.2.2. cannot be used and one must resort to the original equations given in sub-sections 2.3.3. and 2.3.4. In the electron accelerating region the resultant probe current is then given by (8)

$$I_r = \frac{AN_e e\bar{c}_e f}{4} \qquad \ldots(3.11)$$

for a cylindrical probe and $V_p > 0$ and by

$$I_r = \frac{AN_e e\bar{c}_e F}{4} \qquad \ldots(3.12)$$

for a spherical probe and $V_p > 0$ where f and F are defined by

$$f = \left\{\frac{r_s}{r_p}\left[1 - \mathrm{erf}\left(\frac{r_p^2 \frac{eV_p}{kT_e}}{r_s^2 - r_p^2}\right)^{1/2}\right] + \exp\left(\frac{eV_p}{kT_e}\right)\mathrm{erf}\left(\frac{r_s^2 \frac{eV_p}{kT_e}}{r_s^2 - r_p^2}\right)^{1/2}\right\} \qquad \ldots(3.13)$$

$$F = \left\{1 - \left[1 - \left(\frac{r_p}{r_s}\right)^2\right]\exp\left[\frac{-r_p^2 \frac{eV_p}{kT_e}}{r_s^2 - r_p^2}\right]\right\} \qquad \ldots(3.14)$$

When $V_p \gg kT_e/e$ and $\frac{3}{2}(r_s kT_e/r_p eV_p)^2 \gg 1$ f reduces to (6), (8)

$$f = \frac{2}{\pi^{1/2}}\left[1 + \frac{eV_p}{kT_e} - \frac{2}{3}\left(\frac{r_p eV_p}{r_s kT_e}\right)^2\right] \qquad \ldots(3.15)$$

and when $V_p \gg kT_e/e$ and $2(r_s kT_e/r_p eV_p)^2 \gg 1$ F reduces to

$$F = \left[1 + \frac{eV_p}{kT_e} - \frac{1}{2}\left(\frac{r_p eV_p}{r_s kT_e}\right)^2\right] \qquad \ldots(3.16)$$

N_e is determined from the slope of the plot of I_r against f or F. The great difficulty of this method is that it is essential to know the sheath radius and plasma potential. As the electron current from the sheath edge to the probe is governed by orbital motion it cannot be considered to be equal to the diode current and so r_s cannot be deduced from the diode equations. Because of the difficulties in determining the sheath radius and plasma potential this method of determining N_e is not very satisfactory.

3.2.4. Electron concentrations when r_s/r_p is approximately unity.

This method is applicable when r_s/r_p is approximately unity, and when $V_p \gg kT_e/e$ equations (3.13) and (3.14) may be greatly simplified. Under these conditions the electron current flowing to a positive probe is space charge limited rather than limited by orbital motion. For a cylindrical probe f reduces to

$$f = \frac{r_s}{r_p} \qquad \ldots(3.17)$$

for $V_p > -2kT_e(1 - \frac{r_s^2}{r_p^2})/e$ to within 5%,

or for $V_p > -3kT_e(1 - \frac{r_s^2}{r_p^2})/e$ to within 1%.

Similarly for a spherical probe F reduces to

$$F = \left(\frac{r_s}{r_p}\right)^2 \qquad \ldots(3.18)$$

for $V_p > -3kT_e(1 - \frac{r_s^2}{r_p^2})/e$ to within 5%, or for $V_p > -5kT_e(1 - \frac{r_s^2}{r_p^2})/e$ to within 1%.

On substituting these limiting values of f and F into equations (3.11) and (3.12) one obtains

$$I_r = \frac{A_s}{4} N_e e \bar{c}_e \qquad \ldots(3.19)$$

for a cylindrical and a spherical probe for $V_p > 0$ and where A_s is the surface area of the sheath.

Under these conditions the sheath radius r_s can be determined from the diode equations. The electron concentration is then obtained from the slope of the plot of probe current against sheath radius providing the electron temperature is known.

It must be remembered that the sheath radius deduced from the diode equation is not exactly the same as the sheath radius that appears in the expression describing the orbital motion of the electrons. The whole concept of a space charge sheath boundary is one of great importance as well as one that creates much difficulty in probe analysis.

3.2.5. Positive ion concentrations.

In a neutral plasma the positive ion concentration is approximately equal to the electron concentration and is best estimated from electron concentmation measurements. If, on the other hand, direct positive ion concentration measurements are desired the methods available are essentially the same as listed in sub-sections 3.2.2. to 3.2.4.

Using the method given in sub-section 3.2.2. Langmuir (18) (10) determined the positive ion concentration and found that on average N_+ was $2.2N_e$. In his determination he assumed the ion temperature at the sheath edge was $T_e/2$. The difference between the positive ion and electron concentrations was originally thought to be due to the presence of negative ions. It was later satisfactorily explained using the idea of a penetrating electric field which produced a directed motion of the ions in the extra sheath region.

In applying Langmuir's probe equations to the collection of positive ions the ion concentration at the sheath edge N should be replaced by $2.2N_+$ where N_+ now refers to the ion concentration outside the extra sheath region in the unperturbed plasma.

Langmuir's analysis cannot be used to describe the collection of positive ions without replacing T by $T_e/2$ and N by N_+. This then reduces his analysis to a semi-empirical level and the accuracy of the resulting expressions is determined solely by the accuracy of the experimentally determined values of T and N in terms of T_e and N_+.

3.3. Plasma potential.

Plasma potential at a point is defined as the potential at that point relative to the potential of an arbitrary reference point; this reference point is usually taken to be an electrode.

When a probe is immersed in a plasma and kept at plasma potential with the aid of some biasing device no space charge sheath surrounds the probe. The resultant current density at the probe is equal to the sum of the random electron and ion current densities in the plasma.

3.3.1. Discontinuity method.

Figure 6 shows a discontinuity in the probe characteristic at plasma potential. This discontinuity or 'knee' is usually more easily detectable if the semi-logarithmic probe characteristic is drawn. As a general rule the plasma potential is the point at which the semi-logarithmic characteristic departs from a straight line. In practice the exact location of the knee is uncertain due to various disturbing effects (see chapter 4). A statistical method of locating the knee has been suggested (19).

A variation of the method is to obtain the first or second derivatives of the characteristic. Figure 6 shows the discontinuity at plasma potential very clearly.

3.3.2. Electron accelerating method.

This method is applicable to spherical and cylindrical probes and involves the measurement of the resultant probe current in the electron accelerating region. As in the method for determining electron concentrations in sub-section 3.2.2. the condition that r_s/r_p tends to infinity must hold.

Substituting $V_p = V - V_o$ into equations (3.9) and (3.10) gives

$$I_r = AN_e e\left(\frac{kT_e}{2\pi m}\right)^{1/2}\left(1 + \frac{eV}{kT_e} - \frac{eV_o}{kT_e}\right) \qquad \ldots(3.20)$$

for a spherical probe and

$$I_r = AN_e e\left(\frac{kT_e}{2\pi m}\right)^{1/2}\frac{2}{\pi^{1/2}}\left(1 + \frac{eV}{kT_e} - \frac{eV_o}{kT_e}\right)^{1/2} \qquad \ldots(3.21)$$

for a cylindrical probe. I_r and I_r^2 are seen to be zero when

$$\left(1 + \frac{eV}{kT_e} - \frac{eV_o}{kT_e}\right) = 0 \qquad \ldots(3.22)$$

that is when

$$V_o = V + \frac{kT_e}{e} \qquad \ldots(3.23)$$

In equation (3.23) V is the intercept of the characteristic on the probe voltage axis (8), (10), (1).

The error in equation (3.20) is less than 1% providing $V > (5kT_e/e + V_o)$ and the error in equation (3.21) is less than 1% providing $V > (3kT_e/e + V_o)$. The characteristics will be linear only in these regions and it will probably be necessary to extrapolate this linear region back to I_r equal to zero.

3.3.3. Electron emitting probe method.

A number of investigators have determined plasma potentials using an electron emitting probe (1), (20), (21), and (22). This probe is so designed that there is some means of producing controlled electron emission from its surface.

If electrons are being emitted from the probe and the probe is held at a positive potential with respect to the surrounding plasma none of the emitted electrons enter the surrounding plasma. This is because they are attracted by the positive potential on the probe. A positive electron emitting probe has, therefore, the same characteristic as a non electron emitting probe.

If electrons are being emitted from the probe and the probe is maintained at a negative potential with respect to the plasma the emitted electrons are repelled by the probe and enter the surrounding plasma. The electron current flowing to a negative electron emitting probe is therefore less than the current flowing to a negative non electron emitting probe.

Plasma potential is given by the probe potential at which the electron emitting probe characteristic parts from the non electron emitting probe characteristic. Errors in the determination of plasma potential by this method may occur if the electron emitting current is not kept small. A large electron emission may seriously disturb the surrounding space charge sheath and plasma.

3.4. Electron velocity and energy distribution functions.

In the theory outlined in the previous sections it has been assumed that the carriers possess a Maxwellian distribution. The justification for this is that a considerable amount of information has been obtained from probe measurements, and other observations, that is consistant with such an assumption. However, not all observations in all types of plasma are consistant with a Maxwellian distribution.

Probe theories assuming the following non-Maxwellian distributions have been considered inthe past (1), (23), (24).

(a) Velocities equal in magnitude and parallel in direction,

(b) Velocities equal in magnitude but with directions distributed at random in space,

(c) A Maxwellian distribution with superimposed drift,

(d) A uniformly accelerated half-Maxwellian distribution,

(e) A pure Maxwellian distribution superimposed on an accelerated half-Maxwellian distribution.

As it is difficult to know which of these theories to apply to the particular plasma being investigated it is desirable to be able to determine the exact form of the electron distribution from the probe's current-voltage characteristic.

3.4.1. Electron velocity distribution.

The electron current flowing to a spherical probe is given by equation (2.17)

$$I_e = 4\pi r_s^2 N_e e \int_{u_1}^{u_2}\int_{q_1}^{q_2} qug(q,u)\,dq\,du \qquad \ldots(3.24)$$

where the limits of u and q are defined by equations (2.18) and (2.19) for $V_p < 0$. When r_s/r_p tends to infinity equation (3.24) may be simplified and integrated with respect to q to give

$$I_e = 2\pi r_p^2 N_e e \int_{u_1}^{u_2} u\left(u^2 + \frac{2eV_p}{m}\right) g(0,u)du \qquad \ldots(3.25)$$

Differentiating equation (3.25) twice respect to V_p gives (2)

$$\frac{d^2I_e}{dV_p^2} = AN_e e\left(\frac{e}{m}\right)^2 g(0,u) \qquad \ldots(3.26)$$

where $$u = \left(-\frac{2eV_p}{m}\right)^{1/2} \qquad \ldots(3.27)$$

Equation (3.26) was first derived by Langmuir and Mott-Smith in 1926. They also considered the case of a cylindrical probe. In 1930 Druyvesteyn (25) considered the determination of the distribution function from the double differentiation of the characteristic of any non-concave probe surface. He shows that the second derivative of the current-voltage characteristic is the same for both plane and cylindrical probes and is given by

$$\frac{d^2I_e}{dV_p^2} = \frac{AN_e e}{4}\left(\frac{e}{m}\right)\frac{1}{|V_p|}\rho(c) \qquad \ldots(3.28)$$

where $$c = \left(\frac{2e|V_p|}{m}\right)^{1/2} \qquad \ldots(3.29)$$

$\rho(c)dc$ is the fraction of electrons per unit volume having a resultant velocity in the range c to c + dc. It is then argued that the same expression would be obtained for any non-concave probe surface. This is seen to be so in Figure 6 in the case of a Maxwellian velocity distribution for plane, cylindrical and spherical probes.

Equation (3.28) expresses the distribution function in terms of c, the resultant velocity, while equation (3.26) expresses the distribution function in terms of u, the velocity component normal to the probe's surface.

3.4.2. Electron energy distribution.

Druyvesteyn (25) showed that for a plane probe

$$I_e = \frac{AN_e e}{4}\int_{(2e|V_p|/m)^{1/2}}^{\infty} \rho(c)c\left[1 - \frac{2e|V_p|}{mc^2}\right] dc \qquad \ldots(3.30)$$

Equation (3.30) may be expressed in terms of electron energy by replacing $mc^2/2$ by eU and $\rho(c)$ by $F(U)$. Equation (3.30) then becomes

$$I_e = \frac{AN_e e}{4}\left(\frac{2e}{m}\right)^{1/2}\int_{|V_p|}^{\infty} U^{\frac{1}{2}}F(U)\left(1 - \frac{|V_p|}{U}\right) dU \qquad \ldots(3.31)$$

Differentiating equation (3.31) twice with respect to $|V_p|$ gives

$$\frac{d^2 I_e}{dV_p^2} = \frac{AN_e e}{4}\left(\frac{2e}{m|V_p|}\right)^{1/2} F(U)_{u=|V_p|} \qquad \ldots(3.32)$$

CHAPTER 4

PLASMA AND PROBE PERTURBATIONS

The validity of the probe analysis presented in chapters 2 and 3 depends on the probe not disturbing the plasma in which it is immersed. The analysis also requires a stable plasma. If the probe disturbs the plasma or if instabilities occur in the plasma errors may result in the interpretation of the probe characteristics. The effect of these perturbations will now be examined.

Perturbations may be classified as follows:

(a) Due to the probe supports;

(b) Due to the probe geometry;

(c) Due to the probe surface;

(d) Due to plasma instabilities.

4.1. Perturbations due to the probe supports.

The insulating probe supports aquire a negative potential with respect to the plasma. This negative potential will cause an enhancement of the electric field on the side closest to the anode and a reduction of the electric field on the side closest to the cathode. The effect may be observed as a local increase in the intensity of the discharge on the anode side of the probe supports (26), (27).

4.2. Perturbations due to the probe geometry.

One of the criteria, listed in sub-section 2.1.4., that must be satisfied for Langmuir's theory to be valid is that the carrier mean free path ℓ must be greater than the sheath thickness x_s.

$$x_s < \ell = \ell_o/p \qquad \ldots(4.1)$$

where l_o is the carrier mean free path at unit pressure and p is the pressure. The criterion for the application of the orbital theory is

$$\frac{r_s}{r_p} \to \infty \; ; \; \left(\frac{x_s + r_p}{r_p}\right) \to \infty \; ; \; x_s \ggg r_p \qquad \ldots(4.2)$$

Combining (4.2) with (4.1) gives

$$\ell_o/p > x_s \ggg r_p \qquad \ldots(4.3)$$

The criterion for the application of the space charge theory is

$$\frac{r_s}{r_p} \approx 1 \qquad \ldots(4.4)$$

Combining (4.4) with (4.1) gives

$$\ell_o/p > r_s \approx r_p \qquad \ldots(4.5)$$

In equations (4.3) and (4.5) there is no indication of the magnitude of $\ell_o/(pr_p)$ necessary for the accurate application of the theory. If no collisions occur in the sheath region and if $\ell_o/(pr_p)$ is of the order of 10 it can be shown that the carrier concentration, one mean free path from the probe's surface, may be reduced by approximately 1%. The various perturbing effects of the probe's geometry on the carrier concentration will now be considered.

4.2.1. Obscuration effect.

If the carriers are moving randomly and if x_s/r_p is very much less than unity and $\ell_o/(pr_p)$ is greater than unity, the concentration, due to the obscuration of part of the plasma by a spherical probe, at a distance ℓ_o/p from the centre of the probe is less than the unperturbed concentration by a fraction (28)

$$\pi\left(\frac{r_p p}{\ell_o}\right)^2 \Big/ 4\pi \qquad \ldots(4.6)$$

The perturbed concentration N_ℓ, one mean free path from the probe's surface is thus

$$N_\ell = N\left[1 - \left(\frac{r_p p}{2\ell_o}\right)^2\right] \qquad \ldots(4.7)$$

For the carrier perturbation to be less than 1% one requires

$$\frac{\ell_o}{pr_p} > 5 \qquad \ldots(4.8)$$

4.2.2. Carrier drain effect.

If the probe area and/or the probe potential is large the current taken by the probe may be an appreciable fraction of the plasma discharge current. Langmuir (9) reports that the effect of this is to reduce the carrier concentration and thereby seriously perturb the probe characteristic. This perturbation is especially noticeable in the neighbourhood of plasma potential where the characteristic shows a smoothing out of the knee. The outcome of this is a difficulty in locating the plasma potential.

4.2.3. Carrier diffusion effect.

Davydv and Zmanovkaja (29) show that for the fractional drop in carrier concentration, in the neighbourhood of the probe, to be small one requires

$$\frac{\ell_o}{p} \gg r_p \qquad \ldots(4.9)$$

for spherical and cylindrical probes and

$$\frac{\ell_o}{p} \gg R \qquad \ldots(4.10)$$

for a plane disc probe of radius R. If these conditions are not satisfied noticeable diffusion currents occur and there results a drop in carrier concentration.

A quantitative analysis has been given by Bohm et als (28). They assume that the carrier motion to the probe takes place by diffusion except for the last free path when the current is carried by free motion. Assuming the diffusion coefficient D is incorrectly given by

$$D = \frac{\ell \bar{v}}{4} \qquad \ldots(4.11)$$

they show that the perturbed carrier concentration is

$$N_\ell = \frac{N}{\left\{1 + \left[\frac{1}{\frac{\ell_o}{p r_p}\left(1 + \frac{\ell_o}{p r_p}\right)}\right]\right\}} \qquad \ldots(4.12)$$

For the carrier perturbation to be less than 1% one requires

$$\frac{\ell_o}{pr_p} > 9.5 \qquad ...(4.13)$$

Swift (30) has considered the effect of the probe's size on the electron concentration as a function of electron energy. Making the same assumptions as Bohm et als above but with D given by

$$D = \frac{\ell \bar{v}}{3} \qquad ...(4.14)$$

the electron distribution one mean free path from the probe's surface is shown to be

$$f_\ell(U) = \frac{f(U)}{\left\{1 + \left[\dfrac{1 - |V_p|/U}{\frac{4}{3}\frac{\ell_o}{pr_p}\left(1 + \frac{\ell_o}{pr_p}\right)}\right]\right\}} \qquad ...(4.15)$$

where V_p is < 0 and $f(U)$ is the unperturbed electron energy distribution function. When $V_p = 0$ the probe is at plasma potential and N_ℓ is given by

$$N_\ell = \frac{N}{\left\{1 + \left[\dfrac{1}{\frac{4}{3}\frac{\ell_o}{pr_p}\left(1 + \frac{\ell_o}{pr_p}\right)}\right]\right\}} \qquad ...(4.16)$$

For the carrier perturbation to be less than 1% one requires

$$\frac{\ell_o}{pr_p} > 8.2 \qquad ...(4.17)$$

If the electron concentration is deduced from the electron energy distribution function obtained from a measure of the second differential given by equation (3.32) it is necessary to consider the dependence of f_ℓ (U) on the probe potential. Swift shows

$$\frac{d^2 I_e}{d|V_p|^2} = g\,|V_p|^{-1/2}\left[f_\ell(u)\right]_{u=|V_p|} = g\,|V_p|^{-1/2}\left[f(u)\right]\left[1-\psi\theta\right]_{u=|V_p|} \qquad \ldots(4.18)$$

where
$$\psi \equiv \frac{1}{\left[\frac{2}{3}\frac{\ell_o}{pr_p}\left(1+\frac{\ell_o}{pr_p}\right)\right]}$$

$$\theta \equiv \int_{|V_p|}^{\infty} \frac{u^{-3/2} f(u)\,du}{\left[1+\frac{\psi}{2}\left(1-\frac{|V_p|}{u}\right)\right]^{3/2}} \Bigg/ |V_p|^{-1/2}\left[f(u)\right]_{u=|V_p|}$$

$\psi\theta$ is the fractional change produced by the disturbance and is a function of V_p. The fractional error F in the electron concentration obtained from the measurement of the distribution function is given by

$$F = \psi\int_0^{\infty}\theta f(u)\,du \Big/ \int_0^{\infty} f(u)\,du \qquad \ldots(4.19)$$

Calculations show that if $f(u)$ is a Maxwellian or Druyvesteyn distribution F is given to a good approximation by

$$F = 0.65\psi \qquad \ldots(4.20)$$

For the carrier perturbation to be less than 1% one requires

$$\frac{\ell_o}{pr_p} > 9.4 \qquad \ldots(4.21)$$

4.3. Perturbations due to the probe surface.

Experimentally determined probe characteristics may possess errors due to inaccuracies in current and potential measurements brought about by disturbing effects at the probe surface.

4.3.1. Probe current errors.

(a) Carrier reflection.

In the electron retarding region electron reflection will cause an apparent reduction in electron current to the probe (1). In the electron accelerating region reflected electrons will be drawn back to the attracting probe and there is no change in electron current. For a positive ion accelerating probe reflected ions will be drawn back to the probe and there is no change in positive ion current.

It is possible, however, that in the case of a carrier accelerating probe carrier reflection may produce a partial neutralization of the space charge sheath surrounding the probe. If this occurs the sheath must expand in order to restore its original screening effect. This increase in sheath thickness results in an increase in the effective collecting area and hence an increase in the probe current.

(b) Carrier emission.

Depending on the probe potential the emission of electrons (thermionic or secondary) from the probe surface will appear as an increase in ion current or decrease in electron current (1). A similar effect occurs if the probe is contaminated with barium oxide and negative oxygen ions are emitted (31) . The effects of secondary emission from the probe are difficult to differentiate from the effects of electron reflection.

The emission of secondary electrons may be caused by the impingement of metastable atoms, photons, electrons and ions. Measurements made by Boyd (32) indicate that, for a platinum probe in an argon discharge, electron emission may contribute to 10% of the observed saturation ion current.

(c) Probe area.

Uncertainties in the surface area of the probe produce errors in the estimation of the current density flowing to the probe. The estimated area may be too large if insulating patches form on the surface of the probe. These patches may be caused by the formation of an oxide layer of the probe material or the deposition of some insulating material (33). These insulating patches may be removed by heating the probe by either positive ion bombardment (34) or by electron bombardment (27).

An under estimation of the probe area may result if the effective collecting surface area has in some way increased. This may occur if the probe expands appreciably on heating (33), if the probe comes into contact with the probe supports on which may have been deposited sputtered material (27), or if mercury droplets condense on the probe surface (35), (36). Errors due to sputtering of the probe material onto the probe supports may be eliminated by careful probe construction (27) and errors due to condensed mercury may be eliminated by heating the probe immediately before each measurement.

(d) Gas impurities.

If oxide coated cathodes are used in the discharge being investigated the presence of barium and barium oxide on or near the probe may produce errors in the determination of plasma parameters. It has been reported by Coulter and Higginson (31) that the presence of barium vapour near the probe can give rise to a high energy peak in the determination of the electron energy distribution function. Wehner and Medicus (33) have reported that the build up of barium on the probe will produce a reduction in the work function of the probe and so cause electrons to be accelerated towards the probe. This increased acceleration may result in an increase in ionization and hence an increase in probe current.

4.3.2. Probe potential errors.

The surface effect that will produce errors in probe potential measurements is that due to changes in the probe's work function and has been reported by a number of investigators (27), (33) to (36). For accurate probe potential measurements it is essential that the nature of the probe's surface remains constant at the time of measurement. Verweij (27) suggests that this may be achieved by heating the probe by electron bombardment with a current equal to or greater than the maximum current taken by the probe in measuring the characteristic. Waymouth (34) claims that if the probe is held at a strongly negative potential prior to each measurement the nature of the probe's surface will also remain constant as a result of ion bombardment.

Medicus (37) has considered the effect on the probe characteristic of slight variations of work function over the surface of the probe. He assumed that the work function could be represented by a Gaussian distribution about a mean value $\overline{W}$. At any point the work function W is given by

$$W = \overline{W} + w \qquad \ldots(4.22)$$

where the area of the probe having a variation in work function in the range w to w + dw is given by

$$dA = \frac{A}{\pi^{1/2}} \exp\left[-\left(\frac{w}{w_o}\right)^2\right]\frac{dw}{w_o} \qquad \ldots(4.23)$$

and where w_o is a measure of the spread of the work function distribution.

By calculating the current flowing to an area dA of the probe and then integrating over the whole surface of the probe Medicus determined the probe characteristics and their second derivatives for plane and spherical probes. His analysis shows that a spread in work function can cause a rounding of the knee of the plane probe characteristic. Both plane and spherical probe characteristics are identical to the constant work function probe characteristics only in the region of strong electron retarding potentials. The electron temperature can therefore only be obtained from the semi-logarithmic probe characteristic remote from plasma potential. Plasma potential can be determined accurately if the spread in work function w_o is known.

4.4. Perturbations due to plasma instabilities.

This section is concerned with the effect of fluctuations in certain plasma parameters on the static probe characteristic. The three parameters of interest are the electron temperature, the electron concentration and the plasma potential.

Garscadden and Emeleus (38) have examined this problem for a plasma having a Maxwellian electron distribution with pure sine - wave fluctuations. Let the amplitude of the fluctuations be denoted by $\hat{V}_e$, $\hat{V}_o$ and $\hat{N}_e$ and let their angular frequency be p_1, p_2 and p_3. Here it is convenient to express the electron temperature in electron volts defined by equation (2.4). At any given time t the instantaneous electron temperature, plasma potential and probe to plasma potential is given by

$$V_e(t) = V_e + \hat{V}_e \sin p_1 t \qquad \ldots(4.24)$$

$$V_o(t) = V_o + \hat{V}_o \sin p_2 t \qquad \ldots(4.25)$$

$$V_p(t) = V_p + \hat{V}_o \sin p_2 t \qquad \ldots(4.26)$$

When fluctuations in V_e and V_o occur simultaneously and there exists zero phase difference between the fluctuation the electron current flowing to a retarding probe is

$$I_e + I_e(t) = I_{eo} \exp\left[- \frac{|(V_p + V_o \sin p_2 t)|}{(V_e + V_e \sin p_1 t)} \right] \qquad \ldots(4.27)$$

where $V_p(t) < 0$ for all t. The current recorded by a d.c. instrument is the time averaged value of equation (4.27) and may be denoted by $\langle I_e \rangle$.

When only fluctuation in electron temperature occur $\langle I_e \rangle$ is given by

$$\frac{\langle I_e \rangle}{I_e} = J_o\left(\frac{\hat{V}_e |V_p|}{V_e^2}\right) \qquad \ldots(4.28)$$

where I_e is given by equation (3.1) and J_o () is the modified Bessel function of the first kind of zero order with pure imaginary argument. As $J_o()$ is a function of V_p it is clear from equation (4.28) that ln $\langle I_e \rangle$ against V_p is non-linear. However, as the probe approaches plasma potential V_p tends to zero and $J_o()$ tends to unity and the semi-logarithmic characteristic approaches the unperturbed characteristic: the electron temperature can thus be obtained from the slope of the characteristic in the region of plasma potential.

When only fluctuation in plasma potential occur $\langle I_e \rangle$ is given by

$$\frac{\langle I_e \rangle}{I_e} = J_o\left(\frac{\hat{V}_o}{V_e}\right) \qquad \ldots(4.29)$$

As the right hand side of equation (4.29) is independent of V_p the slope of the semi-logarithmic characteristic is unaffected by fluctuations in plasma potential. However the knee of the characteristic occurs at a potential V_o below plasma potential given by

$$\Delta V_o = V_e \ln\left[J_o\left(\frac{\hat{V}_o}{V_e}\right)\right] \qquad \ldots(4.30)$$

Crawford (39) has carried out a more thorough analysis of this problem of fluctuations by taking into account the effect of cross modulation between the three time varying parameters. The electron current I_e in a stable plasma may be represented as a function of N_e, V_o and V_e thus

$$I_e = f\left[N_e, V_o, V_e\right] \qquad \ldots(4.31)$$

The electron current in an unstable plasma will be time dependent and may be represented by

$$I_e + I_e(t) = f\left[(N_e + N_e(t)), (V_o + V_o(t)), (V_e + V_e(t))\right] \qquad \ldots(4.32)$$

Expanding by Taylor's theorem and averaging the fluctuations over one period the time averaged electron current becomes

$$\frac{\langle I_e\rangle}{I_e} = 1 + \frac{1}{2I_e}\left\{\langle \hat{N}_e(t)\rangle^2 \frac{\partial^2 I_e}{\partial N_e^2} + \langle \hat{V}_o(t)\rangle^2 \frac{\partial^2 I_e}{\partial V_o^2} + \langle \hat{V}_e(t)\rangle^2 \frac{\partial^2 I_e}{\partial V_e^2}\right\} + \frac{1}{I_e}\left\{\langle \hat{N}_e(t)\hat{V}_o(t)\rangle \frac{\partial^2 I_e}{\partial N_e \partial V_o} + \langle \hat{V}_o(t)\hat{V}_e(t)\rangle \frac{\partial^2 I_e}{\partial V_o \partial V_e} + \langle \hat{V}_o(t)\hat{N}_e(t)\rangle \frac{\partial^2 I_e}{\partial V_e \partial N_e}\right\} \qquad \ldots(4.33)$$

For a Maxwellian electron distribution in the region of electron retarding probe potentials and assuming the separate fluctuations are uncorrelated so that the cross product terms vanish on averaging, equation (4.33) reduces to

$$\frac{\langle I_e\rangle}{I_e} = 1 + \frac{1}{2}\left(\frac{\langle \hat{V}_o(t)\rangle}{V_e}\right)^2 - \frac{1}{8}\left(\frac{\langle \hat{V}_o(t)\rangle}{V_e}\right)^2 + \frac{1}{2}\frac{V_p}{V_e}\left(\frac{\langle \hat{V}_e(t)\rangle}{V_e}\right)^2 + \frac{1}{2}\left(\frac{V_p}{V_e}\right)^2\left(\frac{\langle V_e(t)\rangle}{V_e}\right)^2 \qquad \ldots(4.34)$$

When fluctuations in electron temperature are small compared with fluctuations in plasma potential and electron density and the effects of cross modulation are not neglected equation (4.33) reduces to

$$\frac{\langle I_e \rangle}{I_e} = 1 + \frac{1}{2}\left(\frac{\langle \hat{V}_o(t) \rangle}{V_e}\right)^2 - \frac{1}{N_e V_e}\,\frac{1}{T}\int_0^T \hat{N}_e(t)\hat{V}_o(t)\,dt \qquad \text{...(4.35)}$$

If $\hat{N}_e(t) = \hat{N}_e \sin(p_3 t + \lambda)$ ...(4.36)

and $\hat{V}_o(t) = \hat{V}_o \sin p_2 t$ ...(4.37)

and if $p_2 = p_3 = p$ equation (4.35) reduces to

$$\frac{\langle I_e \rangle}{I_e} = 1 + \frac{1}{2}\left(\frac{\hat{V}_o}{V_e}\right) + \frac{\hat{V}_o \hat{N}_e}{V_e N_e} \cos\lambda \qquad \text{...(4.38)}$$

The instantaneous electron concentration, probe potential, and electron probe current are respectively given by

$$N_e(t) = N_e + \hat{N}_e \sin(pt + \lambda) \qquad \text{...(4.39)}$$

$$V_p(t) = V_p + \hat{V}_o \sin pt \qquad \text{...(4.40)}$$

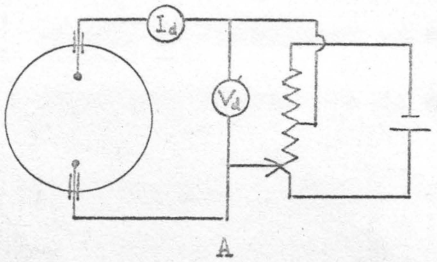
where $V_p(t) < 0$ for all t

$$I_e + I_e(t) = AN_e(t)e\left(\frac{eV_e}{2\pi m}\right)^{1/2} \exp\left[\frac{-|V_p(t)|}{V_e}\right] \qquad \text{...(4.41)}$$

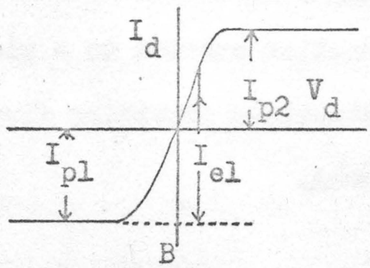
Substituting equations (4.39) and (4.40) into equation (4.41) and time averaging gives

$$\frac{\langle I_e \rangle}{I_e} = J_o\left(\frac{\hat{V}_o}{V_e}\right) + \left(\frac{\hat{N}_e}{N_e}\right)\cos\lambda\, J_1\left(\frac{\hat{V}_o}{V_e}\right) \qquad \text{...(4.42)}$$

If $\hat{N}_e$ is zero equation (4.42) reduces to equation (4.29) originally derived by Garschadden and Emeleus. Crawford also analyses the effect of plasma fluctuations on the saturation electron currents to plane, cylindrical and spherical probes. Only the saturation current to a plane probe is unaffected by plasma fluctuations providing fluctuations in electron temperature can be ignored.

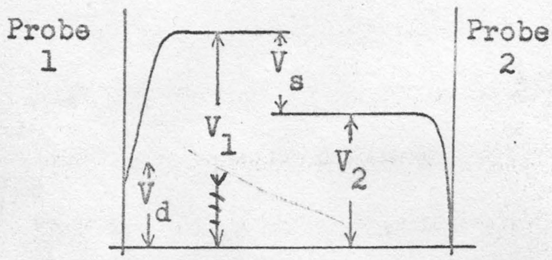


Floating double
probe circuit



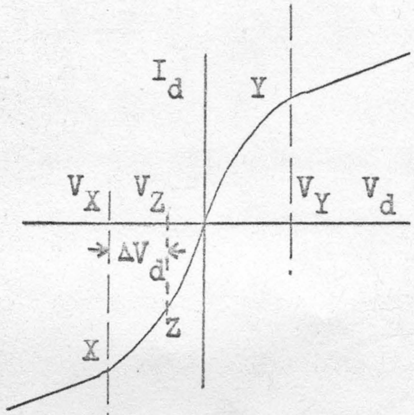
Double probe
characteristic

Figure 7



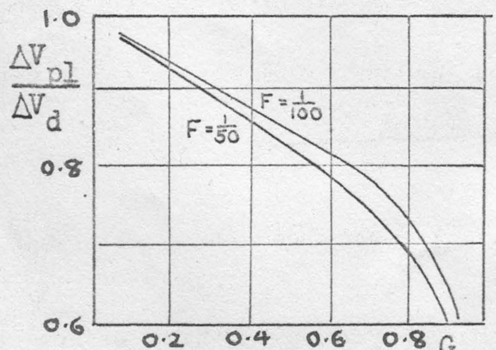
Potential distribution in
a floating double probe
arrangement

Figure 8



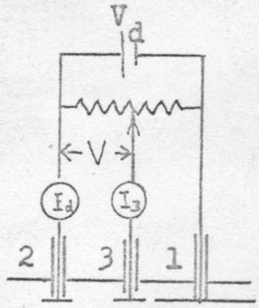
Practical double probe
characteristic

Figure 9



Dependence of $\Delta V_{p1}/\Delta V_d$ on G (40)

Figure 10



Floating triple
probe circuit

Figure 11

CHAPTER 5

FLOATING PROBE SYSTEMS

In Chapter 4 it was mentioned that fluctuations in plasma potential can cause perturbations to a Langmuir probe characteristic. The effect of these fluctuations may be partly overcome by using a floating probe system. Such a system consists of two or three probes connected together by a variable voltage supply. By varying the potential between the probes the probe current will also vary and by analysing the resulting probe characteristic information concerning the plasma parameters can be obtained.

The floating probe system has the advantage over the single probe, whose potential is varied relative to one of the discharge electrodes, in that it can be used in an electrodeless discharge.

Figure 7 shows the basic double probe measuring circuit together with an idealized double probe characteristic. V_d is the differential voltage applied between the two probes and I_d is the resultant current flowing in the external circuit.

5.1. Principles of the floating double probe system.

Providing the area ratio of the two probes is not very great both the probes will be at a negative potential with respect to the plasma for all values of V_d. The ion current flowing to a negative probe is practically independent (compared with the electron current) of the probe to plasma potential; any dependence that may occur would be due to changes in sheath area.

If the probe potential is varied with respect to the plasma by applying a potential difference between the two probes the probe to which a positive potential is applied will move closer to plasma potential. The decrease in probe to plasma potential results in an increase in electron current to the first probe and a decrease in electron current to the second probe. As the first probe is made less and less negative with respect to the plasma a point will eventually be reached where the second probe has become so negative that no electrons at all can reach it. In order to satisfy the floating conditions in this case it is necessary that the electron current I_{e1} flowing to the first probe is equal to the sum of the ion currents flowing to both the probes, hence

$$I_{e1} = I_{+1} + I_{+2} \qquad \ldots(5.1)$$

Making the second probe even more negative with respect to the first probe cannot bring about a further reduction in electron current as all the electrons are now repelled by the negative probe. This means that there cannot be an increase in electron current to the first probe. This part of the double probe characteristic is known as the Saturation region. If, however, the positive ion sheath surrounding the strongly negative second probe increases in area with increasingly more negative probe potential I_{+2} increases.

In this region it is seen from equation (5.1) that an increase in I_{+2} must also result in an increase in I_{e1} and the characteristic possesses a slope in the so called saturation region. There will also result a very slight decrease in I_{+1}.

5.2 Formulation of the floating double probe equations.

The analysis of the floating double probe system was first given by Johnson and Malter (40), (41). Figure 8 shows the potential distribution in the plasma when a floating double probe system has a differential potential V_d applied between the two probes. Let V_{p1} and V_{p2} be the probe to plasma potentials for probes one and two respectively and let V_s be the difference in plasma potential in the neighbourhood of the two probes. Let I_{+1} and I_{+2} be the positive ion currents flowing to probes one and two respectively. The positive ion current flowing to a negative probe is only slightly dependent on the probe to plasma potential. On the other hand the electron current varies exponentially with probe to plasma potential. Let I_{e1} and I_{e2} be the electron currents flowing to probes one and two respectively.

$$I_{e1} = I_{eo1} \exp\left(\frac{eV_{p1}}{kT_e}\right) = j_{eo1}A_1 \exp\left(\frac{eV_{p1}}{kT_e}\right) \quad \ldots(5.2)$$

$$I_{e2} = I_{eo2} \exp\left(\frac{eV_{p2}}{kT_e}\right) = j_{eo2}A_2 \exp\left(\frac{eV_{p2}}{kT_e}\right) \quad \ldots(5.3)$$

where V_{p1} and $V_{p2} < 0$ and A_1 and A_2 are the surface areas of probes one and two respectively.

To satisfy the floating conditions of the double probe system it is necessary that the following equality holds

$$I_{+1} + I_{+2} = I_{e1} + I_{e2} \qquad \ldots(5.4)$$

I_{+1} and I_{+2} will be assumed to remain constant with probe to plasma potential.

$$I_{+1} + I_{+2} = \Sigma I_{+} = \text{Constant} \qquad \ldots(5.5)$$

In actual fact ΣI_{+} is not a constant and its dependence on V_d is discussed in the next sub-section.

Substituting equations (5.2), (5.3) and (5.5) into equation (5.4) gives

$$\Sigma I_{+} = j_{eo1}A_1 \exp\left(\frac{eV_{p1}}{kT_e}\right) + j_{eo2}A_2 \exp\left(\frac{eV_{p2}}{kT_e}\right) \qquad \ldots(5.6)$$

From Figure 8 it is seen that

$$V_{p2} = V_{p1} + V_d - V_s \qquad \ldots(5.7)$$

Substituting equation (5.7) into equation (5.6), dividing both sides by I_{e1} and rearranging gives

$$\frac{\Sigma I_{+}}{I_{e1}} - 1 = \left(\frac{j_{eo2}\,A_2}{j_{eo1}\,A_1}\right) \exp\left[\frac{e}{kT_e}\,(V_d - V_s)\right] \qquad \ldots(5.8)$$

Taking natural logarithms of both sides gives

$$\ln\left[\frac{\Sigma I_{+}}{I_{e1}} - 1\right] = \ln\left[\frac{j_{eo2}\,A_2}{j_{eo1}\,A_1}\right] + \frac{eV_d}{kT_e} - \frac{eV_s}{kT_e} \qquad \ldots(5.9)$$

If V_s remains constant the electron temperature may be deduced from a plot of the left hand side of equation (5.9) against V_d.

5.3 Variation of ΣI_p with V_d.

A practical double probe characteristic shown in Figure 9 usually has a slope in the saturation regions and it may be difficult to estimate the points at which I_{e1} and I_{e2} are zero. The current I_d, flowing in the external circuit is given by

$$I_d = I_{e1} - I_{+1} \qquad \ldots(5.10)$$

$$I_d = I_{+2} - I_{e2} \qquad \ldots(5.11)$$

In the regions where I_{e1} or I_{e2} are zero I_d is equal to $- I_{+1}$ and I_{+2} respectively. It is found that in these saturation regions I_{+1} and I_{+2} vary approximately linearly with V_d. V_{p1} and V_{p2} also vary, to a very good approximation, linearly with V_d. However, once an appreciable electron current flows to the probe the probe to plasma potential no longer varies linearly with V_d and one cannot simply extrapolate the linear saturation region beyond the saturation region in order to estimate the value of the ion current outside this region. If equation (5.9) is to be applied to a practical double probe characteristic it is necessary to know how $\sum I_+$ depends on V_d outside the saturation regions.

In Figure 9 let X correspond to the start of the saturation region such that the electron current contribution is 1/100th to 1/50th of the total ion current. Johnson and Malter (40) show that the change in V_{+1} due to a change in V_d is given by

$$\Delta V_{+1} = \Delta V_d \frac{\ln(F/G)}{\ln (F - 1)/(G - 1)} \qquad \ldots(5.12)$$

where F and G are defined by

$$F = \frac{I_{e1}}{\sum I_+} \quad \text{at X} \qquad \ldots(5.13)$$

$$G = \frac{I_{e1}}{\Sigma I_+} \quad \text{at } Z \qquad \ldots(5.14)$$

and ΔV_d is given by $(V_Z - V_X)$. The point X is defined by an F value of between 1/100th and 1/50th and the variation of $\Delta V_{+1}/\Delta V_d$ with G, for these two values of F, is shown in Figure 10. What is the value of G in the unsaturated region? For a symmetrical double probe arrangement Johnson and Malter (40) state that G = 0.5 at V_d = zero whilst a more recent analysis by Burrows (42) shows that in general for an asymmetrical double probe arrangement G = 0.5 at the point of inflection of the characteristic. Figure 10 shows that for $G = 0.5$ $\Delta V_{+1}/\Delta V_d = 0.85$ or

$$\Delta V_{+1} = 0.85\,\Delta V_d \qquad \ldots(5.15)$$

If a linear dependence of I_{+1} on V_{+1} can be assumed outside the saturation region equation (5.15) enables ΣI_+ to be estimated at the point of inflection of the double probe characteristic.

5.4 Electron temperature by the equivalent resistance method.

Equation (5.8) may be written the form

$$\frac{\Sigma I_+}{I_{e1}} - 1 = \sigma \exp\left(\frac{eV_d}{kT_e}\right) \qquad \ldots(5.16)$$

where

$$\sigma = \left(\frac{j_{eo2}A_2}{j_{eo1}A_1}\right) \exp\left(\frac{-eV_s}{kT_e}\right) \qquad \ldots(5.17)$$

Solving equation (5.16) for I_{e1} gives

$$I_{e1} = \Sigma I_+ \left[\sigma \exp\left(\frac{eV_d}{kT_e}\right) + 1\right]^{-1} \qquad \ldots(5.18)$$

Rearranging equation (5.10) and differentiating with respect to V_d assuming I_{+1} is independent of V_d gives

$$\frac{dI_{e1}}{dV_d} = \frac{dI_d}{dV_d} \qquad ...(5.19)$$

Differentiating equation (5.18) with respect to V_d, substituting into equation (5.19) and putting V_d equal to zero gives

$$\left.\frac{dI_d}{dV_d}\right)_{V_d=0} = \Sigma I_+ \frac{e}{kT_e} \frac{\sigma}{(\sigma+1)^2} \qquad ...(5.20)$$

From equation (5.18) it is seen that when V_d equals zero σ is given by

$$\sigma = \left(\frac{\Sigma I_+}{I_{e1}} - 1\right)_{V_d=0} = \left(\frac{1}{G} - 1\right) \qquad ...(5.21)$$

where G is defined by equation (5.14) with $V_Z = V_d = 0$,

$$G = \left.\frac{I_{e1}}{\Sigma I_+}\right)_{V_d=0} \qquad ...(5.22)$$

and may be measured directly from the double probe characteristic.

$1/(dI_d/dV_d)$ has the dimensions of resistance and may be put equal to an Equivalent resistance R_o. The electron temperature is thus given by (40)

$$T_e = (G - G^2) \frac{e}{k} R_o \Sigma I_+ \qquad ...(5.23)$$

where $$R_o = \left[1 \Big/ \left(\frac{dI_d}{dV_d}\right)\right]_{V_d=0} \qquad ...(5.24)$$

In the case of a symmetrical double probe characteristic G is equal to 0.5 at $V_d = 0$ and equation (5.23) becomes

$$T_e = \frac{e}{4k} R_o \Sigma I_+ \qquad ...(5.25)$$

In deriving these expressions for T_e Johnson and Malter (40) have assumed that I_{+1} is independent of V_d and also that V_d equal to zero does not fall within the saturation regions of the characteristic.

The problem of a symmetrical double probe system in which the saturation ion current is no longer independent of V_d has been considered by Yamamoto and Okunda (43). Under these conditions they show that at $V_d = 0$

$$\left.\frac{dI_{+2}}{dV_d}\right)_{V_d=0} = \left.\frac{dI_{+1}}{dV_d}\right)_{V_d=0} = \frac{S}{2} \qquad \ldots(5.26)$$

where S is the slope of the saturation ion current regions (which are assumed to be equal). The electron temperature is then given by

$$T_e = \frac{e}{4k} \Sigma I_+ \Big/ \left(\frac{1}{R_o} - \frac{S}{2}\right) \qquad \ldots(5.27)$$

where R_o is defined by equation (5.24).

The analysis of a strongly asymmetrical double probe characteristic in which the saturation regions do not necessarily have equal slopes has been made by Burrows (42). In this case it is shown that T_e is given by

$$T_e = \frac{e}{4k}\left(\Sigma I_{+s} - 0.85\, S\Delta V_d\right) \Big/ \left(\frac{1}{R_1} - \frac{S}{2}\right) \qquad \ldots(5.28)$$

where ΣI_{+s} is the sum of the ion currents in the saturation regions, S is the mean slope of the saturation regions and R_1 is defined by

$$R_1 = \left[1 \Big/ \left(\frac{dI_d}{dV_d}\right)\right]_{V_d = V_1} \qquad \ldots(5.29)$$

where V_i is the differential potential corresponding to the point of inflection of the characteristic. ΔV_d is the potential difference between the two points on the characteristic at which the characteristic just begins to deviate from the linear saturation regions.

5.5 Fraction of electrons sampled by a double probe system.

The maximum electron current flowing in a floating double probe circuit is equal to the sum of the saturation positive ion currents. For a symmetrical probe arrangement this is clearly very much less than the total random electron current that would flow to a probe held at plasma potential. The ratio of the electron current flowing in the probe circuit to the random electron current is a measure of the fraction of the total electrons sampled by the system. For a symmetrical probe arrangement it can be shown (40) that this ratio is given by

$$\frac{I_{e1}}{I_{eo}} = 2 \exp\left(\frac{eV_f}{kT_e}\right) \qquad \ldots(5.30)$$

where $V_f < 0$ and is the floating potential of probe 1, i.e. the probe to plasma potential when I_d is zero. It can be shown that V_f is given by

$$V_f = \left(\frac{kT_e}{2e}\right) \ln\left(\frac{T_e m}{T_+ M}\right) \qquad \ldots(5.31)$$

where T_+ is the positive ion temperature at the positive ion sheath edge. By substituting typical values into equations (5.30) and

(5.31) Johnson and Malter (40) show that for a symmetrical double probe system less than 1% of the random electron current is sampled. In the case of an asymmetrical double probe system (44) in which the area ratio is F equation (5.30) becomes

$$\frac{I_{e1}}{I_{eo}} = (1 + F) \exp\left(\frac{eV_f}{kT_e}\right) \qquad \ldots(5.32)$$

Okunda and Yamamoto (44) point out that the whole of the electron energy distribution may be sampled if the ratio (1 + F) is equal to or greater than the ratio of random electron to random ion currents. Under these circumstances the smaller probe behaves essentially as a single probe. The determination of T_e by the equivalent resistance method (sub-section 5.4) has been shown to be consistant with the single probe method described in sub-section 3.1 (42), (45), (46) and (47).

5.6 Floating probe systems for determining electron energy distributions

The determination of the electron energy distribution depends on the determination of the second differential of the electron current as a function of probe to plasma potential. To carry out these measurements it is necessary for the whole of the electron energy distribution to be sampled and for the probe to plasma potential to be determined.

5.6.1. Double probe method.

The whole of the electron energy distribution may be sampled, using a floating double probe system, by having the area ratio of the two probes F very large, say F>1000 (44). If F is sufficiently great it can be shown that as V_d is varied there is a negligible change in the probe to plasma potential of the larger probe and it may therefore be taken as a reference electrode. The double probe characteristic under these conditions should correspond to the single probe characteristic but unfortunately measurements carried out by Okunda and Yamamoto (44) do not confirm this. They claim the reason for this is due to excessive electron drain from the plasma.

5.6.2. Triple probe method.

Normally the floating double probe characteristic gives no indication as to the variation of the electron current with probe to plasma potential as there is no fixed reference potential with which to compare the probe potential. This problem may be overcome by the introduction of a third probe whose potential is always kept constant with respect to the plasma (43), and (44). The constant potential is conveniently taken to be the floating potential of the third probe.

The circuit for the floating triple probe is shown in Figure 11. The method of measuring the characteristic is to apply a potential V_d between the probes 1 and 2, of large area ratio. The tapping of the potential divider is then adjusted so that there is zero current flowing to probe 3. Probe 3 is then at floating potential and the potential of probe 1 with respect to floating potential can then be measured directly and hence a plot of I_d against V obtained. The electron energy distribution is then obtained from a measure of d^2I_d/dV^2 assuming I_d is entirely electronic.

Both the double and the triple probe techniques described above rely on the electron energy distribution being found from the second differential of the experimentally determined probe characteristic. Crawford et als (48) have devised a double single-probe technique which involves only a single differentiation of the characteristic. The method consists of holding two identical single probes at slightly different potentials with respect to the plasma and measuring directly the difference in electron current to the probes as a function of potential difference between the two probes. This can be shown to be a direct measure of dI_e/dV. The advantage of this method over the single probe method is that is is the difference in potential between the two probes that is important and any fluctuations in plasma potential clearly cancel one another out.

CHAPTER 6

THE COLLECTION OF POSITIVE IONS IN A COLLISIONLESS PLASMA

Here a collisionless plasma is taken to be one in which there are no collisions between positive ions and other particles once the ions move towards a probe under the influence of the probe's electrostatic field.

Langmuir found that his simple orbital theory would not account satisfactorily for the observed positive ion characteristics and was forced to replace his theoretical expression

$$I_+ = AN_+e\left(\frac{kT_+}{2\pi M}\right)^{1/2} = 0.40\ AN_+e\left(\frac{kT_+}{M}\right)^{1/2} \qquad \ldots(6.1)$$

by the semi-empirical expression

$$I_+ = 2.20\ AN_+e\left(\frac{kT_e/2}{2\pi M}\right)^{1/2} = 0.62\ AN_+e\left(\frac{kT_e}{M}\right)^{1/2} \ldots(6.2)$$

for the positive ion current to a cylindrical and spherical probe when $r_s \approx r_p$ and to a plane probe when $x_s \geqslant 0$.

The replacement of T_+ by T_e and the change in the numerical constant is the result of the penetrating electrostatic field beyond the boundary of the space charge sheath into the so called Extra-sheath region.

Considerable experimental evidence supports Langmuir's belief that it is the electron temperature rather than the ion temperature that controls the ion current to a negative probe. It has been shown, however, that the ion temperature does influence, to some extent, the ion current to a negative probe. The effect of this is especially important in plasmas where the ion temperature is comparable to the electron temperature.

6.1. Effective collecting area of a probe.

According to Langmuir's probe theory the effective collecting area A' of a probe, when r_s/r_p tends to infinity, is given by

$$A' = A, \text{ for a plane probe,}$$
$$A' = 4\pi h_s^2, \text{ for a spherical probe, and} \qquad \ldots(6.3)$$
$$A' = 2\pi h_c L, \text{ for a cylindrical probe}$$

where $h_{s,c}$ is defined as the impact parameter. If the ions possess a Maxwellian ion energy distribution h_c is given by

$$h_c = \frac{2r_p}{\pi^{1/2}}\left(1 - \frac{V_p}{V_+}\right)^{1/2} \qquad \ldots(6.4)$$

where V_+ is related to the ion temperature by

$$eV_+ = kT_+ \qquad \ldots(6.5)$$

and $V_p < -2V_+$ and h_s is given by

$$h_s = r_p\left(1 - \frac{V_p}{V_+}\right)^{1/2} \qquad \ldots(6.6)$$

when the ions are mono-energetic $h_c = h_s = h$ where h is given by

$$h = r_p\left(1 - \frac{V_p}{V_+}\right)^{1/2} \qquad \ldots(6.7)$$

h is a measure of the distance to which the electrostatic field extends from the probe. It would appear from equation (6.7) that if an ion approached the probe at a distance less than h it would be collected by the probe. Langmuir (1) and Bohm et als (28) have pointed out that for an ion, approaching the probe at a distance less than h, to be collected by the probe it is necessary that the field surrounding the probe satisfies certain conditions. Equation (6.7) represents only the maximum possible value for the effective collecting radius and it is conceivable that under certain conditions the effective collecting radius is less than h. This is seen to be so in equation (3.19) when practically all of the potential drop occurs across a thin space charge sheath surrounding the probe. In this case there are no long range forces and the increase in ion current with V_p will be less than that obtained by using equation (6.7). The probe current calculated from the value of h given by equation (6.7) only represents an upper limit.

When the random ion current density is large r_s/r_p approaches unity and I_+ can be calculated from the space charge equations. When the random ion current density and the probe radius is small and the ions have high initial velocities r_s/r_p is large and I_+ is governed by orbital motion and equation (6.7) defines the effective collecting radius of the probe. For the orbital theory to apply the potential distribution in the space surrounding the probe must satisfy (1) the inequality

$$V_1 > \left(\frac{r_s^2 - r_1^2}{r_s^2 - r^2}\right) \frac{r^2}{r_1^2} \cdot V_1 \qquad \ldots(6.8)$$

where $r_p < r_1 < r$ and V_1 is the potential at r_1.

For an ion to approach within a radius r the impact parameter will be given by

$$h = r\left(1 - \frac{V}{V_+}\right)^{1/2} \qquad \ldots(6.9)$$

Bohm et als (28) argue that h must increase monotonically with r in order that the ions, approaching the probe with an impact parameter h, shall reach the probe. Expressing this mathematically the criterion becomes

$$\frac{dh}{dr} \geq 0 \quad \text{for} \quad r > r_p \qquad \ldots(6.10)$$

Differentiating equation (6.9) gives an expression for the field distribution surrounding the probe that must be satisfied for h to represent the effective collecting radius of the probe.

$$\frac{r}{2V_+}\frac{dV}{dr} \leq \left(1 - \frac{V}{V_+}\right) \qquad \ldots(6.11)$$

Allen et als (49) show that when there exists a distribution of ion energies the potential distribution must satisfy

$$\frac{V}{V_p} > \left(\frac{r_p}{r}\right)^2 \qquad \ldots(6.12)$$

If this inequality does not hold, which may well be the case in a dense plasma, an absorption radius r_A will exist that is less than h.

6.2 Effects of a penetrating field.

In developing the theory of the collection of positive ions all the potential drop between the probe and plasma has been assumed to occur across the sheath region. This assumption is inconsistant with experimental observations and it has been found necessary to assume that a small fraction of the applied potential drop penetrates some distance into the neutral plasma. The region over which this occurs is known as the Extra sheath.

The case of when the ions are assumed to have zero energy outside the extra sheath region and a monoenergetic value of eV_+ at the sheath edge has been considered by Bohm (50) and by Schultz et al (51). Boyd (32), (52) has considered the more general case of when the ions have an energy distribution at the sheath edge.

In the monoenergetic case it is assumed that the field at the sheath edge is effectively zero and that the ions fall through the extra sheath region and aquire an energy eV_+ on reaching the sheath edge. The sheath edge may be defined as the point at which the resultant charge density ρ is zero. At the sheath edge $d\rho/dx$ may also be taken to be zero (51). The ion velocity in the sheath region of a plane probe is

$$u_+ = \left(\frac{2e}{M}\right)^{1/2} \left(V_+ - V\right)^{1/2} \qquad \ldots(6.13)$$

where $V < 0$. The ion concentration in terms of the ion current density j_+ is thus

$$N_+ = \frac{j_+}{e} \left(\frac{M}{2e}\right)^{1/2} \left(V_+ - V\right)^{-1/2} \qquad \ldots(6.14)$$

and the electron concentration is

$$N_e = N_{eo} \exp\left(\frac{V}{V_e}\right) \qquad \ldots(6.15)$$

The resultant charge density is therefore

$$= j_+ \left(\frac{M}{2e}\right)^{1/2}\left(V_+ - V\right)^{-1/2} - eN_{eo} \exp\left(\frac{V}{\bar{V}_e}\right) \qquad \ldots(6.16)$$

Equating both ρ and $d\rho/dx$ to zero at the sheath edge where also V and dV/dx are zero gives

$$V_+ = \frac{V_e}{2} \qquad \ldots(6.17)$$

In Boyd's analysis (52) the ions are assumed to have an energy distribution f(E) at the sheath edge. If N_{+s} is the total ion concentration at the sheath edge the contribution to the ion current, at the sheath edge, by ions in the energy range E to E + dE is

$$N_{+s}\, f(E)\, e\left(\frac{2e}{M}\right)^{1/2} E^{1/2}\, dE \qquad \ldots(6.18)$$

Here E is taken to be the energy associated with the velocity component normal to the sheath edge. At a point just inside the sheath edge where the potential is δV (< 0) with respect to the sheath edge the ions have an energy $(E - \delta V)$ and a velocity $(2e/M)^{1/2}(E - \delta V)$. If these ions contribute dN_+ to the ion concentration the contribution, by the ions in the energy range E to E + dE at the sheath edge, to the ion current just inside the sheath is

$$dN_+\, e\left(\frac{2e}{M}\right)^{1/2}(E - \delta V) \qquad \ldots(6.19)$$

As the ion current at the sheath edge is equal to the ion current just inside the sheath equations (6.18) and (6.19) are equal. Equating and solving for dN_+ gives

$$dN_+ = N_{+s} \; f(E)E^{1/2}(E - \delta V)^{-1/2} dE \qquad \ldots(6.20)$$

Hence the total ion concentration at the point δV just inside the sheath edge is

$$N_+ = N_{+s} \int_0^\infty f(E)E^{1/2}(E - \delta V)^{-1/2} dE \qquad \ldots(6.20)$$

The electron concentration at the same point is

$$N_e = N_{es} \; \exp\left(\frac{\delta V}{V_e}\right) \qquad \ldots(6.21)$$

For a stable positive ion sheath to form $N_+ \geqslant N_e$

Hence

$$\int_0^\infty f(E)E^{1/2}(E - \delta V)^{-1/2} dE \geqslant \exp\left(\frac{\delta V}{V_e}\right) \qquad \ldots(6.22)$$

where the sheath edge has been defined as the point where $N_{+s} \equiv N_{es}$.

If δV is small and negative (6.22) reduces to

$$V_+ \geqslant \frac{V_e}{2} \qquad \ldots(6.23)$$

where V_+ is the reciprocal of the mean inverse ion energy and is given by

$$V_+ = \left[\frac{\int_0^\infty f(E)\frac{1}{E}dE}{\int_0^\infty f(E)dE}\right]^{-1} \qquad \ldots(6.24)$$

6.3 <u>Positive ion characteristics.</u>

The interpretation of probe characteristics requires a knowledge of the state of the plasma being investigated. The state of the plasma determines the distance over which an electric field can influence carriers. A convenient measure of this distance is the Debye length λ_D and is defined by the relation

$$\lambda_D = \left(\frac{kT_e}{4\pi N_{eo} e^2}\right)^{1/2} = \left(\frac{V_e}{4\pi N_{eo} e}\right)^{1/2} \qquad \ldots(6.25)$$

Assuming λ_d is also a measure of the sheath thickness the following table summarizes the range of plasma states over which the various proposed theories should be applicable.

TABLE I

State of plasma	Ion energy	Author and reference
$\frac{r_p}{\lambda_D} \to 0$	Maxwellian distribution of ion energy with V_+ at sheath edge equal to that in neutral plasma. Orbital theory.	Langmuir (Chapters 2 and 3)
$1.4 \leq \frac{r_p}{\lambda_D} \leq 13$	Probe potential V_p sufficiently negative for $\lvert V_p \rvert \gg V_+$	Allen et als (<u>49</u>)
$4 \leq \frac{r_p}{\lambda_D} \leq 15$	V_+/V_e equal to 0 and 0.01 to 0.1. V_+ monoenergetic.	Bernstein et al (<u>53</u>).
$\frac{r_p}{\lambda_D} \to \infty$	Maxwellian distribution of ion energy with V_+ at sheath edge equal to that in neutral plasma. Space charge theory.	Langmuir (Chapter 2 and 3)
$\frac{r_p}{\lambda_D} \to \infty$	V_+/V_e equal to 0, 0.01 and 0.5. V_+ monoenergetic.	Bohm et als (<u>28</u>)
$\frac{r_p}{\lambda_D} \to \infty$	V_+/V_e equal to 0.1 to infinity. V_+ monoenergetic	Lam (<u>54</u>)

6.3.1. r_p/λ_D small for a spherical probe.

The theories to be presented in this sub-section are those proposed by Allen et als (49) and by Bernstein et al (53) and are based on the numerical solution of Poisson's equation in spherical coordinates. The chief problem is to derive an expression for the ion concentration in terms of position and potential. Bernstein et al expression for N_+ is obtained from a detailed analysis of the ion's orbital motion around a spherical probe.

In the case of a central field the ion's orbit must lie in a plane passing through the centre of the probe. Its motion is characterized completely by its energy E, its angular momentum J, and the orientation of the plane of the orbit α. As all orientations are equally probable the distribution function may be denoted by f(E,J). f(E,J) remains constant along the ion's trajectory as long as no collisions occur. Therefore

$$f(E,J) = \text{const.} \quad \text{for} \quad r_p \leq r \leq \ell \qquad \ldots(6.27)$$

E and J are functions of velocity and position and may be expressed in terms of the coordinates w, α and u of a cylindrical coordinate system. u is the radial velocity component and w is the velocity component perpendicular to u. α is the azimuthal angle defining the angle of orientation of the orbit. The elementary velocity space is

$$du\, dw\, w\, d\alpha \qquad \ldots(6.28)$$

u and w may be expressed in terms of E and J as follows

$$E = \frac{M}{2}(u^2 + w^2) + eV \qquad \ldots(6.29)$$

where $V < 0$,

$$J = Mrw \qquad \text{...(6.30)}$$

The elementary velocity space now becomes

$$\frac{J\,dJ\,dE\,d\alpha}{M^2r^2\left[2M(E - eV) - \frac{J^2}{r^2}\right]^{1/2}} \qquad \text{...(6.31)}$$

The ion concentration at a radius r is thus

$$N_+ = \frac{2\pi}{M^2r^2}\iint\frac{f(E,J)\,J\,dJ\,dE}{\left[2M(E - eV) - \frac{J^2}{r^2}\right]^{1/2}} \qquad \text{...(6.32)}$$

where the integration over all possible values of α from 0 to 2π has been carried out. The limits of integration of E and J can be deduced from a study of the limiting values of E and J for the various permitted ion orbits. This involves a consideration of the ion's motion towards the probe as well as the reflected motion of some of the ions at some radius r_o. The distribution function is independant of J in the case of a Maxwellian distribution and so the distribution function may be written as f(E). The integration in equation (6.32) is insensitive to the form of the distribution function and so for simplicity a monoenergetic distribution will be assumed. This is conveniently represented by the Dirac δ- function.

$$f(E) = \frac{M^2N_{+o}}{(4\pi eV_+)^{1/2}}\,.\,\delta(E - eV_+) \qquad \text{...(6.33)}$$

where V_+ is the ion energy in the undisturbed plasma at a distance $r \geq \ell$. The ion concentration at a point r is found by substituting equation (6.33) into equation (6.32) and integrating between the appropriate limits.

$$N_+ = \frac{N_{+o}}{2}\left[\left(1 - \frac{V}{V_+}\right)^{1/2} \pm \left(1 - \frac{V}{V_+} - \frac{I_+}{I_r}\right)^{1/2}\right] \qquad \ldots(6.34)$$

where $V < 0$ and

$$I_r = \pi r^2 N_{+o}\, e \left(\frac{2eV_+}{M}\right)^{1/2} \qquad \ldots(6.35$$

The positive sign corresponds to $r \geqslant r_o$ and the negative sign to $r \leq r_o$ where r_o is defined by

$$\left(1 - \frac{V_o}{V_+} - \frac{I_+}{I_{r,o}}\right) = 0 \qquad \ldots(6.36$$

$$\frac{d}{dr_o}\left(1 - \frac{V_o}{V_+} - \frac{I_+}{I_{r,o}}\right) = 0 \qquad \ldots(6.37)$$

Subscript 0 corresponds to V and I_r values at $r = r_o$.

The electron concentration at r where the potential is V is

$$N_e = N_{eo} \exp\left(\frac{V}{V_e}\right) \qquad \ldots(6.38)$$

where $V < 0$ and $N_{+o} = N_{eo}$.

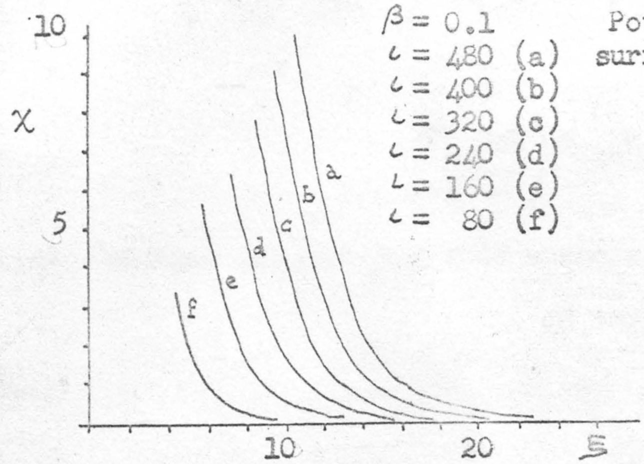
The next step is to substitute the above expressions for N_+ and N_e into Poisson's equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -4\pi e\left(N_+ - N_e\right) \qquad \ldots(6.39)$$

Equation (6.38) then reduces to

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\chi}{d\xi}\right) = \frac{1}{2}\left(1 + \frac{\chi}{\beta}\right)^{1/2} \pm \frac{1}{2}\left(1 + \frac{\chi}{\beta} - \frac{L}{\beta^{1/2}\xi^2}\right)^{1/2} - e^{-\chi} \qquad \ldots(6.40)$$

where the following transforms have been made



Potential distribution surrounding a spherical probe (53)

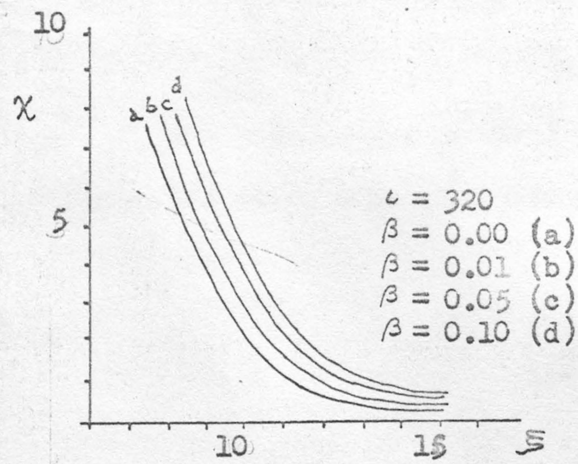
$\chi = -V/V_e$

$\xi = r/\lambda_D$

$\iota = \frac{4I_+}{e}\left(\frac{M}{2eV_e}\right)^{1/2}$

$\beta = V_+/V_e$

Figure 12



Potential distribution surrounding a spherical probe (53)

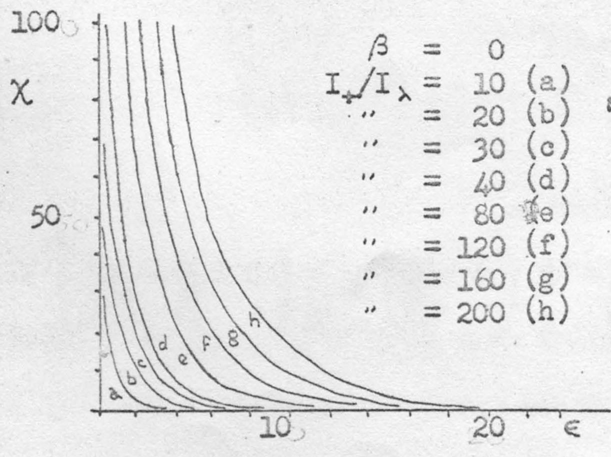
$\chi = -V/V_e$

$\xi = r/\lambda_D$

$\iota = \frac{4I_+}{e}\left(\frac{M}{2eV_e}\right)^{1/2}$

$\beta = V_+/V_e$

Figure 13



Potential distribution surrounding a spherical probe (49)(55)

$\chi = -V/V_e$

$\epsilon = 2^{\frac{1}{2}} r/\lambda_D$

$I_\lambda = \frac{(eV_e)^{3/2}}{2(2M)^{1/2}}$

$\beta = V_+/V_e$

Figure 14

$$\left.\begin{aligned}
\xi &= \frac{r}{\lambda_D}, \quad \xi_0 = \frac{r_o}{\lambda_D}, \quad \xi_p = \frac{r_p}{\lambda_D} \\
\chi &= -\left(\frac{V}{V_e}\right), \quad \chi_p = -\left(\frac{V_p}{V_e}\right) \\
\beta &= \frac{V_+}{V_e} \\
\iota &= \frac{4I_+}{V_e}\left(\frac{M}{2eV_e}\right)^{1/2}
\end{aligned}\right\} \qquad \ldots(6.41)$$

The positive sign applies when $\xi \geq \xi_0$ and the negative when $\xi \leq \xi_0$ where ξ_0 is defined by

$$\left(1 + \frac{\chi_o}{\beta} - \frac{\iota}{\beta^{1/2}\xi_0^2}\right) = 0 \qquad \ldots(6.42)$$

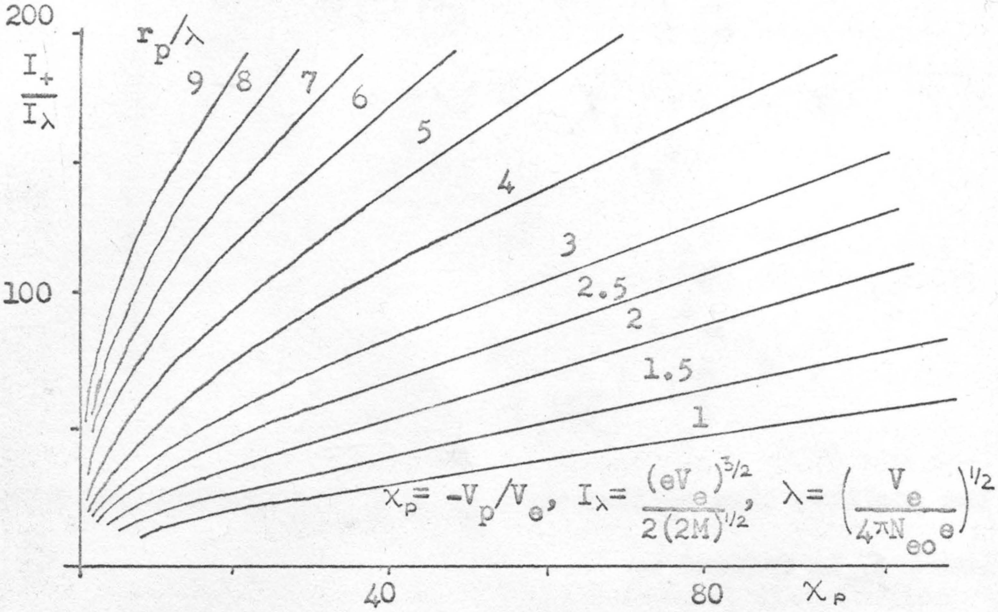
$$\frac{d}{d\xi_0}\left(1 + \frac{\chi_o}{\beta} - \frac{\iota}{\beta^{1/2}\xi_0^2}\right) = 0 \qquad \ldots(6.43)$$

The dependence of χ on ξ for constant ι and β may be obtained by integrating equation (6.40) and is shown in Figures 12 and 13 (53). The positive ion current characteristic, that is the dependence of ι on χ, may be determined for $\beta = 0.10$ by cross plotting the curves in Figure 12 for any desired value of $\xi = \xi_p$ (the probe radius).

When the positive ion energy is very much less than the electron energy β tends to zero and equation (6.40) reduces to

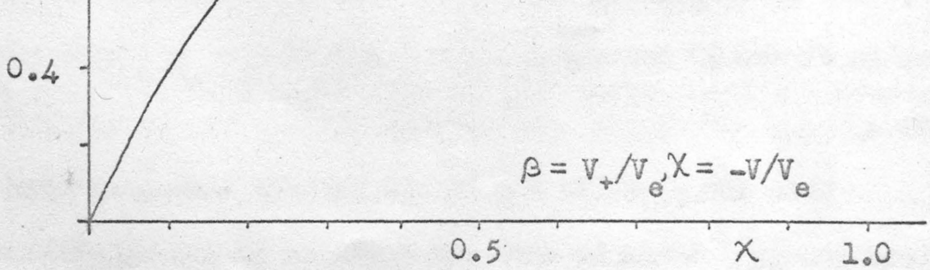
$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\chi}{d\xi}\right) = \frac{\iota}{4\chi^{1/2}\xi^2} - e^{-\chi} \qquad \ldots(6.44)$$

Rearranging we have



The computed positive ion current-voltage characteristics of a spherical probe for $V_+/V_e = 0$ (49)(55)

Figure 15



Dependence of $(r_A/r_s)^2$ on χ for a spherical probe for $\beta = 0.5$ (28)

Figure 16

$$\iota = 4\chi^{\frac{1}{2}}\xi^2\left[e^{-\chi} + \frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\chi}{d\xi}\right)\right] \quad \ldots(6.45)$$

If the following transforms are made

$$\iota = 2I_+/I_\lambda$$
$$\xi = \epsilon/2^{\frac{1}{2}} \quad \ldots(6.46)$$

equation (6.45) becomes

$$\frac{I_+}{I_\lambda} = \chi^{\frac{1}{2}}\epsilon^2\left[e^{-\chi} + 2\left(\frac{2}{\epsilon}\frac{d\chi}{d\epsilon} + \frac{d^2\chi}{d\epsilon^2}\right)\right] \quad \ldots(6.47)$$

Equation (6.47) was originally derived by Allen et als using a more straight forward analysis and assuming V_+ is effectively zero. Instead of substituting equation (6.34) and (6.38) into the expression for the resultant charge density in the right hand side of equation (6.39) the expression given by (6.16) is used with $V_+ = 0$. Equation (6.39) immediately reduces to equation (6.47) when the following transforms are made

$$\epsilon = 2^{\frac{1}{2}}r/\lambda_D , \quad \epsilon_p = 2^{\frac{1}{2}}r_p/\lambda_D$$
$$\chi = - V/V_e \quad \ldots(6.48)$$
$$I_\lambda = \frac{(eV_e)^{3/2}}{e(2M)^{\frac{1}{2}}}$$

The dependence of χ on ϵ for constant I_+/I_λ may be obtained by integrating equation (6.47) and is shown in Figure 14 (49)(55). The positive ion current characteristics may be determined by cross plotting the curves in Figure 14 for any desired value of $\epsilon = \epsilon_p$. These characteristics are shown in Figure 15 and may be used for the determination of N_o. The procedure is to plot the experimental characteristic in the form I_+/I_λ against χ and to estimate the value of ϵ_p by comparison with the theoretical curves. Once ϵ_p has been determined N_{eo} is determined from equations (6.25) and (6.48) providing V_e and r_p are known.

6.3.2. r_p/λ_D tending to infinity for a spherical probe.

This sub-section deals with the case when the sheath region surrounding a spherical probe is very thin. In the first instant the probe potential is assumed to be strongly negative and the sheath region is assumed to be so thin that any variation in thickness with probe potential produces a negligible change in the effective collecting area of the probe. As a first approximation this effective collecting area may be taken to be equal to the surface area of the probe. In the second instant the dependence of the positive ion current on both slightly negative and strongly negative probe potentials is considered.

For $r_s \approx r_p$ and $|V_p| > 0$ Langmuir (chapters 2 and 3) shows that the positive ion current is independent of V_p and depends only on the sheath radius and the mean radial velocity component at the sheath edge. When $r_s \approx r_p$ and the variation in sheath thickness with probe potential is negligible $A_s \approx A$ and I_+ is given by

$$I_+ = AN_+e\,\bar{v}_+ \qquad \ldots(6.49)$$

where N_+ and $\bar{v}_+$ refer to values at the sheath edge. In Langmuir's original analysis N_+ and $\bar{v}_+$ were computed assuming no extra sheath region and that the ion concentration and ion temperature at the sheath edge were the same as in the undisturbed plasma. In this case

$$\bar{v}_+ = \frac{\bar{c}_+}{4} = \left(\frac{kT_+}{2\pi M}\right)^{1/2} = \left(\frac{eV_+}{2\pi M}\right)^{1/2} \qquad \ldots(6.50)$$

When an extra sheath region is assumed to exist it has been shown (sub-section (6.2)) that if V_+/V_e is very small the potential drop developed across the extra sheath region is of the order of $V_e/2$. This is also the ion's radial energy component (assumed to be mono-energetic) at the sheath edge. In this case

$$\bar{v}_+ = \left(\frac{2eV_+}{M}\right)^{1/2} = \left(\frac{eV_e}{M}\right)^{1/2} \qquad \ldots(6.51)$$

The electron concentration in the neighbourhood of a negative spherical probe is, in general, given by (54)

$$N_e = N_{eo}\, e^{-\chi}\left\{1 - \frac{1}{2g(0)}\left[g(\chi_p-\chi)-(1-z^2)^{\frac{1}{2}} g\left(\frac{\chi_p-\chi}{1-z^2}\right)\exp\left[z^2\left(\frac{\chi_p-\chi}{1-z^2}\right)\right]\right]\right\}$$
$$\ldots(6.52)$$

where χ and χ_p are defined by equation (6.41) and z is given by

$$z = r_p/r \qquad \ldots(6.53)$$

and $$g(x) = \int_x^\infty y^{\frac{1}{2}} \exp(-y)\,dy \qquad \ldots(6.54)$$

When the probe is held at a strongly negative potential χ_p is very much greater than χ, the potential in the extra sheath region, and equation (6.52) reduces to

$$N_e = N_{eo} \exp\left(\frac{V}{V_e}\right) \qquad \ldots(6.55)$$

As plasma neutrality exists in the extra sheath region up to the sheath edge N_+ at the sheath edge is

$$N_+ = N_{+o} \exp\left(-\tfrac{1}{2}\right) \qquad \ldots(6.55')$$

where $N_{+o} = N_{eo}$ and $V = -V_e/2$. On substituting equations (6.51) and (6.55') into (6.49) one obtains

$$I_+ = 0.61\, AN_{+o}e\left(\frac{eV_e}{M}\right)^{1/2} \qquad \ldots(6.56)$$

which is in excellent agreement with Langmuir's semi-empirical equation (6.2).

In the extra sheath region dV/dr and d^2V/dr^2 are approximately zero and under these conditions equation (6.47) reduces to the plasma solution

$$\frac{I_+}{I_\lambda} = \chi^{\frac{1}{2}} \epsilon^2 e^{-\chi} \qquad \ldots(6.57)$$

The sheath edge is taken as the point where $d\chi/d\epsilon$ tends to infinity. Differentiating equation (6.57) shows this to occur as χ tends to $\frac{1}{2}$. Under these conditions equation (6.57) reduces to equation (6.56) providing $r_s \approx r_p$. This solution is valid providing I_+/I_λ is greater than, or equal to, 10^8 (49).

So far no account has been taken of the positive ion energy in the undisturbed plasma. Bohm et als (28) first took account of the ion energy by considering the effect of the energy on the absorption radius. An ion reaching a radius $r_p < r < r_A$ will be absorbed by the probe. The effective collecting radius which results in ions reaching a radius r_A is

$$h = r_A\left(1 - \frac{V_A}{V_+}\right)^{1/2} \qquad \ldots(6.58)$$

and the effective collecting area is therefore

$$A' = 4\pi r_A^2 \left(1 - \frac{V_A}{V_+}\right) \qquad \ldots(6.59)$$

The positive ion current flowing to a negative probe is

$$I_+ = A' N_+ e \bar{v}_+ \qquad \ldots(6.60)$$

where equation (6.60) is evaluated at a radius r=h. For mono-energetic ions

$$\bar{v}_+ = \left(\frac{eV_+}{8M}\right)^{1/2} \qquad \ldots(6.61)$$

and N_+ may be put equal to N_{+o}. The positive ion concentration outside the sheath region is given by

$$N_+ = N_{+o} \exp\left(\frac{V}{V_e}\right) \qquad \ldots(6.62)$$

and Bohm et als also show

$$N_+ = \frac{N_{+o}}{2}\left[\left(1 - \frac{V}{V_+}\right)^{1/2} \pm \left(1 - \frac{V}{V_+} - \frac{r_A^2}{r^2}\left(1 - \frac{V_A}{V_+}\right)\right)^{1/2}\right] \qquad \ldots(6.63)$$

It should be noted that equation (6.63) is identical with equation (6.34) derived by Bernstein et al (53). The positive sign is taken when $r > r_A$ and the negative when $r < r_A$. The dependence of r_A on V_+/V_e is found as follows. Equate equations (6.62) and (6.63) to give

$$\left(\frac{r_A}{r}\right)^2 \left(1 - \frac{V_A}{V_+}\right) = 4 \exp\left(\frac{V}{V_e}\right)\left[\left(1 - \frac{V}{V_+}\right)^{1/2} - \exp\left(\frac{V}{V_e}\right)\right] \qquad \ldots(6.64)$$

and set $r = r_A$ and $V = V_A$. Equation (6.64) then reduces to

$$2 \exp\left(\frac{V_A}{V_e}\right) = \left(1 - \frac{V_A}{V_+}\right)^{1/2} \qquad \ldots(6.65)$$

Equation (6.65) may be solved for V_A in terms of V_+/V_e and it can be shown that when

$$\left.\begin{aligned} V_+/V_e &= 0.01, \quad V_A = -2.8\,V_+ \\ V_+/V_e &= 0.50, \quad V_A = -0.79\,V_+ \end{aligned}\right\} \quad \ldots(6.66)$$

Substitute the values of V_+/V_e and V_A given by equation (6.66) into equation (6.64). Set $r = r_s$ and plot the dependence of $(r_A/r_s)^2$ on V/V_e. Figure 16 shows this dependence for $V_+/V_e = 0.50$. The sheath edge is defined as the radius where dV/dr_A tends to infinity. Using this definition one finds that for

$$\left.\begin{aligned} V_+/V_e &= 0.01, \quad (r_A/r_s)^2 = 4.20 \\ V_+/V_e &= 0.50, \quad (r_A/r_s)^2 = 1.17 \end{aligned}\right\} \quad \ldots(6.67)$$

When the sheath region is very thin r_s may be put equal to r_p. Making this assumption and combining equations (6.59) to (6.61), (6.66) and (6.67) gives for

$$\left.\begin{aligned} V_+/V_e &= 0.01, \quad I_+ = 0.56\,AN_{+o}e\left(\frac{eV_e}{M}\right)^{1/2} \\ V_+/V_e &= 0.50, \quad I_+ = 0.52\,AN_{+o}e\left(\frac{eV_e}{M}\right)^{1/2} \end{aligned}\right\} \quad \ldots(6.68)$$

Lam (54) was the first to investigate the variation of positive ion current with probe potential. His analysis also takes into account the effect of the ion energy on the ion current. For very small probe potentials the electron and positive ion concentrations in the extra sheath region are given by equations (6.52) and (6.32) respectively. Equating N_+ and N_e at the probe's surface gives

$$I_+ = AN_{+o}e\left(\frac{eV_+}{8M}\right)^{1/2}\exp\left(\frac{V_p}{V_e}\right)\left[2\left(1 - \frac{V_p}{V_+}\right)^{1/2} - \exp\left(\frac{V_p}{V_e}\right)\right] \quad \ldots(6.69)$$

Equation (6.69) is valid only for small probe potentials when no space charge developes around the probe. It clearly breaks down when the probe becomes so negative that plasma neutrality no longer exists. That is, $|V_p|$ must be less than the potential at which dV/dr tends to infinity. The value of this limiting potential for a given value of V_+/V_e may be obtained by differentiating (6.69) with respect to V_p and then setting $dI_+/dV_p = 0$. This maximum value of $|V_p|$ may be denoted by $V_{p,MAX}$ and its dependence on V_+/V_e is shown in Table 2.

TABLE 2

$\frac{V_+}{V_e}$	$\frac{V_{p,MAX}}{V_e}$	$\frac{V_{EX}}{V_e}$	$\frac{r_o j^{\frac{1}{2}}}{j_m}$	j_m
0.0	0.50	0.50	1.31	1.72
0.1	0.54	0.68	1.18	1.40
0.5	0.44	0.75	1.10	1.21
1.0	0.33	0.75	1.07	1.14
2.0	0.21	0.74	1.04	1.08
5.0	0.10	0.72	1.02	1.04
10.0	0.05	0.71	1.01	1.02
∞	0.0	0.69	1.00	1.00

TABLE 3

$F \frac{j^{\frac{1}{2}}}{j_m^{1/2}}$	$\frac{j}{j_m}$
0.00000	1.000
0.00370	1.020
0.00929	1.040
0.01590	1.061
0.02327	1.082
0.03125	1.102
0.07775	1.210
0.13189	1.590
0.19131	1.440
0.25473	1.562

We will now consider the case when $|V_p| \gg 0$. In the extra sheath region the electron concentration given by equation (6.55) may be equated to the ion concentration given by equation (6.34). After some rearrangement

$$I_+ = 4\pi r^2 N_{+o} e\left(\frac{2eV_+}{M}\right)^{1/2}\exp\left(\frac{V}{V_e}\right)\left[\left(1 - \frac{V}{V_+}\right)^{1/2} - \exp\left(\frac{V}{V_e}\right)\right] \quad \ldots(6.70)$$

This is an expression for the positive ion current crossing a surface of radius r and potential V (≤ 0) in the extra sheath region. It is only valid for $|V_{EX}| > |V| \geq 0$ where V_{EX} is the potential at which dV/dr tends to infinity. The radius at which $V = V_{EX}$ is denoted by r_{EX}. The dependence of V_{EX}/V_e and $r_p j^{\frac{1}{2}}/r_{EX}$ on V_+/V_e is shown in Table 2 where j is defined as

$$j = \frac{I_+}{\pi r_p^2 N_{+o}\, e\left(\frac{2eV_e}{M}\right)^{1/2}\left(1 + \frac{V_+}{V_e}\right)^{1/2}} \qquad \ldots(6.71)$$

If V_{EX} is substituted for V in equation (6.70) and r is put equal to r_{EX} one obtains an expression for the ion current crossing the transition boundary of the extra sheath region. Using the value for V_{EX} given in Table 2 for $V_+/V_e = 0.50$ equation (6.70) becomes

$$I_+ = 0.52\, A_{EX} N_{+o}\, e\left(\frac{eV_e}{M}\right)^{1/2} \qquad \ldots(6.72)$$

When $r_{EX} \approx r_s \approx r_p$ equation (6.72) becomes identical to equation (6.68). Note that unlike in the derivation of Bohm's equation the derivation of equation (6.72) has not required the calculation of r_A in terms of V_+/V_e.

Lam next considers the transition between the extra sheath region and the sheath region. This region falls between $r = r_s$ and $r = r_{EX}$ where r_s is defined as

$$r_s = r_p\left(\frac{j}{j_m}\right)^{1/2} \qquad \ldots(6.73)$$

When $r_p/\lambda_D \gg 1$, $r_{EX} \approx r_s$ and so $j_m^{\frac{1}{2}}$ is given by

$$j_m^{\frac{1}{2}} = \frac{r_p}{r_{EX}}\, j^{\frac{1}{2}} \qquad \ldots(6.74)$$

The dependence of j_m on V_+/V_e is shown in Table 2. In the sheath region dV/dr and d^2V/dr^2 can no longer be neglected and Lam equates I_+ to V_p by the following equation

$$I_+ = |V_p|^{3/2}\left(\frac{2e}{M}\right)^{1/2}\frac{1}{\left[F(j^{\frac{1}{2}}/j_m^{\frac{1}{2}})\right]^{3/2}} \qquad \ldots(6.75)$$

where $F(j^{\frac{1}{2}}/j_m^{\frac{1}{2}})$ is a universal function and is tabulated in terms of j/j_m in Table 3. F( ) can be found experimentally from the slope of the plot of $I_+^{2/3}$ against $|V_p|$; the corresponding value of j/j_m is then given by Table 3. Providing V_+ and V_e are known j can be found from Table 2 and then N_{+o} from equation (6.71). When $r_s \approx r_p$ and is effectively independent of V_p j/j_m is unity. If V_+/V_e is zero Table 2 shows $j = 1.72$. Substituting this value of j into equation (6.71) and then solving for I_+ gives equation (6.56).

CHAPTER 7

PROBE CHARACTERISTICS IN A COLLISION DOMINATED PLASMA

Here we consider the case where the carriers come under the influence of the probe's field at a distance of several carrier mean free paths from the probe. In deriving expressions for N_e and N_+ in terms of I_e and I_+ it is necessary to approach the problem from ambipolar diffusion considerations.

7.1. Ambipolar diffusion equations.

In the presence of an electric field and carrier density gradient the carrier flux densities are

$$\Gamma_e = -D_e \text{ grad } N_e - \mu_e N_e E \quad \ldots(7.1)$$

$$\Gamma_+ = -D_+ \text{ grad } N_+ + \mu_+ N_+ E \quad \ldots(7.2)$$

where D is the carrier diffusion coefficient and μ is the carrier mobility. D and μ are conveniently related to one another by Einstein's relation

$$D = \mu \frac{kT}{e} = \mu V \quad \ldots(7.3)$$

If ν is defined as the total number of ionizing collisions an electron makes per second per unit volume $N_e \nu$ represents the increase in the number of electrons in unit volume per second. Assuming $N_e = N_+ = N$ and that N remains constant

$$\text{div } \Gamma_e = \text{div} \Gamma_+ = N\nu \quad \ldots(7.4)$$

Combining equation (7.4) with equations (7.1) and (7.2)

$$\mathrm{div}\,\Gamma_e = -D_e\nabla^2 N - \mu_e\,\mathrm{div}\,(NE) = N\nu \qquad \ldots(7.5)$$

$$\mathrm{div}\,\Gamma_+ = -D_+\nabla^2 N + \mu_+\,\mathrm{div}\,(NE) = N\nu \qquad \ldots(7.6)$$

Eliminating E between equations (7.5) and (7.6) gives

$$\nabla^2 N + \frac{\nu}{D_a} N = 0 \qquad \ldots(7.7)$$

where

$$D_a = \frac{D_e\mu_+ + D_+\mu_e}{\mu_+ + \mu_e} \qquad \ldots(7.8)$$

7.2 Diode equations.

The ambipolar diffusion equations take account of the carrier concentration gradients that must exist in the neighbourhood of a carrier absorbing probe. If these diffusion effects are small the motion of the carriers is determined solely by the electric field. Therefore, for a strongly negative or a strongly positive probe, when carriers of only one sign reach the probe, the probe current may be calculated from the diode space charge equation derived from mobility considerations.

7.2.1. Plane probe.

If it is assumed that the carriers in the sheath have a drift velocity very much less than their thermal velocity their motion is mobility controlled and the carrier current is

$$I = ANe\mu\frac{dV}{dx} \qquad \ldots(7.9)$$

Assuming $dV/dx = 0$ at the sheath edge and that μ is a constant independent of the field the integration of Poisson's equation gives (56)

$$I = A \frac{9}{8} \frac{\mu}{4\pi} \frac{V_p^2}{x_s^3} \qquad \ldots(7.10)$$

7.2.2. Cylindrical probe.

Making the same assumptions as in the last sub-section the current flowing to a cylindrical probe is

$$I = A \frac{\mu}{4\pi} \frac{V_p^2}{\gamma^2 r_p^3} \qquad \ldots(7.11)$$

where γ is given by (51)

$$\gamma = \left[\left(\frac{r_s}{r_p}\right)^2 - 1\right]^{\frac{1}{2}} - \left(\frac{r_s}{r_p}\right) \ln\left[\frac{r_s}{r_p} + \left\{\left(\frac{r_s}{r_p}\right)^2 - 1\right\}^{\frac{1}{2}}\right] \qquad \ldots(7.12)$$

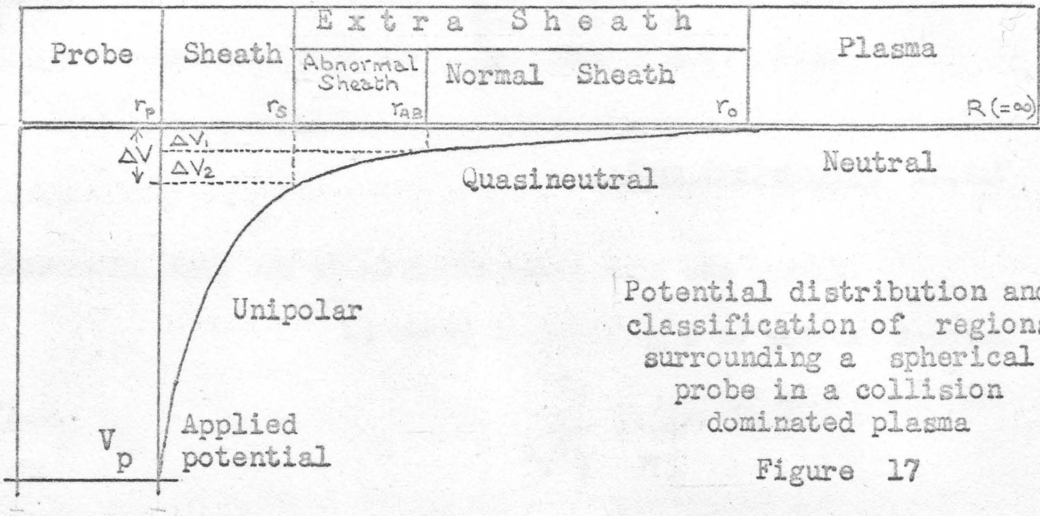
when $2 < r_s/r_p < 10$ equation (7.12) simplifies to (57)

$$\gamma = -\frac{1}{4}\left(\frac{r_s}{r_p}\right)^2 \qquad \ldots(7.13)$$

7.2.3. General diode equations

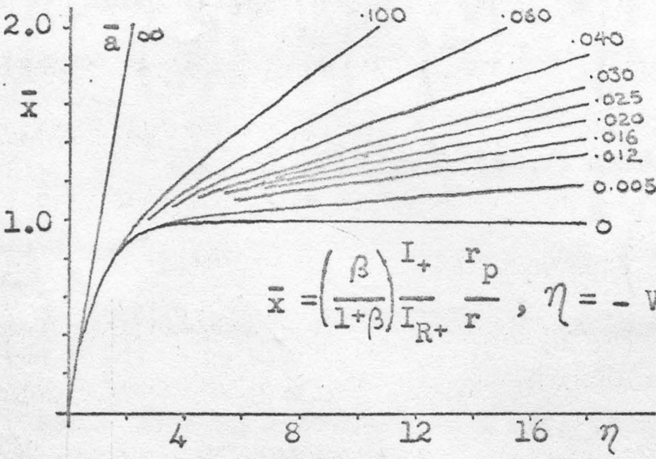
Weinstein and Kenty (58) have derived a general space charge equation which can be applied to plane, cylindrical and spherical diodes. They assume the motion of ions is mobility controlled and that the field at the sheath edge is approximately zero.

Miyajima and Yamamoto (59) have developed a general space charge theory for plane diodes that is applicable at all pressures. Their equations reduce to the low pressure and the high pressure case when the carrier transit time through the sheath satisfies certain specified conditions.



Potential distribution and classification of regions surrounding a spherical probe in a collision dominated plasma

Figure 17

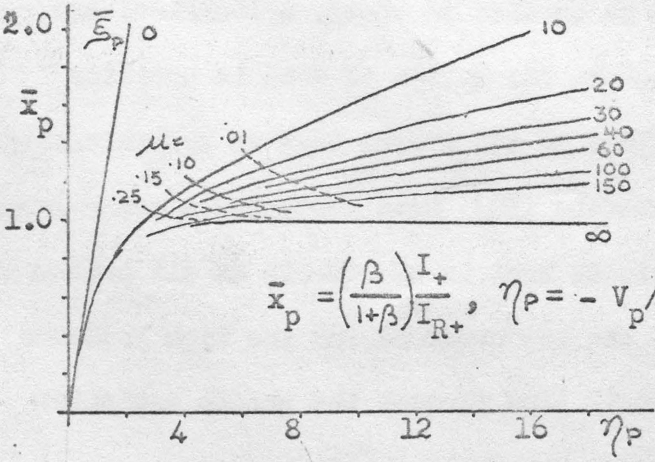


Dependence of $\bar{x}$ on η for a spherical probe (63)

$\beta = V_+/V_e$, $I_{R+} = 4\pi r_p e N_o D_+$

$$\bar{x} = \left(\frac{\beta}{1+\beta}\right)\frac{I_+}{I_{R+}}\,\frac{r_p}{r}\,,\quad \eta = -\,V/V_e\,,\quad \bar{a} = \frac{(1+\beta)^{\frac{1}{2}}}{\beta}\,\frac{I_{R+}}{I_+}\,\frac{\lambda_D}{r_p}$$

Figure 18



Computed positive ion current-voltage characteristics of a spherical probe (63)

$\beta = V_+/V_e$, $I_{R+} = 4\pi r_p e N_o D_+$

$$\bar{x}_p = \left(\frac{\beta}{1+\beta}\right)\frac{I_+}{I_{R+}},\quad \eta_p = -\,V_p/V_e,\quad \bar{\xi}_p = (1+\beta)^{\frac{1}{2}} r_p/\lambda_D\,.$$

Figure 19

Ciobanu and Iovitzu Popescu (60) have considered the case where ions pass through a sheath and suffer many collisions which result in a resonant charge transfer. They derive a generalized Poissón's equation applicable to plane, cylindrical and spherical diodes, which can be reduced to the low pressure case but which in general requires to be solved by numerical integration.

7.3 Probe characteristics.

The potential distribution surrounding a spherical probe in a collision dominated plasma is shown in Figure 17. Table 4 indicates the state of the plasma for which the various probe theories, to be considered, may be applied. The theories fall into two distinct groups: (i) when no collisions occur in the sheath region and (ii) when collisions occur in the sheath region.

The theories in group (i) involve deriving expressions for the current in the collision dominated extra sheath region and in the free fall sheath region and then matching them at the boundary of the two regions. Theories in this group may be classified as 'Diffusion plus Free fall' theories.

The theories in group (ii) involve the simultaneous solution of Poisson's equation with the ion flux and electron flux ambipolar diffusion equations. Theories in this group may be classified as 'Diffusion' theories.

7.3.1. Diffusion plus free fall theories.

TABLE 4

State of plasma	Sheath	Extra Sheath		Ion energy	Characteristic	Author & ref.
		Abnormal	Normal			
$(r_s - r_p) \lesssim \ell$ $r_p \gg \ell$	$(r_s - r_p) \lesssim \ell$ Free fall motion	$r_s = r_{As}$	$r_s \ll r_o \ll R$ Ambipolar diffusion $V_+/V_e \to 0$	V_+ finite	+ve ion & elec.	Davydv & Zmannskaja (29)
$(r_s - r_p) \lesssim \ell$ $r_p \gg \ell$	$(r_s - r_p) \lesssim \ell$ Free fall motion	$r_s = r_{As}$	$r_o/r_s \to \infty$ Ambipolar diffusion V_+/V_e finite	V_+ finite	+ve ion & elec.	Waymouth (61)
$(r_s - r_p) \lesssim \ell$ $r_p \gg \ell$	$(r_s - r_p) \lesssim \ell$ $\ell \left(\frac{dV}{dr}\right)_{r_s} \gtrsim V_e$ Free fall motion	Mobility controlled ion motion $v_+ \propto \left(\frac{dV}{dr}\right)^{\frac{1}{2}}$	$v_+ \propto \left(\frac{dV}{dr}\right)$	V_+ finite	+ve ion	Boyd (62)
$10 \leq \frac{r_p}{\lambda_D} \leq 150$ $\left(\frac{\ell}{r_p}\right)^{\frac{1}{3}} \left(\frac{\ell}{\lambda_D}\right)^{\frac{2}{3}}$ 1 $8V_e \ll \lvert V_p \rvert \ll 18V_e$	$(r_s - r_p) \gg \ell$	Considers transition regions between sheath & extra sheath	Essentially ambipolar diffusion with collisions up to probe's surface	V_+ finite Theory for V_p/V_e tending to ∞	+ve ion	Su & Lam (63)
$50 \leq \frac{r_p}{\lambda_D} \leq 1600$ $\left(\frac{\ell}{r_p}\right)^{\frac{1}{3}} \left(\frac{\ell}{\lambda_D}\right)^{\frac{2}{3}} \ll 1$ $0 \leq \lvert V_p \rvert \leq 14V_e$	$(r_s - r_p)$	Considers transition regions between sheath & extra sheath	Essentially ambipolar diffusion with collisions up to probe's surface	$V_+ =$ 0, 0.01 0.10, Theory for $r_p/{}_D$ tending to	+ve ion & elec.	Cohen (64)

The theories presented in this subsection are due to Davydv and Zmannskaja (29), Waymouth (61) and Boyd (62). They all assume the sheath thickness is less than a carrier mean free path thick and is also very much less than the probe radius.

Sheath region.

The carrier flux density at the probe's surface is

$$\Gamma_e = - N_s \bar{v}_e \epsilon_e \qquad \ldots(7.14)$$

$$\Gamma_+ = - N_s v_+ \epsilon_+ \qquad \ldots(7.15)$$

where N_s is the carrier concentration at the sheath edge, $\bar{v}_e$ and $\bar{v}_+$ are given by

$$\bar{v}_e = \left(\frac{eV_e}{2\pi m}\right)^{\frac{1}{2}} \quad ; \quad \bar{v}_+ = \left(\frac{eV_+}{2\pi M}\right)^{\frac{1}{2}} \qquad \ldots(7.16)$$

and $$\epsilon_e = 1 \text{ for } V_p > 0 \qquad \ldots(7.17)$$

$$\epsilon_e = \exp\left(\frac{+V_p}{V_e}\right) \text{ for } V_p < 0 \qquad \ldots(7.18)$$

and $$\epsilon_+ = 1 \text{ for } V_p < 0 \qquad \ldots(7.19)$$

$$\epsilon_+ = \exp\left(-\frac{V_p}{V_+}\right) \text{ for } V_p > 0 \qquad \ldots(7.20)$$

V_+ and V_e refer to the positive ion and electron energy at the sheath edge. If the carrier mobility and diffusion coefficients are assumed to be constant in space Einstein's relation, equation (7.3), shows that V_+ and V_e at the sheath edge are identical with V_+ and V_e in the undisturbed plasma.

In this sub-section it is assumed that no collisions occur in the sheath region. Boyd (65) shows that if his criterion for the formation of a stable positive ion sheath is applied (see equation (6.24)) no collisions will occur in the sheath providing

$$\ell\left(\frac{dV}{dr}\right)_{r\xi} \geqslant V_e \qquad \ldots(7.21)$$

When no collisions occur in the sheath region the sheath thickness may be estimated using the diode equations given in sub-section (2.2)

Extra sheath region.

This is a region of quasi-neutrality and the motion of the carriers is described by ambipolar diffusion. The carrier flux density at the sheath edge is

$$\Gamma_e = \left[-D_e \frac{dN_e}{dr} - \mu_e N_e E\right]_{r\xi} \qquad \ldots(7.22)$$

$$\Gamma_+ = \left[-D_+ \frac{dN_+}{dr} + \mu_+ N_+ E\right]_{r\xi} \qquad \ldots(7.23)$$

The theories of Davydv et al (29) and Waymouth (61) involve matching equations (7.14) and (7.15) with equations (7.22) and (7.23).

Boyd's analysis (62) is concerned solely with the collection of positive ions by a strongly negative probe. He assumes that the ions' diffusion motion is negligable in comparison with their mobility motion. He further assumes that the ions' drift velocity is proportional to the electric field strength in the outer regions of the extra sheath where the field is assumed to be small. On the other hand in the inner regions of the extra sheath (the abnormal sheath region), where the electric field strength is high, the ions' drift velocity is proportional to the square root of the field strength.

Perturbation of carrier density.

We will now calculate the carrier concentration as a function of radius from the centre of a spherical probe and as a function of the applied potential across the sheath region.

From equations (7.14), (7.15), (7.22) and (7.23) we have

$$-N_s\bar{v}_e\epsilon_e = \left[-D_e\frac{dN_e}{dr} - \mu_e N_e E\right]_{r_s} \qquad \ldots(7.24)$$

$$-N_s\bar{v}_+\epsilon_+ = \left[-D_+\frac{dN_+}{dr} + \mu_+ N_+ E\right]_{r_s} \qquad \ldots(7.25)$$

Writing $N_+ = N_e = N$ and eliminating E between equations (7.24) and (7.25) gives

$$\frac{1}{N_s}\left(\frac{dN}{dr}\right)_{r_s} = \frac{Q}{r_s} \qquad \ldots(7.26)$$

where $$Q = Q_e + Q_+ \qquad \ldots(7.27)$$

and $$Q_e = \frac{r_s v_e \epsilon_e}{\mu_e(V_e + V_+)} \qquad \ldots(7.28)$$

$$Q_+ = \frac{r_s v_+ \epsilon_+}{\mu_+(V_e + V_+)} \qquad \ldots(7.29)$$

The general solution of equation (7.7) is

$$N = \frac{1}{wr}\left(A\sin wr + B\cos wr\right) \qquad \ldots(7.30)$$

where $$w^2 = \frac{\nu}{D_a} \qquad \ldots(7.31)$$

In the absence of the probe, i.e. when r tends to zero, there is no disturbance at $r = 0$ and so $N = N_o$. Also $N = 0$ at $r = R$.

Applying these boundary conditions gives

$$A = N_o, \quad B = 0, \quad \text{and } w = \frac{\pi}{R}$$

In the presence of a probe if $r_p \ll R/\pi$, ν remains unaltered and w is still given by

$$w = \left(\frac{\nu}{D_a}\right)^{\frac{1}{2}} = \frac{\pi}{R}$$

In the extra sheath region $r/R \ll 1$ and so $\sin(r/R) \approx (r/R)$ and $\cos(r/R) \approx 1$. Equation (7.30) then simplifies to

$$N = A + \frac{B}{wr} \qquad \ldots(7.32)$$

Differentiating with respect to r

$$\frac{dN}{dr} = -\frac{B}{wr^2} \qquad \ldots(7.33)$$

Substituting equations (7.32) and (7.33) into equation (7.26) and solving for (B/A) gives

$$\frac{B}{A} = -wr_s\frac{Q}{(1+Q)} \qquad \ldots(7.34)$$

Now $A = N_o$ so $B = -N_o wr_s \frac{Q}{(1+Q)}$ and equation (7.32) becomes

$$N = N_o\left[1 - \frac{r_s}{r}\left(\frac{Q}{1+Q}\right)\right] \qquad \ldots(7.35)$$

We will now examine the form of equation (7.35) for a number of limiting conditions.

When $r = r_s$

$$N_s = \frac{N_o}{(1+Q)} \qquad \ldots(7.35a)$$

and $$N_s = \frac{N_o}{(1 + Q_+)} \quad \text{if } V_p \ll 0 \qquad \ldots(7.35b)$$

$$N_s = \frac{N_o}{(1 + Q_e)} \quad \text{if } V_p \gg 0 \qquad \ldots(7.35c)$$

When $V_p = 0$ the sheath region vanishes and $r_s = r_p$. Equation (7.35a) then becomes

$$N_p = N_o\left[1 + \frac{r_p}{(V_e + V_+)}\left(\frac{\bar{\bar{v}}_e}{\mu_e} + \frac{\bar{\bar{v}}_+}{\mu_+}\right)\right]^{-1} \qquad \ldots(7.35d)$$

Making use of equation (7.3) and the relation

$$D = \frac{\ell_o \bar{c}}{p\,3} = \frac{4\,\ell_o \bar{\bar{v}}}{3p}$$

reduces equation (7.35d) to

$$N_p = N_o\left[1 + \frac{3\,p\,r_p}{4(V_e + V_+)}\left(\frac{V_e}{\ell_{eo}} + \frac{V_+}{\ell_{+o}}\right)\right]^{-1} \qquad \ldots(7.35e)$$

When $V_e \gg V_+$ and ℓ_{eo} is of the same order as ℓ_{+o} equation (7.35e) reduces to

$$N_p = N_o\left[1 + \frac{3pr_p}{4\ell_{eo}}\right]^{-1}$$

and when $(\ell_{eo}/p) \ll r_p$

$$N_p = \frac{4\,N_o \ell_{eo}}{3\,p\,r_p} = \frac{4\,N_o \ell_e}{3\,r_p} \qquad \ldots(7.35f)$$

This last equation is identical with equation (4.16) when (ℓ_o/pr_p) is very much less than unity. Equation (7.35) was derived by Waymouth (61). When certain simplifying assumptions are made this equation reduces to the forms (7.35a) to (7.35f) and these are shown

to be identical with equations derived earlier by Davydv et al (29).
In the analysis of Davydv et al they have assumed that

$$V_e \gg V_+, \quad \mu_e \gg \mu_+ ,$$

$$r_p \gg \ell_{e,+} \quad r_p \approx r_s ,$$

and the fraction of electrons reflected at the probe's surface and escape is δ. They show the carrier concentration at the sheath edge is given by

$$N_s = N_o\left[\frac{D_e - r_s\bar{\bar{v}}_e i_e}{D_e + r_s\bar{\bar{v}}_e\chi}\right] \text{ for } V_p < 0 \qquad \ldots(7.36)$$

where
$$i_e = \frac{N_s}{N_o}\left[\frac{(1-\delta)\exp(V_p/V_e)}{1+\delta+(1-\delta)\phi(-V_p^{\frac{1}{2}}/V_e^{\frac{1}{2}})}\right] \qquad \ldots(7.37)$$

and
$$N_s = N_o\left[i_e\left(\frac{1+\delta}{1-\delta}\right)\frac{\exp(-V_p/V_+)}{1+\phi(V_p^{\frac{1}{2}}/V_+^{\frac{1}{2}})}\right] \text{ for } V_p > 0 \qquad \ldots(7.38)$$

where
$$i_e = \left(\frac{1-\delta}{1+\delta}\right)\left[\frac{D_e - r_s\bar{\bar{v}}_e i_e}{D_e + r_s\bar{\bar{v}}_e\chi}\right] \qquad \ldots(7.39)$$

and where χ and $\phi(x)$ are defined by

$$\chi = \frac{\bar{\bar{v}}_+\mu_e}{\bar{\bar{v}}_e\mu_+} \qquad \ldots(7.40)$$

$$\phi(x) = \frac{2}{\pi^{1/2}}\int_0^x \exp(-u^2)du \qquad \ldots(7.41)$$

$$\phi(0) = 0 \text{ and } \phi(\infty) = 1$$

It is readily shown that when $V_p \ll 0$ and $\delta = 0$ equation (7.37) reduces to $i_e = 0$ and equation (7.36) then reduces to (7.35b) providing $V_e \gg V_+$.

Perturbation of plasma potential.

The presence of a probe in a plasma produces a so called 'extra sheath' region. In general the potential drop across this region is given by

$$\Delta V = V_s - V_o = \int_{r_s}^{r_o} E \, dr \qquad \ldots(7.42)$$

and represents a perturbation of the original plasma potential. No serious error is introduced if the upper limit of integration is taken as infinity providing $r_s \ll r_o \ll R$. This approximation is made by Waymouth (61) and by Boyd (62) while Davydov et al carry out their analysis assuming r_o to be finite. Boyd analyses the perturbation in both the normal and abnormal sheath regions.

Following the analysis of Waymouth the field in the extra sheath region may be found by subtracting equation (7.5) from (7.6) and then integrating

$$\mathrm{div}\,(NE) = -\left[\frac{D_e - D_+}{\mu_e + \mu_+}\right]\nabla^2 N \qquad \ldots(7.43)$$

If $V_e \gg V_+$ and $\mu_e \gg \mu_+$

$$\mathrm{div}\,(NE) = -V_e \nabla^2 N \qquad \ldots(7.44)$$

which, in spherical coordinates, becomes

$$\frac{1}{r^2}\frac{d}{dr}(r^2 NE) = -\frac{V_e}{r^2}\frac{d}{dr}\left(r^2 \frac{dN}{dr}\right) \qquad \ldots(7.45)$$

Integrating between $r = r_s$ and $r = r$ and solving for E

$$E = -\frac{V_e}{N}\frac{dN}{dr} + \left(\frac{r_s}{r}\right)^2 \frac{1}{N}\left[NE + V_e \frac{dN}{dr}\right]_{r_s} \qquad \ldots(7.46)$$

After some working the expression in brackets can be shown to be given by

$$\frac{N_o Q_e (V_e + V_+)}{r_s(1 + Q)}$$

Substituting equation (7.46) into (7.42) then gives for $r_o = \infty$

$$\Delta V = -\int_{r_s}^{\infty} \frac{V_e}{N}\, dN + \int_{r_s}^{\infty} \frac{r_s}{r^2} \frac{Q_e(V_e + V_+)\, dr}{(1+Q)\left[1 - \left(\frac{Q}{1+Q}\right)\frac{r_s}{r}\right]} \qquad \ldots(7.47)$$

$$\Delta V = -V_e\left[1 - \frac{Q_e}{Q}\left(1 + \frac{V_+}{V_e}\right)\right]\ln(1 + Q) \qquad \ldots(7.48)$$

The corresponding expression derived by Davydv et al, who assume r_o to be finite, is

$$\Delta V = V_e\left[1 - \frac{C}{bN_o}\right]\ln\left[\frac{r_o}{r_s}\left(\frac{r_s - b}{r_o - b}\right)\right] \qquad \ldots(7.49)$$

where $$C = \left(\frac{r_s \bar{\bar{v}}_e}{\mu_e V_e}\right) r_s i_e N_o \qquad \ldots(7.50)$$

$$b = r_s\left(1 - \frac{N_s}{N_o}\right) \qquad \ldots(7.51)$$

and i_e is defined by equations (7.37) and (7.39). When $V_p = 0$ and it is assumed that

$$r_o \gg r_s, \qquad V_e \gg V_+, \text{ and } \mu_e \gg \mu_+$$

both equations (7.48) and (7.49) reduce to

$$\Delta V = -V_e\left[1 - \frac{r_p \bar{\bar{v}}_e}{\mu_e V_e (1 - N_p/N_o)}\right]\ln\left(\frac{N_o}{N_p}\right) \qquad \ldots(7.52)$$

or $$\Delta V = -V_e\left[1 - \frac{1}{1 - \frac{4\ell}{3r_p}}\right]\ln\left(\frac{3r_p}{4\ell_e}\right) \qquad \ldots(7.53)$$

It can also be shown that when $V_p \ll 0$

$$\Delta V = -V_e \ln\left(\frac{3r_s}{4\ell_+}\right) \qquad \ldots(7.54)$$

and when $V_p \gg 0$

$$\Delta V = V_+ \ln\left(\frac{3r_s}{4\ell_e}\right) \qquad \ldots(7.55)$$

ΔV is independent of V_p when $V_p \ll 0$ and so V_e may be obtained from the slope of ln I_e against V_p for strongly negative values of V_p. Also knowing V_+ the positive ion concentration can be estimated from equation (7.15) providing N_s is replaced by

$$N_s = \frac{N_o}{\left(1 + \frac{3r_s V_+}{4\ell_+ V_e}\right)} \qquad \ldots(7.56)$$

Boyd (62) shows that the potential drop across the normal sheath region is given by

$$\Delta V_1 = -V_e \ln\left(\frac{1}{1 - \alpha/r_{As}}\right) \qquad \ldots(7.57)$$

for a strongly negative probe where

$$\alpha = \frac{j_+ r_s^2}{N_o e \mu_+ V_e} \qquad \ldots(7.58)$$

and

$$j_+ = N_{+As}\, e\, \bar{v}_{+As}\left(\frac{r_{As}}{r_s}\right)^2 \qquad \ldots(7.59)$$

When the abnormal sheath region is negligibly thin $r_{As} = r_s$ and

$$\Delta V_1 = \Delta V = -V_e \ln\left[\frac{1}{1 - \frac{N_{+s}\bar{v}_+ r_s}{N_o \mu_+ V_e}}\right] \qquad \ldots(7.60)$$

Making use of equations (7.29) and (7.35b) equation (7.60) reduces to

$$\Delta V = - V_e \ln (1 + Q_+) \qquad \ldots(7.61)$$

If the abnormal sheath region cannot be neglected Boyd (62) shows that the potential drop across the abnormal sheath is given by

$$\Delta V_2 = - V_e \ln\left[\frac{1}{1 - (\beta/r_s)^3}\right] \qquad \ldots(7.62)$$

where
$$\beta = \frac{2}{3V_e}\left[\frac{j_+ p^{\frac{1}{2}}}{N_o \, e \mu_+^{'}}\right]^2 \qquad \ldots(7.63)$$

where $(\mu_+^{'}/p^{\frac{1}{2}})$ is the mobility constant for high field strengths at a pressure p and where

$$j_+ = N_{+s} e \bar{v}_+ \qquad \ldots(7.64)$$

Boyd's sub-division of the extra sheath region into two distinct regions is somewhat arbitrary and is therefore difficult to apply in the interpretation of an experimentally determined characteristic.

7.3.2. Diffusion theories.

The theories presented in this sub section are due to Su and Lam (63) and to Cohen (64). They assume that the carrier mean free path is sufficiently small for there to be numerous collisions in the sheath region.

If the effect of carrier concentration gradients can be ignored the probe current is given by the diode equations in sub section (7.2) for either strongly negative or strongly positive probe potentials. These equations enable an estimate of the sheath thickness to be made and hence the carrier current density at the sheath edge can be calculated. Knowing V_+ the ion concentration at the sheath edge can be found.

Unfortunately there is no way of relating this concentration to the unperturbed concentration when carriers undergo numerous collisions in the sheath region. This approach is also unsatisfactory as the diode equations derived in sub section (7.2) assume that the field strength at the sheath edge is zero. The finite field strength in the extra sheath region results in the ambipolar diffusion of carriers towards the probe. This motion of carriers in the extra sheath region should be taken into account when solving Poisson's equation and it is just this that has been done by Su et al and by Cohen.

When the probe is close to plasma potential the sheath thickness is shown by Su et al (63) to be of the order

$$x_s = \frac{r_p \beta^{1/3}}{\xi_{p+}^{2/3}(1+\beta)^{1/3}} \qquad \ldots(7.65)$$

where β and ξ_{p+} are defined by equations (6.41) but with λ_D replaced by λ_{D+} where

$$\lambda_{D+} = \left(\frac{V_+}{4\pi N_{+o} e}\right)^{1/2} \qquad \ldots(7.66)$$

The criterion for the applicability of the diffusion theory is thus

$$\frac{r_p \beta^{1/3}}{\xi_{p+}^{2/3}(1+\beta)^{1/3}} \gg \ell \qquad \ldots(7.68)$$

or $$(1+\beta)^{1/3}\left(\frac{\ell}{r_p}\right)^{1/3}\left(\frac{\ell}{\lambda_{D+}}\right)^{2/3} \ll 1 \qquad \ldots(7.68)$$

Poisson's equation

The generalized Poisson's equation is

$$\nabla^2 V = -4\pi e(N_+ - N_e) \qquad \ldots(7.69)$$

which expressed in spherical coordinates becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -4\pi e(N_+ - N_e) \qquad \ldots(7.70)$$

Equation (7.70) may be put into dimensionless form using the following transforms

$$\eta = -V/V_e, \qquad \eta_p = -V_p/V_e$$

$$z = r_p/r \qquad \ldots(7.71)$$

$$n_e = N_e/N_o, \qquad n_+ = N_+/N_o$$

thus $$\frac{V_e}{4\pi N_o e\, r_p^2} z^4 \frac{d^2\eta}{dz^2} = n_+ - n_e \qquad \ldots(7.72)$$

or $$\frac{z^4}{\xi_p^2}\frac{d^2\eta}{dz^2} = n_+ - n_e \qquad \ldots(7.73)$$

where $$\xi_p = r_p/\lambda_D \qquad \ldots(7.74)$$

To solve Poisson's equation the dependence of N_+ and N_e on V must be known. This is achieved by solving Poisson's equation simultaneously with the ambipolar flux equations (7.1) and (7.2). If I_+ and I_e are the carrier currents to a spherical probe equations

(7.1) and (7.2) may be rewritten in the form

$$\frac{\Gamma_e}{D_e} = \frac{-I_e}{D_e 4\pi r^2 e} = -\left(\frac{dN_e}{dr} - \frac{N_e}{V_e}\frac{dV}{dr}\right) \qquad \ldots(7.75)$$

$$\frac{\Gamma_+}{D_+} = \frac{-I_+}{D_+ 4\pi r^2 e} = -\left(\frac{dN_+}{dr} + \frac{N_+}{V_+}\frac{dV}{dr}\right) \qquad \ldots(7.76)$$

Using the transforms given in (7.71) together with

$$\beta = V_+/V_e \qquad \ldots(7.77)$$

equations (7.75) and (7.76) reduce to

$$\frac{dn_e}{dz} + n_e \frac{d\eta}{dz} = \frac{-I_e}{4\pi e r_p N_o D_e} = -J_e \qquad \ldots(7.78)$$

$$\beta\frac{dn_+}{dz} - n_+ \frac{d\eta}{dz} = \frac{-\beta I_+}{4\pi e r_p N_o D_+} = -\beta J_+ \qquad \ldots(7.79)$$

J is equal to the carrier current I divided by the corresponding random carrier current I_R. I_R is defined as the carrier current flowing to a probe when at plasma potential. Providing $\ell \ll r_p$

$$I_R = 4\pi r_p^2 . \frac{N_o e\bar{c}}{4} . \frac{4\ell}{3r_p} \qquad \ldots(7.80)$$

or $$I_R = 4\pi r_p e N_o D \qquad \ldots(7.81)$$

Equations (7.73), (7.78) and (7.79) may be put in an alternative form by defining a new variable x such that

$$x = \frac{br_p}{r} = bz \qquad \ldots(7.82)$$

Equation (7.73) then becomes

$$\frac{x^4}{b^2 \xi_p^2}\frac{d^2}{dx^2} = n_+ - n_e \qquad \ldots(7.83)$$

or $$a^2x^4 \frac{d^2\eta}{dx^2} = n_+ - n_e \qquad \ldots(7.84)$$

where $$a^2 = \frac{1}{b^2\xi_p^2} \qquad \ldots(7.85)$$

hence $$x = \frac{\lambda_D}{ar} \qquad \ldots(7.86)$$

Equation (7.79) becomes

$$\beta\frac{dn_+}{dx} - n_+\frac{d\eta}{dx} = -\frac{\beta J_+}{b} = -\beta J_+ a \xi_p \qquad \ldots(7.87)$$

a is so defined that

$$\beta J_+ a \xi_p = 1 \qquad \ldots(7.88)$$

or $$a = \frac{\lambda_D 4\pi e N_o D_+}{I_+\beta}$$

Equations (7.79) and (7.78) may then be rewritten as

$$\beta\frac{dn_+}{dx} - n_+\frac{d\eta}{dx} = -1 \qquad \ldots(7.90)$$

$$\frac{dn_e}{dx} + n_e\frac{d\eta}{dx} = -\mu \qquad \ldots(7.91)$$

where $$\mu = \frac{I_e\mu_+}{I_+\mu_e} \qquad \ldots(7.92)$$

Spherical probe positive ion characteristics for intermediate values of r_p/λ_D.

For this analysis it is convenient to use equations (7.84), (7.90) and (7.91) together with the boundary conditions that at $\quad x = x_p, \quad n_+ = n_e = 0$ and $\eta = \eta_p$ ...(7.93)

and at $\quad x = 0, \quad n_+ = n_e = 1$ and $\eta = 0$ ...(7.94)

This analysis is due to Su and Lam (63). Integrating equation (7.91) gives

$$n_e = e^{-\eta}\left[1 - \mu\int_0^x e^{\eta}\,dx\right] \qquad ...(7.95)$$

where
$$\mu = \left[\int_0^{x_p} e^{\eta}\,dx\right]^{-1} \qquad ...(7.96)$$

Close to the probe's surface x is very nearly equal to x_p and equation (7.95) shows that n_e is very much less than $\exp(-\eta)$. However if $\eta \gg 0$ and one is interested in the region some distance away from the probe's surface equation (7.95) reduces to Boltzman's equation

$$n_e = e^{-\eta} \qquad ...(7.97)$$

Eliminating n_e and n_+ between equations (7.84), (7.90) and (7.97) gives

$$a^2x^4\eta'' = \frac{1 + \beta(a^2x^4\eta'')'}{\eta'} - (1+\beta)\exp(-\eta) \qquad ...(7.98)$$

where the primes indicate differentiation with respect to x. On replacing $x/(1+\beta)$ by $\bar{\bar{x}}$ and $a(1+\beta)^{\frac{1}{2}}$ by $\bar{\bar{a}}$ equation (7.98) reduces to

$$\bar{\bar{a}}^2\bar{\bar{x}}^4\eta'' = \frac{1 + \beta\bar{\bar{a}}^2(\bar{\bar{x}}^4\eta'')'}{\eta'} - \exp(-\eta) \qquad ...(7.99)$$

where the primes now indicate differentiation with respect to $\bar{\bar{x}}$. If $\eta_p \gg 1$ and β is of the order of unity $\beta\bar{\bar{a}}^2(\bar{\bar{x}}^4\eta'')'$ may be neglected compared with unity and equation (7.99) reduces to

$$\bar{a}^2\bar{x}^4\eta'' = \frac{1}{\bar{\eta}'} - \exp(-\eta) \qquad \ldots(7.100)$$

The dependence of $\bar{x}$ on η for constant $\bar{a}$ may be found by integrating equation (7.100) and applying the boundary conditions that

at $\quad \bar{x} = 0, \ \eta = 0 \qquad \ldots(7.101)$

and at $\quad \bar{x} = \bar{x}_p, \ \eta = \eta_p \qquad \ldots(7.102)$

This integration has been carried out numerically and the dependence of $\bar{x}$ on η for constant $\bar{a}$ is shown in Figure 18.

From the definition of $\bar{x}$ and $\bar{a}$ it is seen that

$$\frac{1}{\bar{x}_p\bar{a}} = (1+\beta)^{\frac{1}{2}}\xi_p = \left(1 + \frac{V_+}{V_e}\right)^{1/2}\frac{r_p}{\lambda_D} \qquad \ldots(7.103)$$

By cross plotting the curves in Figure 18 the dependence of $\bar{x}$ on η_p for constant $1/\bar{x}_p\bar{a}$ can be found. These characteristics are the positive ion characteristics as a function of probe radius and are shown in Figure 19. These curves have been obtained assuming that the electron concentration is given by Boltzman's equation (7.97) rather than the function given by equations (7.95) and (7.96). This is justified only if $\mu \ll 1$. Figure 18 may be used to find η as a function of $\bar{x}$ and $\bar{a}$ and hence μ may be found as a function of $\bar{x}_p$ using the modified definition of μ

$$\mu = \left[(1+\beta)\int_0^{\bar{x}_p} \exp(\eta)d\bar{x}\right]^{-1} \qquad \ldots(7.104)$$

Contours of constant μ are also shown in Figure 19. It is seen that for constant η_p the characteristics are more accurate as the probe radius decreases.

<u>Spherical probe positive ion characteristics for</u> $r_p/\lambda_D \ll 1$.

From equations (7.81) and (7.89) it follows that

$$a = \frac{I_{R+}}{I_+}\frac{\lambda_D}{\beta r_p} = \frac{I_{R+}}{I_+}\frac{1}{\beta \xi_p} \qquad \ldots(7.105)$$

Therefore when $\xi_p \ll 1$ a is very large and equation (7.84) may be solved by neglecting its right hand side to give (63)

$$\frac{d\eta}{dx} = A \qquad \ldots(7.106)$$

$$\eta = Ax + B \qquad \ldots(7.107)$$

Boundary condition (7.94) shows that B is zero. Hence

$$\eta = Ax \qquad \ldots(7.108)$$

Substitute (7.106) into (7.90) and solve for n_+ to give

$$n_+ = \frac{1}{A}\left[1 + (A - 1)e^{\eta/\beta}\right] \qquad \ldots(7.109)$$

where the constant of integration has been found using boundary condition (7.94) and Ax has been put equal to η according to equation (7.108). The constant A may be found by applying the boundary condition (7.93). It is then found that

$$A = \left[1 - \exp\left(-\frac{\eta_p}{\beta}\right)\right] \qquad \ldots(7.110)$$

Substitute (7.110) into (7.108) and solve fpr x_p to give

$$x_p = \frac{\eta_p}{\left[1 - \exp\left(-\frac{\eta_p}{\beta}\right)\right]} \qquad \ldots(7.111)$$

or

$$I_+ = -\frac{4\pi e N_o r_p \mu_+ V_p}{\left[1 - \exp(V_p/V_+)\right]} \qquad \ldots(7.112)$$

When V_p is very negative $\exp(V_p/V_+)$ tends to zero and equation (7.113) reduces to

$$I_+ = 4\pi e N_o r_p \mu_+ |V_p| \qquad ...(7.113)$$

or

$$I_+ = I_R \left|\frac{V_p}{V_+}\right|$$

Spherical probe characteristics for large values of r_p/λ_D

For this analysis it is convenient to use equations (7.73), (7.78) and (7.79) (64).

Equation (7.73) shows that as r_p/λ_D tends to infinity $(n_+ - n_e)$ must tend to zero. Therefore assuming that n_+ equals n_e and that $0 \leq z \leq 1$ equation (7.78) and (7.79) may be solved simultaneously to give the quasi-neutral solutions

$$n = 1 - \left[\frac{\beta J_+ + J_e}{1 + \beta}\right] z \qquad ...(7.115)$$

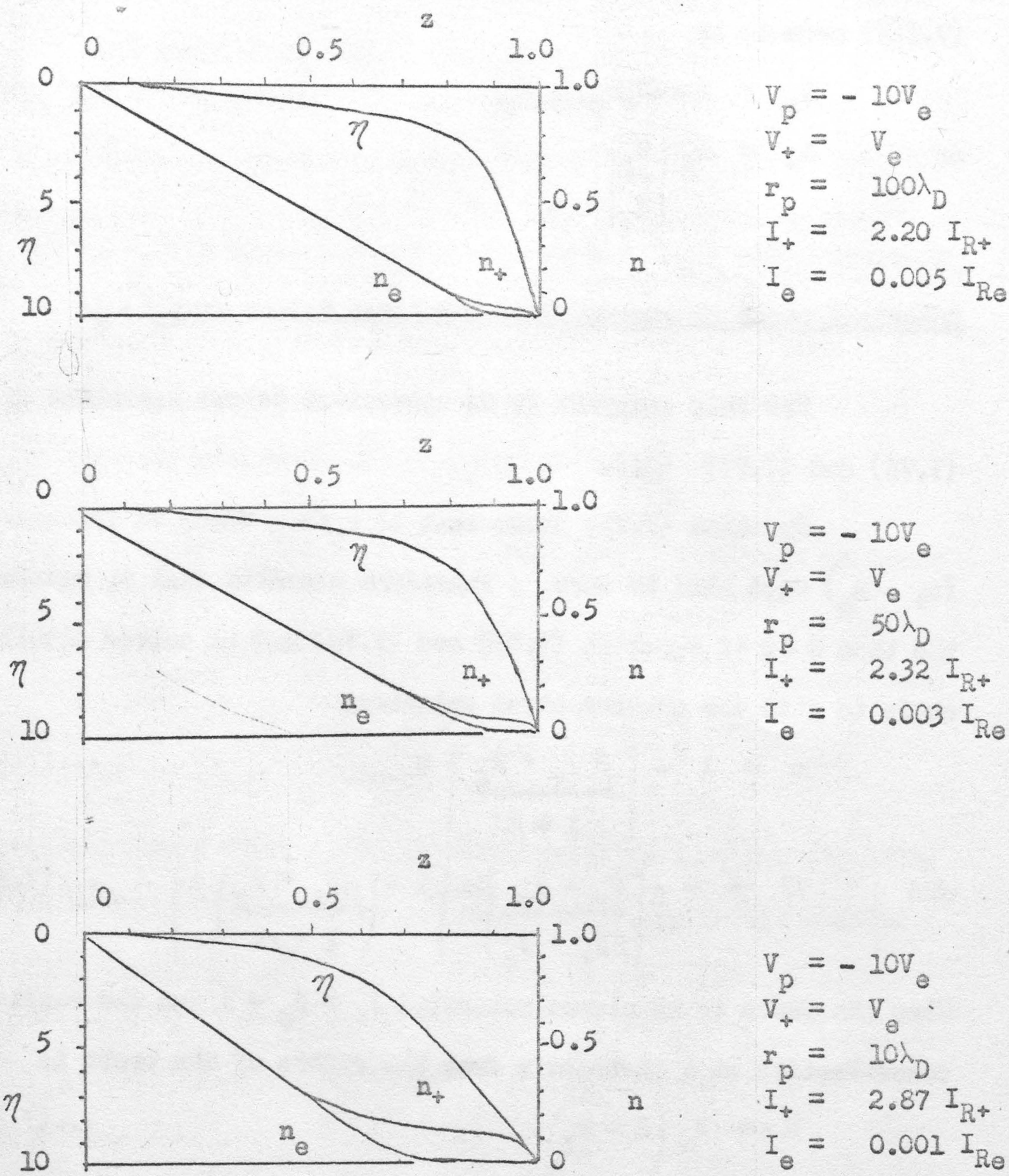
and

$$\eta = -\beta\left[\frac{J_+ - J_e}{\beta J_+ + J_e}\right]\ln\left[1 - \left(\frac{\beta J_+ + J_e}{1 + \beta}\right) z\right] \qquad ...(7.116)$$

When the probe is at plasma potential $J_+ = J_e = 1$ and the carrier concentration at a distance r from the centre of the probe is

$$N = N_o\left(1 - \frac{r_p}{r}\right) \qquad ...(7.117)$$

Providing $V_e \gg V_+$ and V_p is zero equation (7.117) can easily be shown to be identical with equation (7.35). This is reasonable as both theories should be identical when the probe is at plasma potential and no sheath region is present. Equation (7.116) is valid only for



$\eta = -V/V_e$, $z = r_p/r$, $n = N/N_o$

Potential and carrier concentration distributions in an infinite plasma surrounding a negative spherical probe (64)

Figure 20

$$z < \left(\frac{1+\beta}{\beta J_+ + J_e}\right) \qquad \ldots(7.118)$$

When the probe is at plasma potential η is zero for $0 < z < 1$. This too is identical with the result obtained for the potential drop across the extra sheath region when $V_p = 0$ and $\ell \ll r_p$ in the 'Free fall plus diffusion' theory and given by equation (7.53).

Computed curves showing the variation of n_e, n_+, and η in the quasi-neutral plasma and in the space immediately surrounding a spherical probe are shown in Figure 20 for various values of ξ_p, J_+ and J_e for $\beta = 1$ and $\eta_p = 10$.

In computing the probe characteristics for very large values of ξ_p it is convenient to replace $-d\eta/dz$ by E in equations (7.73), (7.78) and (7.79). After eliminating n_+ and n_e one obtains

$$\frac{1}{\xi_p^2}\left[\frac{(z^4E')'}{E}\right]' + \frac{(\beta^{-1}-1)(z^4E')'}{\xi_p^2} - \frac{z^4 EE'}{\beta\xi_p^2}$$

$$= -\left(J_+ + \frac{J_e}{\beta}\right) - \frac{E'}{E^2}(J_+ - J_e) \qquad \ldots(7.119)$$

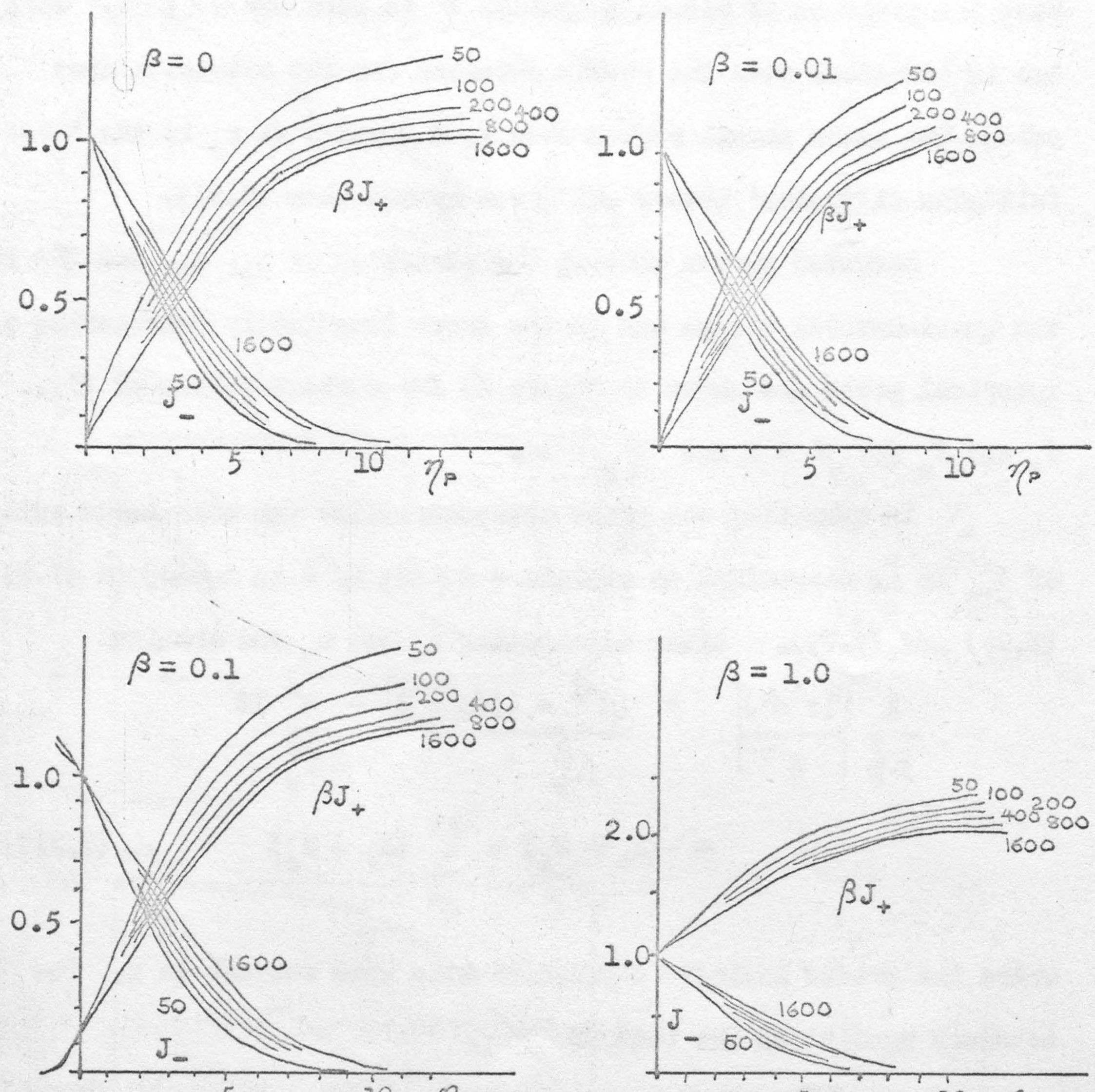
where the primes indicate differentiation with respect to z. The boundary conditions are that at

$$z = 0, \quad dE/dz \text{ exists and } E = -\beta\left(\frac{J_+ - J_e}{1+\beta}\right) \qquad \ldots(7.120)$$

and at $\quad z = 1, \quad dE/dz = 0$ and $d^2E/dz^2 = \xi_p^2(J_+ - J_e) \quad \ldots(7.121)$

Also $\quad \eta_p = -\int_0^1 E\,dz \qquad \ldots(7.122)$

On setting $\xi_p = \infty$ in equation (7.119) one recovers the quasi-neutral solutions (7.115) and (7.116)



$J_+ = I_+/4\pi e r_p N_o D_+$ $\quad$ $J_e = I_e/4\pi e r_p N_o D_e$

The computed positive ion and electron current-voltage characteristics of a spherical probe (64)

Figure 21

For large but finite values of ξ_p there will be a region where $d^2\eta/dz^2$ divided by ξ_p^2 is of the order of n_+ or n_e and the quasi-neutral solutions are no longer valid. This region becomes progressively thinner as ξ_p tends to infinity. To analyse this case it is necessary to rescale the variables so that the sheath and electric field is finite. These rescaled variables are

$$s = \left[\frac{(\beta J_+ + J_e)^5}{\beta(1+\beta)^4}\right]^{1/3} \xi_p^{2/3}(z_s - z) \qquad \ldots(7.123)$$

where
$$z_s = \frac{1+\beta}{\beta J_+ + J_e} \qquad \ldots(7.124)$$

and
$$E(z) = \left[\frac{(\beta J_+ + J_e)^5}{\beta(1+\beta)^4}\right]^{1/3} \xi^{2/3} F(s) \qquad \ldots(7.125)$$

Using these rescaled variables equation (7.119) transforms to

$$-\left(\frac{F''}{F}\right)' + \left(\frac{1}{\beta} - 1\right)F'' + \frac{F F'}{\beta} = -1 + \frac{\lambda F'}{F^2} \qquad \ldots(7.126)$$

where
$$\lambda = \frac{\beta(J_+ - J_e)}{\beta J_+ + J_e} \qquad \ldots(7.127)$$

and the primes represent differentiation with respect to s. Integrating (7.126) and applying the appropriate boundary conditions enable the dependence of J_+ and J_e on η to be computed for constant values of β and ξ_p. These spherical probe characteristics are shown in Figure 21. It is seen that the characteristics never saturate for large probe bias although they more nearly tend to do so as ξ_p becomes larger. The reason for this is that the screening by the space charge sheath surrounding the probe is incomplete and the electric field penetrates into the extra sheath region.

CHAPTER 8

THE COLLECTION OF POSITIVE IONS IN A COLLISIONLESS ELECTRONEGATIVE PLASMA

In the analyses presented so far it has been assumed that the plasma consists solely of positive ions and electrons in approximately equal quantities. If negative ions are present the criterion for plasma neutrality is

$$N_+ = N_- + N_e \qquad \text{...(8.1)}$$

where N_- is the negative ion concentration. The presence of negative ions in the plasma modifies the criterion for the formation of a stable positive ion sheath given by equation (6.23)

Define

$$\alpha = \frac{N_-}{N_e} \qquad \text{...(8.2)}$$

and

$$\gamma = \frac{V_e}{V_-} \qquad \text{...(8.3)}$$

8.1. Criterion for formation of stable positive ion sheath.

For a stable positive ion sheath to form Boyd et al (52) point out that in the sheath

$$N_+ \geqslant N_- + N_e \qquad \text{...(8.4)}$$

Equation (6.22) becomes

$$N_{+s} \int_0^{\infty} f(E)E^{\frac{1}{2}} (E - \delta V)^{-\frac{1}{2}} dE \geqslant N_{-s} \exp\left(\frac{\delta V}{V_-}\right) + N_{es} \exp\left(\frac{\delta V}{V_e}\right) \qquad \text{...(8.5)}$$

where the first term on the right hand side takes into account the presence of the negative ions. If δV is small and negative (8.5) reduces to

$$V_+ \geqslant \frac{V_e}{2}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right) \qquad \ldots(8.6)$$

where V_+ is defined by equation (6.24) and α_s is the value of at the sheath edge.

The potential at the sheath edge adjusts itself until the ion energy distribution satisfies equation (8.6). If the ions are assumed to be at rest at the outer edge of the extra sheath region surrounding a spherical probe and if no collisions occur in the extra sheath region the potential at the sheath edge with respect to the unperturbed plasma corresponds to the gain in ion energy on passing through the extra sheath region. The potential at the sheath edge is thus

$$\Delta V = -\frac{V_e}{2}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right) \qquad \ldots(8.7)$$

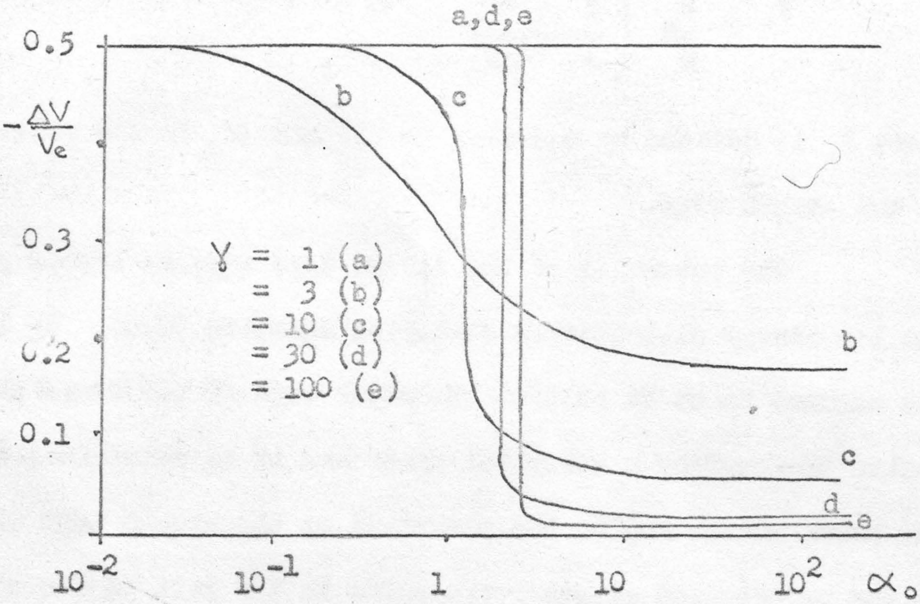
8.2. Dependence of sheath potential on α_s and γ

γ is assumed to be independent of position while α_s depends on the potential drop across the extra sheath region.

$$N_{es} = N_{eo} \exp\left(\frac{\Delta V}{V_e}\right) \qquad \ldots(8.8)$$

$$N_{-s} = N_{-o} \exp\left(\frac{\Delta V}{V_-}\right) \qquad \ldots(8.9)$$

therefore $$\alpha_s = \alpha_o \exp\left[\frac{\Delta V}{V_e}\left(\gamma - 1\right)\right] \qquad \ldots(8.10)$$



$\gamma = V_e/V_-, \qquad \alpha_o = N_{-o}/N_{eo}$

The potential drop across the extra sheath region in an electro-negative gas as a function of N_{-o}/N_{eo} and V_e/V_- (52)

Figure 22

Substitute (8.7) into (8.10) to give

$$\alpha_s = \alpha_o \exp\left[-\frac{1}{2}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right)(\gamma-1)\right] \quad \ldots(8.11)$$

The dependence of (α_s/α_o) on α_s for constant γ may be deduced from equation (8.11) and hence the dependence of α_s on α_o for constant γ can be obtained. $(-\Delta V/V_e)$ as a function of α_o for constant γ may then be calculated from equation (8.7) and is plotted in Figure 22. It is seen that $(-\Delta V/V_e)$ is approximately 0.5 for $\alpha_o < 2$ and $\gamma > 30$.

Halogen and halogen compound discharges have large values of α_o and γ and so the potential drop across the extra sheath region is small.

8.3. Positive ion characteristic.

The positive ion current to a spherical probe is

$$I_+ = 4\pi r_s j_+ \quad \ldots(8.12)$$

where j_+ is the positive ion density at the sheath edge. In general j_+ may be given by

$$j_+ = N_+ e v_+ \frac{r^2}{r_s^2} \quad \ldots(8.13)$$

If no collisions occur in the extra sheath region and if the ions may be assumed to have effectively zero thermal velocity at the extra sheath edge

$$v_+ = \left(-\frac{2eV}{M}\right)^{\frac{1}{2}} \quad \ldots(8.14)$$

As plasma neutrality exists in the extra sheath region

$$N_+ = N_e + N_- \qquad \ldots(8.15)$$

where $$N_e = N_{eo} \exp\left(\frac{V}{V_e}\right) \qquad \ldots(8.16)$$

and $$N_- = \alpha_o N_{eo} \exp\left(\frac{\gamma V}{V_e}\right) \qquad \ldots(8.17)$$

Substitute equations (8.14) and (8.15) into (8.13) to give

$$j_+ = N_{eo}\left[\exp\left(\frac{V}{V_e}\right) + \alpha_o \exp\left(\frac{\gamma V}{V_e}\right)\right] e \left(-\frac{2eV}{M}\right)^{\frac{1}{2}} \frac{r^2}{r_s^2} \qquad \ldots(8.18)$$

Equation (8.18) is the potential distribution in the extra sheath region. At $r = r_s$, $V = \Delta V$ and equation (8.18) becomes

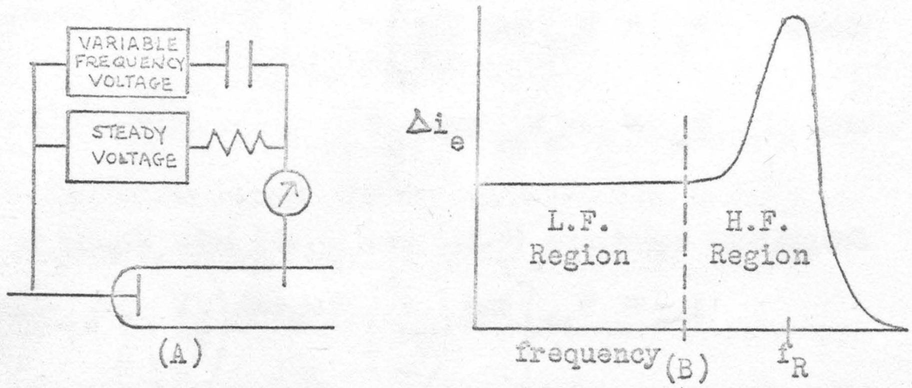
$$j_+ = N_{eo}\left[\exp\left(\frac{\Delta V}{V_e}\right) + \alpha_o \exp\left(\gamma \frac{\Delta V}{V_e}\right)\right] e \left(-\frac{2e\Delta V}{M}\right)^{\frac{1}{2}}$$

Substitute equation (8.19) into equation (8.12) and replace ΔV by equation (8.7) to give

$$I_+ = 4\pi r_s N_{eo} e \left[\exp\left\{-\frac{1}{2}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right)\right\} + \alpha_o \exp\left\{-\frac{\gamma}{2}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right)\right\}\right]$$

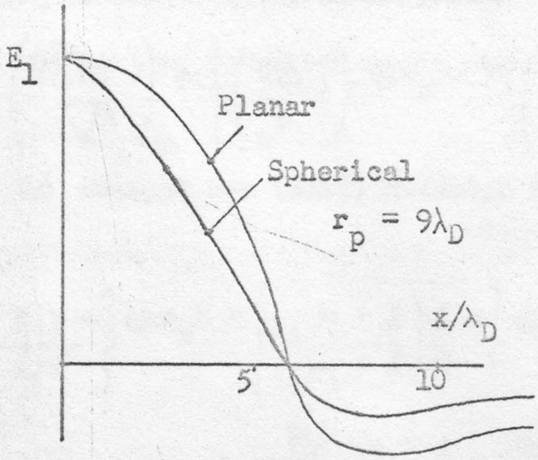
$$\ldots\ldots\ldots \times \left[\frac{eV_e}{M}\left(\frac{1+\alpha_s}{1+\gamma\alpha_s}\right)\right]^{\frac{1}{2}} \qquad \ldots(8.20)$$

When α_s and γ are zero equation (8.20) reduces to equation (6.56) the positive ion characteristic in an electropositive plasma. It is only valid when r_s is of the order of r_p as no account has been taken of the variation of sheath thickness with probe potential. In and electronegative plasma equation (6.56) is also valid when $\alpha_o < 2$ and $\gamma > 30$ because under these circumstances ΔV is of the order $- V_e/2$. In general, however, the positive ion current in an electronegative plasma is given by equation (8.20) where α_s is given by equation (8.11).



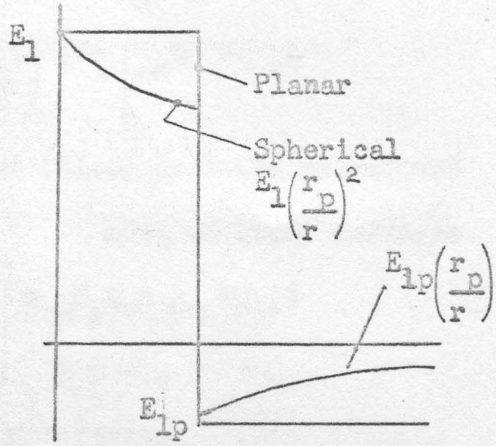
(A) Basic resonance probe circuit, and (B) frequency characteristic

Figure 23



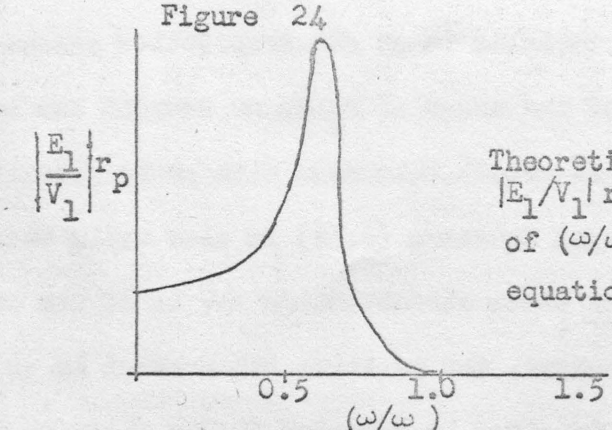
Theoretical planar and derived spherical resonance probe r.f. electric field distributions

Figure 24



Simplified model of the planar and spherical resonance probe (70)

Figure 25



Theoretical variation of $|E_1/V_1|\, r_p$ as a function of (ω/ω_p) according to equation (9.16) (70)

Figure 26

CHAPTER 9

THE RESONANCE PROBE

If an alternating voltage is superimposed on the steady potential applied to a probe an increase in probe current is observed. At low frequancies this increase in current is independent of frequency but at high frequencies the probe current passes through a maximum with increasing frequency. A further increase in frequency results in the increase in probe current falling to zero. Figure 23 shows the basic resonance probe circuit and frequency characteristic.

9.1. Low frequency region.

The steady electron current flowing to a negatively biased probe ($V_p < 0$) is

$$I_e = I_{eo} \exp\left(\frac{-|V_p|}{V_e}\right) \qquad \ldots(9.1)$$

When a sinusoidal voltage is superimposed on the steady probe voltage the time dependent electron current is

$$I_e + I_e(t) = I_{eo} \exp\left(\frac{-|V_p + \hat{V}_p \sin wt|}{V_e}\right) \qquad \ldots(9.2)$$

where $\hat{V}_p$ is the amplitude of the superimposed voltage of angular frequency w. The average electron current flowing in one complete period is

$$\frac{\langle I_e \rangle}{I_e} = J_o\left(\frac{\hat{V}_p}{V_e}\right) \qquad \ldots(9.3)$$

If the increase in electron current is denoted by ΔI_e we have

$$\langle I_e \rangle = I_e + \Delta I_e \qquad \ldots(9.4)$$

therefore equation (9.3) may be rewritten as

$$\frac{\Delta I_e}{I_e} = \left[J_o\left(\frac{\hat{V}_p}{V_e}\right) - 1 \right] \qquad \ldots(9.5)$$

Equation (9.3) is identical with equation (4.29) when $\hat{V}_p$ is put equal to $\hat{V}_o$ (38), (39). Equation (9.5) has been verified experimentally by Garscadden et al (38), by Takayama et als (66) and by Cairns (67).

9.2. High frequenct region.

It was thought, at one time, that the resonance frequency w_R was indentical with the plasma electron resonance frequency w_p (66), (67) and (68).

Ichikwa et al (68) derived an expression for the fractional increase in electron current

$$\frac{\Delta I_e}{I_e} = J_o\left(\frac{\hat{V}_p}{V_e}\right) - 1 + \frac{2^{\frac{1}{2}}\nu}{w\left\{\left[1 - \left(\frac{w_p}{w}\right)\right]^2 + \left(\frac{2\nu}{w}\right)^2\right\}} \frac{\lambda_D w_p}{Lw}\left(\frac{V_p}{V_e}\right)^{\frac{1}{2}} J_1\left(\frac{\hat{V}_p}{V_e}\right) \qquad \ldots(9.6)$$

where ν is the electron collision frequency, L is the effective penetration depth of the applied h.f. field and J_n is the modified Bessel function of the first kind of order n. By inspection it is seen that equation (9.6) has a maximum value when $w = w_p$. In other words according to equation (9.6) $w_R = w_p$.

Crawford et al (69), (70), point out that the assumptions made in deriving equation (9.6) are invalid. They claim that no account has been taken of the variations of the electron concentration, electric field and potential within the plasma sheath region. The h.f. field is assumed to penetrate only to a depth L and the analysis applies only to an infinite plane probe in a semi-infinite plasma. None of these assumptions are justified when applied to a practical probe.

If a high frequency voltage is applied between two floating probes Gibson et al (71) deduced that the coupling between the two probes is purely capacitative. The probe current will thus be proportional to the permittivity of the medium between the probes.

9.2.1. Plasma permittivity.

The complex permittivity of a plasma is given by

$$\epsilon = 1 - \frac{w_p^2}{w(w - j\nu)} \qquad \ldots(9.7)$$

where w_p, the angular plasma electron resonance frequency, is related to the electron concentration by

$$w_p = \left(\frac{N_{eo} e^2 4\pi}{m} \right)^{1/2} \qquad \ldots(9.8)$$

providing the permittivity of the unionized gas is taken to be unity. If $w \gg \nu$ equation (9.7) reduces to

$$\epsilon = 1 - \frac{w_p^2}{w^2} \qquad \ldots(9.9)$$

9.2.2. Resonance conditions

Assuming a purely capacitative coupling between the probes and an effective permittivity ϵ_{eff} the instantaneous probe current is

$$I(t) = \frac{\epsilon_{eff}}{4\pi} \frac{dE}{dt} \qquad \ldots(9.10)$$

where E is the electric field strength in the medium between the probes. If the medium is a homogeneous plasma ϵ_{eff} is given by equation (9.7) and when $w \gg \nu$ ϵ_{eff} is given by equation (9.9). The probe current increases as w increases.

If one assumes two plane parallel probes with a vacuum sheath of thickness x_s in front of each probe and a plasma of thickness p between the two sheaths the effective permittivity of the medium between the probes is

$$\epsilon_{eff} = \frac{(w_p^2 - w) + jw\nu}{\left\{ w_p^2 \left[\frac{2x_s}{2x_s + p} \right] - w^2 \right\} + jw\nu} \qquad \ldots(9.11)$$

where the permittivity of the sheath is assumed to be unity and that of the plasma is given by equation (9.7). The probe current reaches a maximum when

$$w = w_R = \frac{w_p}{\left(1 + \frac{p}{2x_s}\right)^{1/2}} \qquad \ldots(9.12)$$

In this case it is seen that resonance occurs at a frequency below that of the plasma electron resonance frequency.

9.3. High frequency resonance.

The high frequency resonance mechanism has been examined in detail for an infinite plane probe. The analysis has been greatly simplfied and adjusted to fit the case of a spherical probe (69), (70). It is assumed that a parabolic d.c. potential profile exists in the sheath and that the electrons have a Maxwellian velocity distribution.

9.3.1. High frequency potential drop between probe and plasma.

If E_1 is the h.f. electric field the h.f. potential drop between the surface of the plane probe and the plasma is

$$V_1 = \int_0^\infty E_1 \, dx \qquad \ldots(9.13)$$

Equation (9.13) may be extended to the case of a spherical probe, according to Crawford and Harp (69) by writing

$$V_1 = \int_{r_p}^\infty E_1 \left(\frac{r_p}{r}\right)^2 dr \qquad \ldots(9.14)$$

The variation of E_1 with distance from the surface of a plane probe has been derived theoretically and is shown in Figure 24. Also shown in Figure 24 is the variation of $E_1(r_p/r)^2$ with distance from the surface of a spherical probe of radius $r_p = 9\lambda_D$.

In general the dependence of E_1 on x/λ_D can only be obtained by rather complicated analysis involving numerical integration. This approach may be simplified by assuming a vacuum sheath region of constant h.f. electric field E_1 and a plasma region of constant h.f.

electric field E_1/ϵ where ϵ is, in general, given by equation (9.7) or by equation (9.9) when $w \gg \nu$. The extension to the case of a spherical probe is made by multipling the electric field by $(r_p/r)^2$. This simplified variation of h.f. electric field with distance from the probe's surface is shown in Figure 25.

The high frequency potential drop between probe and plasma is given, in the case of a spherical probe, by

$$V_1 = \int_{r_p}^{r_p+x_s} E_1\left(\frac{r_p}{r}\right)^2 dr + \int_{r_p+x_s}^{\infty} \frac{E_1}{\epsilon}\left(\frac{r_p}{r}\right)^2 dr \qquad \ldots(9.15)$$

Assuming ϵ is given by equation (9.7) integrating equation (9.15) gives

$$\left|\frac{E_1}{V_1}\right| r_p = \left[\frac{(w_p^2 - w^2)^2 + (w\nu)^2}{\left\{w_p^2\left[\frac{x_s}{x_s + r_p}\right] - w^2\right\}^2 + (w\nu)^2}\right]^{1/2} \qquad \ldots(9.16)$$

Comparing equation (9.16) with the exact theoretical analysis it is found that the best fit results when $x_s = 5\lambda_D$. The variation of $|E_1/V_1| r_p$ with w/w_p is shown in Figure 26. This variation has been verified expefimentally.

9.3.2. Effect of probe potential on resonance.

When the steady probe to plasma potential is V_p Harp and Crawford (70) show that the high frequency sheath thickness x_s should be replaced by

$$x_s = \left[\left|\frac{10\, V_p}{V_e}\right|^{1/2} - 2\right]\lambda_D \qquad \ldots(9.17)$$

9.3.3. Half-width of resonance peak.

The half-width of the resonance peak described by equation (9.16) can be shown to be equal to the electron collision frequency ν.

9.3.4. Resonance increase in probe current.

The critical electron energy U_c that an electron must have to reach a probe at floating potential V_f where $V_f < 0$ is

$$U_c = e|V_f| \qquad \ldots(9.18)$$

When a potential V_1 is superimposed on the floating potential

$$U_c = e|V_f + V_1| \qquad \ldots(9.19)$$

Now at any given time t V_1 is given by

$$V_1 = \int_0^\infty E_1(x,t)\, dx \qquad \ldots(9.20)$$

If $E_1(x)$ varies sinusoidally with time

$$V_1 = \int_0^\infty E_1 \sin wt.\ F(x)\, dx \qquad \ldots(9.21)$$

where $F(x)$ is the spatial distribution function of the electric field and is a function of frequency w. If $K(w)$ is the maximum value of the integral in equation (9.21) we can write

$$V_1 = E_1 \sin wt.\ K(w) \qquad \ldots(9.22)$$

At a given time t U_c is given by equation (9.19) as

$$U_c = e\left|V_f + E_1 \sin wt.\ K(w)\right| \qquad \ldots(9.23)$$

The instantaneous electron current flowing to the negative probe is

$$I_e + I_e(t) = I_{eo} \exp\left(\frac{-U_c}{eV_e}\right) \qquad \ldots(9.24)$$

where U_c is given by equation (9.23). The time averaged electron current is

$$\frac{\langle I_e \rangle}{I_e} = J_o\left[\left(\frac{\hat{V}_p}{V_e}\right)\left(\frac{E_1}{\hat{V}_p}\right)K(w)\right] \qquad \ldots(9.25)$$

or

$$\frac{\Delta I_e}{I_e} = J_o\left[\left(\frac{\hat{V}_p}{V_e}\right)\left(\frac{E_1}{\hat{V}_p}\right)K(w)\right] - 1 \qquad \ldots(9.26)$$

When $w \ll w_R$ the transit time of an electron is very much less than the period of oscillation of the applied h.f. potential and U_c is then given by

$$U_c = e\left|V_f + \hat{V}_p \sin wt\right| \qquad \ldots(9.27)$$

Equation (9.27) then immediately reduces to equation (9.2). As w tends to w_R it has been shown that E_1/V_1 tends to a maximum. As K(w) varies only slowly with w it follows from equation (9.26) that $\Delta I_e/I_e$ tends to a maximum as w tends to w_R. For $w \gg w_R$ the electric field at the probe's surface is reduced and so $\Delta I_e/I_e$ tends to zero.

CHAPTER 10

SPACE PROBES

Probes have been used to measure plasma parameters in the ionized layers surrounding the earth. Numerous types of probes have been used including the double probe and the resonance probe. Here we will consider the effect of the probe's motion on the collection of positive ions by a moving cylindrical probe.

10.1. Floating potential of a space probe.

The potential of an isolated probe relative to the plasma in which it is immersed is such that the net probe current is zero. In other words the current flowing towards the probe is just equal to the current flowing away from the probe. If I_+ and I_e are the positive ion and electron currents reaching the probe's surface and $I_{e,h\nu}$ and $I_{e,\gamma}$ are the photoelectric and secondary electron currents flowing away from the probe's surface the condition for a probe to be at floating potential is

$$I_e - I_+ = I_{e,h\nu} + I_{e,\gamma} \qquad \ldots(10.1)$$

I_e must always be greater than I_+ in order to compensate for the photoelectric and secondary electron current emitted by the probe. The magnitude of I_e depends on the probe to plasma potential V_f. If $(I_{e,h\nu} + I_{e,\gamma})$ is small $(I_e - I_+)$ is small and V_f is negative.

This situation would arise when the probe is shielded from the ultra violet of the sun either by the shadow of the earth or of the satellite carrying the probe. If the photoemission is large $(I_e - I_+)$ will also have to be large and it may even be necessary for V_f to be positive.

The floating potential will be a function of the probe's orientation and position in its orbit. Both of these variables must be taken into account when analysing space probe characteristics.

When using a double probe system the floating potential of the larger probe will only vary very slightly with orientation and position providing its surface area is very large in comparison with the measuring probe.

10.2. Space charge sheath surrounding a moving probe.

An important problem encountered in the theory of space probes is that of estimating the dimensions of the space charge sheath surrounding a moving probe. There do not appear to be any theories that satisfactorily account for the inevitable distortion of the sheath and one is usually forced to assume that the sheath has the same shape as the probe.

Because the probe's velocity is of the same order of the ion's thermal velocity it is essential to consider its effect on the collection of positive ions.

10.3. Moving cylindrical probe characteristic.

Here the case is considered where a double probe consists of a large reference electrode connected via a variable voltage supply to a much smaller cylindrical electrode. Under these conditions the cylindrical electrode behaves as a single probe.

Consider a rectangular coordinate system x, y, z in which x and z are perpendicular to and tangential to the probe's surface respectively and y is parallel to the probe's axis.

Let u_x, u_y, and u_z be the carrier velocity components and let the velocity vector W of the moving probe be inclined at an angle θ to the y axis and let its projection on the xz plane be inclined to the x axis at an angle β.

Assuming a Maxwellian distribution of velocities with respect to a stationary coordinate system one obtains for the distribution function at the sheath edge of a moving system

$$F(u_x,u_y,u_z)\,du_x du_y du_z = \frac{N}{(\pi C^2)^{3/2}} \exp\left\{-\frac{1}{C^2}\left[(u_x - W\sin\theta\cos\beta)^2 + (u_y - W\cos\theta)^2 + (u_z - W\sin\theta\sin\beta)^2\right]\right\} . du_x du_y du_z \qquad \ldots(10.2)$$

where $\quad C = \left(\frac{2kT}{m}\right)^{1/2}$

The number of carriers crossing the elementary sheath region of area $r_s d\beta L$ with velocities in the range u_x to $u_x + du_x$, u_y to $u_y + du_y$ and u_z to $u_z + du_z$ is

$$L r_s u_x\, F(u_x,u_y,u_z)\, du_x du_y du_z d\beta \qquad \ldots(10.3)$$

where L is the length of the cylindrical probe.

The probe current is obtained by substituting equation (10.2) into equation (10.3), multiplying by the carrier charge, and then integrating between the appropriate limits to give (72)

$$I = \frac{4ANr_s e}{c^2} \exp(-K^2) \int_{u_x=0,\,u_1}^{\infty} \int_{u_z=0}^{p} u_x \exp\left[\frac{-(u_x^2+u_z^2)}{c^2}\right] \times J_0\left[\frac{2K}{C}(u_x^2+u_z^2)^{\frac{1}{2}}\right] du_x du_z \qquad \ldots(10.4)$$

where $K = \frac{W}{C} \sin\theta$

The lower limit of u_x is zero when the probe attracts carriers and is $(2eV_p/\mathcal{M})^{\frac{1}{2}}$ when it repels them. The upper limit of u_z is p where p is given by

$$p^2 = \frac{r_p^2}{r_s^2 - r_p^2}\left(u_x^2 - \frac{2eV_p}{\mathcal{M}}\right) \qquad \ldots(10.5)$$

10.3.1. Accelerating probe potentials

When $eV_p < 0$ equation (10.4) may be integrated to give

$$I = ANe\left(\frac{kT}{2\mathcal{M}\pi}\right)^{\frac{1}{2}} \frac{2}{\pi^{\frac{1}{2}}} \exp(-K^2) \sum_{n=0}^{\infty} \frac{K^n}{n!}\left[e^{-\eta}(-\eta)^{-\frac{1}{2}n}\,\Gamma\left\{n+\frac{3}{2}, -\eta\left(1+\frac{r_p^2}{r_s^2-r_p^2}\right)\right\} \times J_n\left(-2K\eta^{\frac{1}{2}}\right) + \frac{r_s}{r_p}\frac{K^n}{n!}\gamma\left\{n+\frac{3}{2}, \frac{-r_p^2\eta}{r_s^2-r_p^2}\right\}\right] \qquad \ldots(10.6)$$

where $\eta = \frac{eV_p}{kT}$ and in this case $\eta < 0$ ...(10.7)

$$\Gamma(\nu, x) = \int_x^{\infty} e^{-t}\, t^{\nu-1}\, dt \qquad \ldots(10.8)$$

$$\gamma(\nu, x) = \int_0^{x} e^{-t}\, t^{\nu-1}\, dt \qquad \ldots(10.9)$$

The stationary probe characteristic is obtained by putting K = 0 in equation (10.6) which then reduces to equation (2.22).

When r_s/r_p tends to unity equation (10.6) becomes

$$I = ANe\left(\frac{kT}{2\pi m}\right)^{1/2}\frac{2}{\pi^{1/2}}\exp\left(-K^2\right)\left[(1+K^2)J_0\left(\frac{K^2}{2}\right) + K^2J_1\left(\frac{K^2}{2}\right)\right] \quad \ldots(10.10)$$

When r_s/r_p tends to infinity equation (10.6) becomes

$$I = ANe\left(\frac{kT}{2\pi m}\right)^{1/2}\frac{2}{\pi^{1/2}}\exp\left(-\eta-K^2\right) \times\sum_{n=0}^{\infty}\frac{K^n}{n!(-\eta)^{\frac{1}{2}n}}\Gamma(n+\tfrac{3}{2}, -\eta)\, J_n\left[2K(-\eta)^{\frac{1}{2}}\right] \quad \ldots(10.11)$$

10.3.2. Retarding probe potentials

Equation (10.4) may be integrated for $\eta > 0$ to give

$$I = ANe\left(\frac{kT}{2\pi m}\right)^{1/2}\exp\left(-\eta-K^2\right) \times\sum_{n=0}^{\infty}\frac{(2n+1)!\,K^n}{(n!)^2 2^{2n}\eta^{\frac{1}{2}n}}J_n\left(2K\eta^{\frac{1}{2}}\right) \quad \ldots(10.12)$$

When $K = 0$ equation (10.12) becomes

$$I = ANe\left(\frac{kT}{2\pi m}\right)^{1/2}\exp\left(-\eta\right) \quad \ldots(10.13)$$

10.3.3. Practical probe characteristics.

Space probes are normally opperated in the electron retarding and positive ion accelerating regions. To avoid the necessity of knowing how r_s depends on V_p equation (10.10) is used for the collection of positive ions. For the collection of electons

K may be taken to be effectively zero and equation (10.13) is applicable.

10.4. Moving spherical probe.

The advantage of a moving spherical probe over the cylindrical probe is that the probe characteristic is independent of the angle of orientation. Theoretical probe characteristics may be deduced from an analysis similar to that given in sub section 10.3.

A modification to the conventional double probe system is the screened probe. This consists of a spherical probe screened by a concentric grid and has been used by Nagy et als (73).

10.5. Determination of space plasma parameters.

10.5.1. Static technique.

The electron temperature may be deduced in the conventional way from the slope of the $\ln(I_e)$ vrs V_p characteristic.

Carrier concentrations are usually deduced from the positive ion characteristic assuming a value for the ion temperatûre which is thought to be between one half and one times the electron temperature.

10.5.2. Dynamic technique.

The resultant carrier current flowing to a negative probe is

$$I_r = I_{eo} \exp\left(+\frac{V_p}{V_e}\right) - I_+ - I_{e,h\nu} - I_{e,\gamma} \qquad \ldots(10.14)$$

I_+, $I_{e,h\nu}$ and $I_{e,\gamma}$ are insensitive to changes in V_p and so on differentiating equation (10.14) one obtains

$$\frac{dI_r}{dV_p} = I_{eo}\left(\frac{1}{V_e}\right)\exp\left(\frac{V_p}{V_e}\right) \qquad \ldots(10.15)$$

and differentiating again

$$\frac{d^2I_r}{dV_p^2} = I_{eo}\left(\frac{1}{V_e}\right)^2\exp\left(\frac{V_p}{V_e}\right) \qquad \ldots(10.16)$$

Boyd (74) has used the technique of obtaining V_e by measuring the ratio of the first and second derivatives of I_r.

CHAPTER 11

MEASURING TECHNIQUES

In general the theories presented in the preceeding chapters apply to infinite, isotropic, steady plasmas. Practical plasmas rarely approach these ideal conditions and so great care must be taken in interpreting the characteristics.

Any technique employed to measure the plasma parameters should produce the minimum disturbance to the plasma as is possible.

11.1. Probe dimensions.

The probe's dimensions are governed by the size of the vessel containing the plasma, the mean free path of the carriers, the dimensions of the sheath region and the characteristic Debye length of the plasma.

The mean free path ℓ is given by

$$\ell = \frac{1}{N\,q} \qquad \ldots(11.1)$$

where N is the neutral gas concentration and q is the elastic collision cross section for the carrier in the neutral gas. q is, in general, a function of carrier velocity; this is especially so for electrons in argon.

In the case if a collisionless space charge sheath the sheath dimensions may be calculated using the Langmuir diode equations presented in chapter 2.

The Debye length is given by

$$\lambda_D = \left(\frac{kT_e}{4\pi N_{eo}\, e^2}\right)^{1/2} \qquad \ldots(11.2)$$

The calculation of λ_D requires a previous estimation of T_e and N_{eo}.

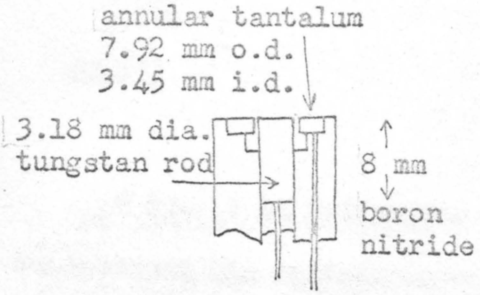
Variations in space of carrier concentration and temperature may well exist and care must be taken to ensure that these variations are small over the range of influence of the probe's electric field. The electron temperature is governed, in part, by a balance between the ionization rates and the recombination rates both at the wall of the vessel and in the volume of the plasma. Any increase in wall area will increase the wall recombination and hence the electron temperature. To reduce this perturbation of electron temperature to a minimum the area of the probe and insulated probe supports should be kept as small as possible.

It is unually assumed that no ionization occurs in the region of influence of the probe's electric field. Allen et als (49) show that the ionization current to a spherical probe is less than 1% of the normal probe current when

$$r_p < 8 \times 10^{-4} R \qquad \ldots(11.3)$$

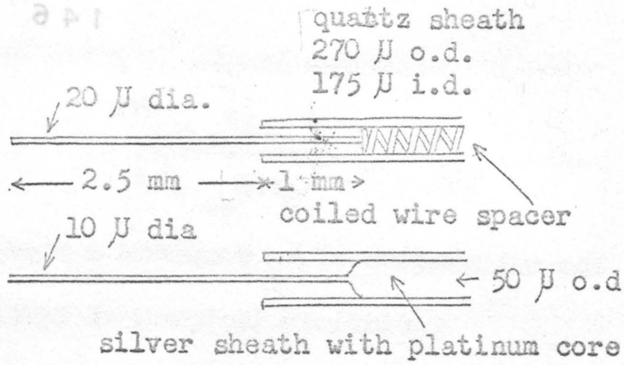
where R is the radius of the vessel.

Knowing ℓ, x_s or r_s, λ_D and R one should then be in a position to select a convenient value of r_p that will fit the theoretical requirements of one of the probe theories presented in the preceding chapters.



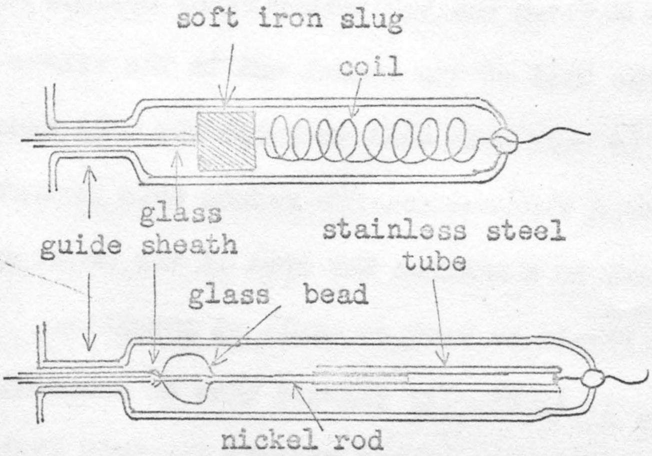
Plane probe with guard ring (75)

Figure 27



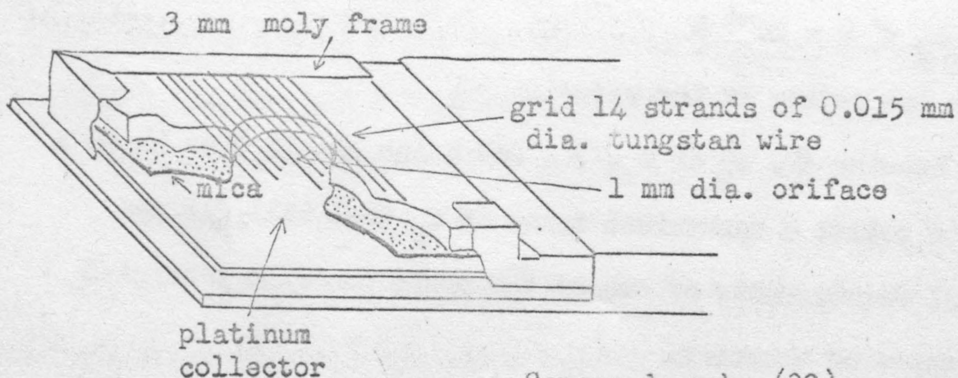
Cylindrical probes (27)

Figure 28



Movable probes (27)

Figure 29



Screened probe (32)

Figure 30

11.2. Probe design.

11.2.1. Plane probe.

To avoid problems caused by edge effects a guard ring should always be used with a plane probe. The gap between the probe and guard ring should be less than the sheath thickness. Janes et al (75) report that they observed a 100% increase in ion current for a 200volt change in probe potential in the absence of a guard ring whilst only a 5% increase was observed in the presence of the guard ring.

The probe design shown in Figure 27 overcomes the problems of changes in sheath surface area as a result of changes in probe potential and of changes in probe surface area as a result of sputtering.

11.2.2. Cylindrical probe.

As one normally considers a cylindrical probe to be infinitely long the practical requirement is that the length should be very much greater than its diameter. End effects may be corrected for by using a probe of variable length (76).

The sputtering of probe material onto the probe's supports is always a problem as this may produce an increase in probe area. Verweij (27) overcame this problem by placing a spacer of finely coiled wire between the probe and the insulating sheath. Another technique for separating the probe from the insulating sheath is to electroplate the wire within the sheath or to use Wollaston wire and then dissolve off the silver from around the length of platinum probe. These two types of cylindrical probe are shown in Figure 28.

11.2.3. Spherical probe.

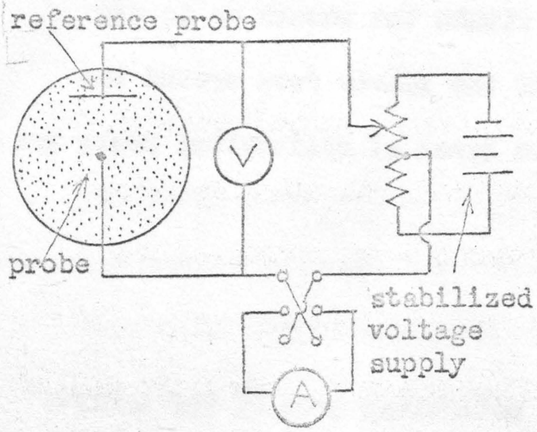
It is necessary that the collecting area of the sphere be very much smaller than the cylindrical length of wire supporting the sphere. Spherical platinum probes may be made by holding the tip of some wire in a non-luminous Bunsen flame. After a little practice perfect spherical probes can be made quite easily.

11.2.4. Movable probes.

Spacial distributions of plasma parameters can conveniently be measured with movable probes. The movement of the probe may be brought about either by the use of a magnet or by tapping. Figure 29 shows two such movable probes (27). Other designs of movable probes have been used by Howe (36).

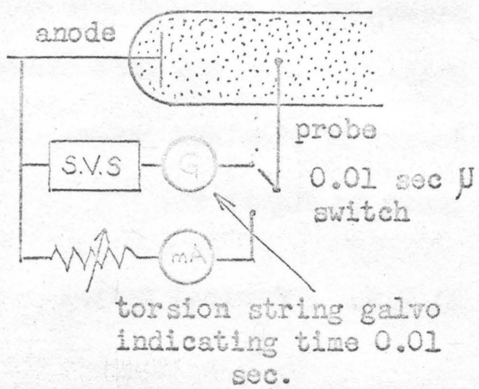
11.2.5. Screened probe.

The positive ion current and the electron current flowing to a plane probe have been separated out by Boyd (32) using a screened probe. The construction details of this are given in Figure 30.



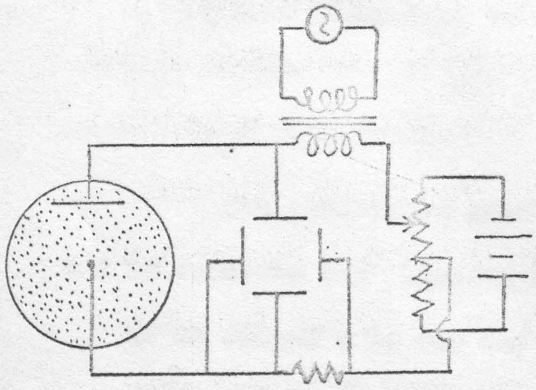
Basic probe circuit

Figure 31



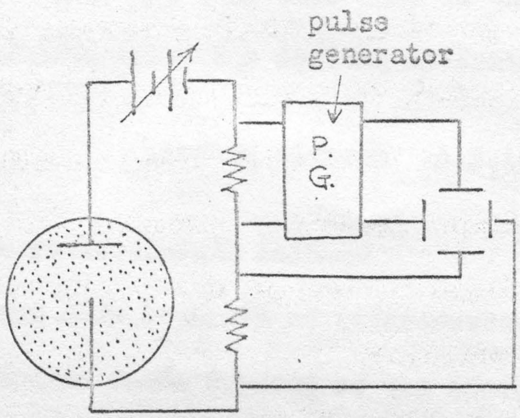
Verweij's probe circuit(27)

Figure 32



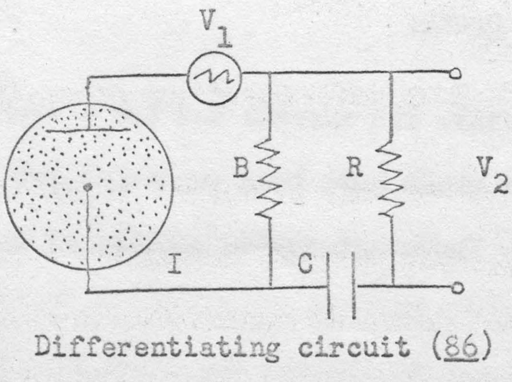
Basic sweep circuit

Figure 33



Basic pulse circuit (77)

Figure 34



Differentiating circuit (86)

Figure 35

11.3. Static probe measurements.

If the plasma under investigation does not suffer from either short term or long term fluctuations the point by point method of measuring the probe characteristic is, perhaps, the simplest. Figure 31 shows the basic probe circuit.

In many plasmas probe contamination may result in serious errors in the measurement of probe characteristics. The circuit shown in Figure 32 was used by Verweij (27) for measuring characteristics in discharges having oxide coated cathodes. Except at the time of taking a reading the probe is biased at such a potential that an electron current flows to the probe that is just equal to the maximum electron current that would flow during the measurement of the characteristic. This ensures that the work function of the probe's surface remains constant in between each measurement. The readings should be taken as rapidly as possible after switching off the clean up current. This requires a fast responding critically damped milli- or micro-ammeter.

11.4. Dynamic probe measurements.

11.4.1. Application of dynamic measuring techniques.

These techniques may be applied to periodically fluctuating plasmas. They involve an instantaneous measurement of a particular point on the characteristic or a measurement of the complete characteristic at a given phase angle of the periodically fluctuating plasma. All measurements must be made in a time interval very much shorter than the time in which fluctuations in plasma parameters occur.

Dynamic probe measurements are usually displayed on a cathode ray oscilloscope and photographed or plotted directly on an x-y recorder.

11.4.2. Equilibrium considerations.

For dynamic probe measurements to be meaningful the rate of change of probe potential must not be so rapid that the carriers in the neighbourhood of the probe do not reach equilibrium with the probe's electric field.

Oskam et als (77) have calculated an approximate expression for a time constant that gives a measure of the time necessary for the carriers to reach equilibrium with the probe's field as a result of an instantaneous change in probe potential. Their analysis assumes a collision dominated plasma and that the electrons reach equilibrium instantaneously whilst the ions reach equilibrium relatively slowly. Their expression for the time constant t_o is

$$t_o \approx \left(\frac{4\pi e N_+ \mu_{o+}}{p} \right)^{-1} \qquad \ldots(11.4)$$

To ensure equilibrium between the ion's motion and the probe's field the scan time must be very much less than t_o.

11.4.3. Sweep methods.

A time varying potential is applied to the probe having sufficient amplitude to scan the whole of the characteristic. This sweep voltage is applied to the x-plates of a cathode ray oscilloscope and the corresponding changes in probe current are applied to the y-plates by monitoring the voltage drop across a resistor placed in the probe current circuit. The basic lay out of a sweep circuit is shown in Figure 33.

In order to obtain any useful information from the characteristic displayed on the oscilloscope using the circuit shown in Figure 33 the trace must first be photographed and then the current readings converted to a logarithmic scale. This task may be eliminated by feeding the voltage drop developed across the current measuring resistor into a logarithmic amplifier before feeding it onto the y-plates of the oscilloscope. The electron temperature can then be found directly by measuring the slope of the oscilloscope trace.

This sweep technique has been used by Jones et al (78), and by Harp (79) who gives full details of his circuit including the logarithmic amplifier. Details of a logarithmic amplifier are also given in reference (80). Crawford (81) describes a circuit that they have used to measure electron temperatures without the necessity of a logaßithmic amplifier.

11.4.4. Pulse methods.

These methods involve biasing the probe with a steady potential and then superimposing a voltage pulse of known amplitude at the same time measuring the corresponding change in probe current. The basic pulse circuit is shown in Figure 34 (77).

Pulse techniques have been used by Bills et al (82) and by Waymouth (34). Waymouth keeps his probe biased strongly negative between pulses. It is hoped that by doing this the high flux of positive ions reaching the probe stabilizes the work function of the probe's surface.

11.4.5. Differentiating methods.

These methods are used for determining the electron energy distribution functions from a measure of d^2I_e/dV_p^2 as a function of V_p.

A number of techniques have been developed for obtaining differentiated probe characteristic. The earliest of these was simply a graphical differentiation of the probe characteristic in the electron repelling region. This method has been used by Medicus (83). It is, however, liable to considerable error and so other techniques have been developed that can measure the differentiated characteristic directly.

It has been shown in sub section (9.1) that if a small alternating voltage is superimposed on the steady voltage bias of the probe an increase in d.c. probe current is observed. The increase in probe current given by equation (9.5) has assumed a

Maxwellian electron energy distribution and a sinusoidal voltage fluctuation.

If in the absence of a superimposed voltage the electron current is given by

$$I_e = f(V_p) \qquad \ldots(11.5)$$

in the presence of a superimposed voltage of amplitude $\hat{V}_p$ it is given by

$$I_e + \Delta I_e = f(V_p + \hat{V}_p(t)) \qquad \ldots(11.6)$$

Expanding equation (11.6) as a Taylor series

$$I_e + \Delta I_e = f(V_p) + \hat{V}_p f'(V_p) + \frac{\hat{V}_p^2}{2!} f''(V_p) + .. \qquad \ldots(11.7)$$

When $V_p(t) = \hat{V}_p \sin wt$ equation (11.7) becomes

$$I_e + \Delta I_e = f(V_p) + \frac{\hat{V}_p^2}{4} f''(V_p) + \frac{\hat{V}_p^4}{64} f^{iv}(V_p) + \ldots$$

$$- \left[\frac{\hat{V}_p}{4} f''(V_p) + \frac{\hat{V}_p^4}{48} f^{iv}(V_p) + \ldots \right] \cos wt$$

$$+ \left[V_p f'(V_p) + \frac{\hat{V}_p^3}{8} f'''(V_p) + \ldots \right] \sin wt \qquad \ldots(11.8)$$

If $\hat{V}_p$ is small equation (11.8) reduces to

$$I_e + \Delta I_e = f(V_p) + \frac{\hat{V}_p^2}{4} f''(V_p) \qquad \ldots(11.9)$$

Hence $$\frac{d^2 I_e}{dV_p^2} = \frac{4\Delta I_e}{V_p^2} \qquad \ldots(11.10)$$

This method for determining the second derivative of the probe characteristic was first used by Sloane et al (84). The method has been extended by Branner et als (85) who consider the relative mèrits of a superimposed sine-wave modulated and square-wave modulated probe potential.

Probe current characteristics may be differentiated directly using differentiating circuits. An example of a differentiating circuit is given in Figure 35 (86). If R>>B and CR is sufficiently small, it can be shown that

$$\frac{dI}{dV_1} = \frac{V_2}{C\,B\,R}\left(\frac{dV_1}{dt}\right)^{-1} \qquad \text{...(11.11)}$$

If (dV_1/dt) is constant (dI/dV_1) is directly proportional to V_2. The process may be repeated to obtain the second derivative. Space potential in general is taken to be at the point where (d^2I/dV^2) is zero; that this is so is shown in Figure 7.

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