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LONG MEMORY IN US REAL OUTPUT PER CAPITA

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Abstract

This paper analyses the long memory properties of quarterly real output per capita in the US (1948Q1 – 2008Q3) using non-parametric, semi-parametric and parametric techniques. The results vary substantially depending on the methodology employed. Evidence of mean reversion is obtained in a parametric context if the underlying disturbances are weakly autocorrelated. We also examine the possibility of a structural break in the data and the results indicate that there is a slight reduction in the degree of persistence after the break that is found to occur in the second quarter of 1978.

JEL Classification: C22, O40
Keywords: Fractional Integration, Long Memory, Convergence

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1. Introduction

Following the seminal work of Barro (1991) and Barro and Sala-i-Martin (1991, 1992, 1995), in the last couple of decades a vast literature on convergence has been produced. This is a key issue to assess the empirical relevance of neoclassical versus endogenous growth models. The standard Solow (1956) model implies that all countries should converge to a level of income that is determined by their respective saving rates and population growth rates, and that over time poor countries or regions tend to grow faster than rich ones. Therefore, evidence of convergence has traditionally been interpreted as supporting the neoclassical model. However, Barro and Sala-i-Martin (1991, 1992) argued that the estimated rate of convergence is only consistent with the neoclassical model if diminishing returns to capital set in very slowly, and that endogenous growth models with constant returns and gradual diffusion of technology also fit the data (see also Mankiw et al., 1992, Quah, 1993, and Sala-i-Martin, 1996). Their approach is based on testing for β-convergence in a regression of the first difference of per capita output against an exogenous rate of technical progress, as well as lagged steady-state and actual per capita output, where β is the slope coefficient. Their conclusion is that the rate of convergence to its steady state value of per capita output is exponential and approximately equal to 2% for most economies.

More recently, though, Michelacci and Zaffaroni (2000) have pointed out that this finding cannot be reconciled with the other stylized facts of unit roots in output (see Nelson and Plosser, 1982), and a fairly smooth trend of output per capita in the OECD economies (see Jones, 1995). They show that extending the standard Solow model to allow for cross-sectional heterogeneity in the adjustment speed results in output exhibiting long memory, and that per capita output is well represented by a mean-reverting long memory process with $0.5 \leq d < 1$, where d is the fractional integration parameter – in other words, it is not covariance stationary, but still mean-reverting, with the implication that standard unit root tests will not reject the null of non-stationarity even when convergence takes place. Their interpretation is that, given the long-memory properties of the output series, convergence does take place, but at a hyperbolic very slow rate rather than an exponential one.

However, their analysis has been criticised by Silverberg and Verspagen (2000), who have argued that it cannot really shed light on the time series properties of output per capita (and therefore take forward the convergence debate), as it relies on questionable filtering of the data, and on using the semiparametric Geweke and Porter-Hudak (GPH, 1983) method as modified by Robinson (1995a), which has been shown to be biased in small samples. Instead, Silverberg and Verspagen (2000) use Beran's (1994) FGN estimator and Sowell's (1992) parametric maximum likelihood estimator, which are not affected by small-sample bias, and show that the evidence of fractional integration in the range [0.5, 1), which is a key result in the paper by Michelacci and Zaffaroni (2000), disappears when these more appropriate methods are used.

In another recent paper Mayoral (2006) examined annual real GNP and GNP per capita in the US for the time period 1869-2001, using several parametric (Sowell, 1992, Mayoral, 2004, Velasco and Robinson, 2000) and semi-parametric (Geweke and Porter-Hudak, 1983 and Teverovsky and Taqqu, 1997) long-memory methods. Her results, though slightly different depending on the technique used, provide evidence that the orders of integration lie in the interval [0.5, 1), implying nonstationarity, high persistence and mean-reverting behaviour.

The present paper makes the following contributions: first, we investigate if mean reversion takes place in quarterly per capita real output in the US (1948-2008) by using a variety of non-parametric, semi-parametric and fully parametric techniques based on fractional integration and long-memory processes. The results indicate that the behaviour of US per capita real output is captured well by a linear trend model with stationary long-memory behaviour, implying that shocks affecting the series will revert to its trend sometime in the future. Moreover, the possibility of structural change is also investigated, and it is found that there has been a decrease in the degree of persistence of the series during the last three decades.

The layout of the paper is as follows. Section 2 discusses the relevance of fractional integration to test for convergence. Section 3 outlines the methods used here. Section 4 presents the empirical results. Section 5 focuses on long memory and structural change. Section 6 summarises the main findings and offers some concluding remarks.

2. Fractional integration and economic growth

Given a covariance stationary process $\{x_t, t = 0, \pm 1, \ldots\}$, with autocovariance function $E(x_t - Ex_t)(x_{t-j} - Ex_t) = \gamma_j$, according to McLeod and Hipel (1978), x_t displays the property of long memory if

$$\lim_{T \to \infty} \sum_{j=-T}^{T} \left| \gamma_j \right|$$

is infinite. An alternative definition, based on the frequency domain is as follows. Suppose that x_t has an absolutely continuous spectral distribution, so that it has a spectral density function, denoted by $f(\lambda)$, and defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \le \pi.$$

Then, x_t displays long memory if the spectral density function has a pole at some frequency λ in the interval $[0, \pi]$. Most of the empirical literature has concentrated on the case where the singularity or pole in the spectrum occurs at the zero frequency.[1] This is the standard case of I(d) models of the form:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \tag{1}$$
$$x_t = 0, \quad t \leq 0,$$

where L is the lag operator ($Lx_t = x_{t-1}$) and u_t is I(0) defined as a covariance stationary process with spectral density function that is positive and bounded at any frequency. Thus, the process u_t could itself be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially; however, it could decay at a much slower rate than exponentially. When d = 0 in (1), $x_t = u_t$, and x_t is said to be "*weakly autocorrelated*" as opposed to the case of "*strongly autocorrelated*" if d > 0. Moreover, if 0 < d < 0.5, x_t is still covariance stationary, but its lag-j autocovariance γ_j decreases very slowly, at the rate of j^{2d-1} as $j \to \infty$, and so the γ_j are non-summable. We say then that x_t has long memory given that $f(\lambda)$ is unbounded at the origin. Also, as d in (1) increases beyond 0.5 and through 1 (the unit root case), x_t can be viewed as becoming "*more nonstationary*" in the sense, for example, that the variance of the partial sums increases in magnitude. Processes of the form given by (1) with positive non-integer d are called fractionally integrated, and when u_t is ARMA(p, q) x_t has been called a fractionally ARIMA (or ARFIMA) model. This type of model provides a higher degree of flexibility in modelling low frequency dynamics which is not achieved by non-fractional ARIMA models.

There are several theoretical arguments that can be put forward to justify long memory (and fractional integration) in aggregate time series (Robinson, 1978; Granger, 1980; Taqqu et al., 1997; Chambers, 1998; Parke, 1999; etc.). Robinson (1978) and

[1] During the 1960s, Granger (1966) and Adelman (1965) pointed out that most aggregate economic time series have a typical shape with the spectral density increasing dramatically as the frequency approaches zero and that differencing the data leads to overdifferencing at the zero frequency.

Granger (1980) showed that if the individual series follow heterogeneous AR(1) processes of the form:

$$x_{i,t} = \alpha_i \, x_{i,t-1} + u_{i,t}, \quad i = 1, 2, ..., N, \quad t = 1, 2, ...,$$

then the aggregate series

$$x_t = \sum_{i=1}^{N} x_{i,t}$$

can exhibit long memory if, for example, the α_i are drawn from a Beta B(p, q) distribution for certain values p and q. With slight variations, this is also the argument employed in Michelacci and Zaffaroni (2000), Silverberg and Verspagen (2000), Mayoral (2006) and others when using long range dependence in aggregate output series. A crucial issue here is then to determine the appropriate order of the series to distinguish between permanent and transitory changes in output. Thus, for example, if it is I(0) stationary, shocks affecting the series will be transitory and the degree of decay will depend on the structure describing the short-run dynamics, being, for example, exponential if they are autoregressive. On the other hand, if the series is I(1), shocks will be permanent and thus persisting forever. By allowing for fractional integration, we permit a much richer degree of flexibility in the dynamic behaviour of the series to analyse the persistence of shocks: if $d \in (0, 0.5)$, the series is covariance stationary and mean reverting, with the effects of shocks disappearing in the long-run, though at a slower (hyperbolic) rate than in the I(0) case; if $d \in [0.5,1)$, the series is no longer covariance stationary but is still mean reverting, while $d \geq 1$ implies that the series is nonstationary and non-mean-reverting.[2]

[2] In the case of d in the interval [0.5, 1) some authors argue that "mean reversion" is a misnomer given the nonstationarity nature of the process (Phillips and Xiao, 1999).

3. Methodology

In this section we briefly describe the methods employed for the empirical analysis in Section 4, which are based on non-parametric, semi-parametric and parametric techniques for modelling long-range dependence. It is well known that the findings on persistence and long memory can vary substantially depending on the method used, and therefore a robustness check is crucial.

3.1. Non-parametric approaches

The two methods presented here test the null hypothesis of short memory (i.e. d = 0 in (1)) against long memory (d > 0) and/or anti-persistence (d < 0).[3] First we describe a procedure developed by Lo (1991). The modified R/S statistic (Lo, 1991) is:

$$Q_T(q) = \frac{1}{\hat{\sigma}_T(q)}\left(\max_{1 \le k \le T} \sum_{j=1}^{k} (x_t - \bar{x}) \; - \min_{1 \le k \le T} \sum_{j=1}^{k} (x_t - \bar{x}) \right),$$

where $\hat{\sigma}_T^2(q) = \hat{\sigma}_x^2 + 2 \sum_{j=1}^{q} \omega_j(q) \hat{\gamma}_j$, and $\omega_j(q) = 1 - \frac{j}{q+1}$, $1 \le j < T$,

and x_t is a stationary series (-0.5 < d < 0.5) of sample size T, with sample mean $\bar{x}$, sample variance $\hat{\sigma}_x^2$, and sample autocovariance at lag j given by $\hat{\gamma}_j$. This statistic was further normalized as:

$$V_T(q) = \frac{Q_T(q)}{\sqrt{T}}. \tag{2}$$

An advantage of the modified R/S statistic is that it allows us to obtain a simple formula for the fractional differencing parameter d, since

$$d = \frac{Log\,(Q_T(q))}{Log\,(T)}.$$

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[3] A process is said to be anti-persistent if it reverses itself more often than a random series would (see Mandelbrot, 1977).

The null hypothesis of I(0) includes ARMA models, though, as pointed out by Haubrich and Lo (2001), it does not contain a trend-stationary model. The limit distribution of $V_T(q)$ is derived in Lo (1991) and the 95% confidence interval with equal probabilities in both tails is [0.809, 1.862]. Several Monte Carlo experiments conducted by Teverovsky et al. (1999) and Willinger et al. (1999) show that this method is biased in favour of accepting the null of no long memory as the bandwidth parameter q increases. Therefore, these authors warned about using Lo's modified method in isolation.[4]

Lee and Schmidt (1996) showed that the KPSS test proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992) has the same power as Lo's statistic against long memory alternatives. Giraitis, Kokoszka, Leipus and Teyssiere (2003) proposed a centering of the KPSS statistic based on the partial sum of the deviations from the mean. They called this method the rescaled-variance V/S statistic, which is given by

$$M_T(q) \;=\; \frac{Var(S_1^*,\, S_2^*,\, \ldots,\, S_T^*)}{T\,\hat{\sigma}_T^2(q)}, \tag{3}$$

where $S_k^* \;=\; \sum_{j=1}^{k}(x_t - \bar{x})$, and $Var(S_1^*,\, S_2^*,\, \ldots,\, S_T^*) = \dfrac{1}{T}\sum_{j=1}^{T}(S_j^* - \bar{S}^*)$ is their sample variance. According to Giraitis et al. (2003) the V/S test is more suitable for series that exhibit high volatility, and various Monte Carlo experiments conducted by these authors show that the V/S test is less sensitive to the choice of the bandwidth number q. These authors showed that the asymptotic distribution of $M_T(q)$ is given by $F_k(\pi c^{1/2})$, where F_k is the limit distribution of the standard Kolmogorov statistic and c is the critical value at a chosen significance level (e.g. the 5% level).

[4] Other papers dealing with the small sample distribution of the R/S statistic are Harrison and Treacy (1997) and Izzeldin and Murphy (2000).

As for the choice of the optimal bandwidth parameter q (q^*) in the two methods described above, q = 0 corresponds to the classic Hurst-Mandelbrot R/S statistic. Haubrich and Lo (2001) suggested using Andrew's (1991) data-dependent procedure to determine the optimal bandwidth, which is given by

$$q^* = \left(\frac{3a^* T}{2} \right)^{\frac{1}{3}}, \qquad (4)$$

with $a^* = \dfrac{4\hat{\rho}^2}{(1-\hat{\rho}^2)^2}$, where $\hat{\rho}$ is the first order AR coefficient.

3.2 Semi-parametric approaches

The main advantage of the semi-parametric methods is that they only specify the I(d) structure, without modelling the d-differenced process, which is merely described as an I(0) process. In doing so they avoid the problem of potential misspecification in the short-run dynamics of the series.

There exist several procedures for estimating the fractional differencing parameter in semiparametric contexts. Of these, the log-periodogram regression estimate proposed by Geweke and Porter-Hudak (1983) has been the most widely used. This method was later modified by Künsch (1986) and Robinson (1995a) and has been analysed, among others, by Hurvich and Ray (1995), Velasco (1999a, 2000) and Shimotsu and Phillips (2002). Based on the Whittle function, Robinson (1995b) proposed another estimator, which is essentially a local 'Whittle estimator' in the frequency domain, using a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{s=1}^{m} \log \lambda_s \right), \qquad (5)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{s=1}^{m} I(\lambda_s)\, \lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{T}, \qquad \frac{m}{T} \rightarrow 0,$$

where $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t\, e^{i \lambda_s t} \right|^2,$$

and $d \in (-0.5, 0.5)$. Under finiteness of the fourth moment and other mild conditions, Robinson (1995b) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \rightarrow_d N(0, 1/4) \quad as\ T \rightarrow \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroskedasticity (Robinson and Henry, 1999) and is more efficient than other semi-parametric competitors.

Although there exist further refinements of this procedure, (Velasco, 1999b, Velasco and Robinson, 2000; Phillips and Shimotsu, 2004, 2005; etc.), these methods require additional user-chosen parameters, and the estimates of d may be very sensitive to the choice of these parameters. In this respect, the method of Robinson (1995b) seems computationally simpler.

3.3 Parametric approaches

Estimating d parametrically along with the other model parameters can be done in the frequency domain or in the time domain. In the time domain, Sowell (1992) analysed the exact maximum likelihood estimator of the parameters of the ARFIMA model, using a recursive procedure that allows quick evaluation of the likelihood function, which is given by:

$$(2\pi)^{-n/2} |\Sigma|^{-1/2} \exp\left(-\frac{1}{2} X_n^{'} \Sigma^{-1} X_n \right),$$

where $X_n = (x_1, x_2, ..., x_n)'$ and $X_n \sim N(0, \Sigma)$.[5] Other parametric methods of estimating d based on the frequency domain were proposed, among others, by Fox and Taqqu (1986) and Dahlhaus (1989). Small sample properties of these and other estimators were examined in Smith et al. (1997) and Hauser (1999). In the first of these articles, several semi-parametric procedures were compared with Sowell's maximum likelihood estimation method, finding that Sowell's (1992) procedure outperforms the semi-parametric ones in terms of bias and mean square errors. Hauser (1999) also compares Sowell's (1992) procedure with others based on the exact and the Whittle likelihood function in the time and the frequency domain, and shows that Sowell's procedure dominates the others in the case of fractionally integrated models.

In this paper we will employ a method suggested by Robinson (1994). There are several reasons for using this method. First, it allows us to include deterministic terms such as an intercept or a linear time trend unlike other methods such as Lo's (1991) non-parametric approach. Another advantage of this method is that it is valid for any real value of d, therefore encompassing stationary (d < 0.5) and nonstationary (d ≥ 0.5) hypotheses, unlike the methods described above that require first differencing to render the series stationary prior to the estimation of d. Moreover, the limit distribution is standard normal and is the most efficient method under Gaussianity of the error term. We employ here the following model,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, ... \tag{6}$$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \tag{7}$$

where u_t is assumed to be I(0), and given the parametric nature of this method, u_t has to be specified in a parametric form, that may be a white noise process, or more generally,

[5] See also Beran (1995), Tanaka (1999), Dolado, Gonzalo and Mayoral (2003) for other parametric methods in the time domain.

some type of weak autocorrelation (i.e, ARMA) structure. In this approach we test the null hypothesis:

$$H_o : \; d = d_o,$$

for any real value d_o in (6) and (7), and the limit distribution is N(0, 1). The functional form of the test statistic is presented in Appendix A.[6]

4. Empirical results

The time series data analysed in this section is real per capita US GDP, quarterly, for the time period 1948Q1 – 2008Q3. We will present results based on both the original time series and its (first-difference) log-transformation. Real GDP data (GDPC96) were obtained from the US Department of Commerce_ Bureau of Economic Analysis (http://www.bea.gov), while those of population were retrieved from the Civilian Noninstitutional Population (CNP160V) obtained from the Department of Labor, Bureau of Labor Statistics (http://www.bls.gov).

[INSERT FIGURE 1 ABOUT HERE]

Figure 1 displays plots of the time series and their first differences. Both series appear to be nonstationary in levels, increasing over time. On the other hand, their first differences have a stationary appearance. First, we perform the non-parametric methods described in Section 3.1. The results based on Lo's (1991) modified R/S statistic and Giraitis et al.'s (2003) procedure are displayed in Table 1.

[6] Empirical applications based on this procedure can be found for example, in Gil-Alana and Robinson, (1997), Gil-Alana (2000, 2005), etc.

[INSERT TABLE 1 ABOUT HERE]

Starting with real per capita GDP (in first differences), we cannot reject the null of I(0) stationarity for any bandwidth parameter when using Lo's (1991) modified R/S statistic. Moreover, the estimated values of d are smaller than 0.10 in all cases. Using the V/S statistic of Giraitis et al. (2003) the same evidence against long memory is obtained for all bandwidth parameters. Performing the same type of analysis on the growth rate series, evidence of short memory is obtained in all cases. Thus, using these non-parametric procedures, there is no evidence of long memory in the first differences of US real output (both the raw series and the one in logs).

[INSERT FIGURE 2 ABOUT HERE]

Next we compute the estimates of d based on the semi-parametric method of Robinson (1995b). The results displayed in Figure 2 refer to $\hat{d}$ in (5) for the whole range of values of the bandwidth number m = 1, 2, …, T/2 (on the horizontal axis).[7] Figure 2a refers to real per capita GDP while Figure 2b displays the estimates for the growth rate, and, in both cases, we present the 95% confidence intervals corresponding to the I(1) and the I(0) hypothesis respectively. The results are consistent in both cases. Evidence of I(1) behaviour in real output (or, alternatively, I(0) in the growth rate) is found if the bandwidth parameter is small. However, if m is higher than T/4, this hypothesis is rejected in favour of higher orders of integration. If we focus on $m = (T)^{1/2} \approx 16$, the I(1) hypothesis cannot be rejected for the original series and the I(0) one for the growth rate. Overall there is strong evidence against mean reversion (d < 1) for real

[7] The choice of the bandwidth is crucial in view of the trade-off between bias and variance: the asymptotic variance is decreasing with m while the bias is growing with m.

per capita GDP, and we found some evidence of long memory (d > 0) in the growth rate series if the bandwidth is large, a result that might be spurious and reflect the neglected ARMA structure for the d-differenced process. However, a limitation of the above approaches is that they do not allow the inclusion of deterministic terms such as intercepts and linear trends. Thus, in what follows, we assume that the series have an intercept and/or a linear time trend, and that it is the demeaned (detrended) series that exhibits long memory. For this purpose we employ the parametric approach of Robinson (1994) described in Section 3.3, assuming that the disturbances are white noise and also autocorrelated. In particular, we consider the set-up in (6) and (7), testing H_o (8) for d_o-values from -0.500 to 1.500 and 0.001 increments in the real per capita output, and from -1.500 to 0.500 in the growth rate. In other words, the tested (null) model is:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad (1 - L)^{d_o} x_t = u_t, \qquad t = 1, 2, \ldots ,$$

with I(0) u_t. Performing the statistic $\hat{r}$ as given in Appendix A we should expect a monotonic decrease in the value of the test statistic with respect to the values of d_o. Such monotonicity is a consequence of the one-sided alternatives employed in this procedure. Thus, for example, we would expect that if H_o (8) is rejected with $d_o = 0.250$ against the alternative H_a: $d > 0.250$, an even stronger rejection occurs when testing H_o with $d_o = 0.200$.

[INSERT FIGURE 3 ABOUT HERE]

Figure 3 shows the values of the test statistic for the two cases of an intercept and a linear time trend under white noise and AR(1) disturbances, as well as the confidence bands for the non-rejection cases. It can be seen that, in the two cases of white noise disturbances, there is a monotonic decrease, and the values of d_o for which H_o cannot be rejected (displayed in Table 2) range, for real per capita GDP, between 1.116 and 1.333 with an intercept and between 1.119 and 1.334 with a linear trend. For the growth rate series, the non-rejection values are constrained between 0.152 and 0.394 with an intercept, and between 0.146 and 0.396 with a linear trend respectively. Thus, although there are slight differences between the raw and the logged series, the implications are the same in the two cases, with values above 1 for the two series in levels.

In the case of autocorrelated errors, we observe a lack of monotonicity in the values of the test statistic with respect to d: we obtain non-rejection values when d is close to 0 and 1 but rejections for values in between. This may be explained by the low power of this method if the roots of the AR polynomials are close to the unit circle. In fact, this happens with all parametric procedures due to the competition between the fractional differencing parameter and the AR parameters in describing the time dependence. When employing higher AR orders we obtain essentially the same results.

[INSERT TABLE 2 ABOUT HERE]

We report in Table 2 the values of d that produce the lowest statistics in absolute value. These values should be approximations to the maximum likelihood estimates since Robinson's (1994) method is based on the Whittle function, which is an approximation to the likelihood function.[8] Thus, we choose the values of d where the test statistic crosses the 0-axis in the plots in Figure 3. For real per capita GDP the most interesting case in view of the LR tests is the one with a linear time trend and AR(1) u_t,

[8] We also computed Sowell's (1992) and Beran's (1994) statistics, and the results were rather similar to those reported here.

with an estimated value of d of 0.312. In case of the growth rate series we observe that the estimated value of d is also very sensitive to the choice of the error term. We obtain values around 0.26 for the white noise case (and thus with values significantly above 1 for the undifferenced log-series), and anti-persistence (d < 0) in case of autocorrelated disturbances with values of -0.622 (with an intercept) and -0.546 (with a linear trend). Thus, according to these two specifications, the log-series are mean-reverting with values of d equal to 0.378 (with an intercept) and 0.454 (with a linear time trend).

Based on the t-tests on the deterministic terms and LR tests on the specification of the autocorrelated structure, we choose as potential models, for the real per capita GDP, the following specification:

$$y_t = \underset{(141.86)}{0.01497} + \underset{(189.85)}{0.00014t} + x_t, \quad (1-L)^{0.312} x_t = u_t, \quad u_t = 0.913 u_{t-1} + \varepsilon_t$$

(t-values in parenthesis), and for the growth series,

$$(1-L)\log y_t = \underset{(190.66)}{0.00497} + x_t, \quad (1-L)^{-0.622} x_t = u_t, \quad u_t = 0.887 u_{t-1} + \varepsilon_t,$$

implying the latter equation an order of integration for the log-real per capita output of about 0.378. Thus, evidence of mean reversion is obtained in the two cases, the unlogged and the logged versions of real per capita US output.[9]

Overall, our findings are partially consistent with those of Mayoral (2006), who using a long span of US data on both GNP and GNP per capita covering 133 years reaches the conclusion that these series are highly persistent but mean-reverting series that can be modelled as fractionally integrated processes, a result which is found to be robust even when allowing for breaks in the deterministic component of the model. As

[9] We also consider AR(k) processes with k = 2, 3 and 4 for the I(0) error term u_t, and the results were in all cases similar to the AR(1) case. Moreover, the orders of integration were in all cases in the interval (0, 0.5) for the two series in levels. We report in the paper the values for the AR(1) case since it produced the lowest statistics.

Mayoral (2006) points out, this evidence is inconsistent with endogenous growth models, in the context of which permanent policy changes should have permanent effects on economic growth. There is an important difference between our findings and those of Mayoral (2006), namely our estimated orders of integration are all in the range (0, 0.5) while in Mayoral (2006) they are in the interval [0.5, 1). An explanation for this may be the different data frequency employed. Since Mayoral's data are annual they are characterised by a higher degree of aggregation which may induce a higher degree of persistence in the data.

5. Long memory and structural change

In this section we take into account the possibility of a structural break in the data. This is a relevant issue in the context of fractional integration since it has been argued by many authors that fractional integration might be an artificial artefact generated by the existence of breaks in the data (see, e.g., Cheung, 1993; Diebold and Inoue, 2001; Giraitis et al., 2001; Mikosch and Starica, 2004; Granger and Hyung , 2004; etc.).

[INSERT TABLE 3 ABOUT HERE]

Table 3 displays for the two series the estimates of the break dates and the deterministic terms along with the fractional differencing parameters using the procedure developed by Gil-Alana (2008).[10] We employ models with intercept and intercepts with linear trends combined with white noise and AR(1) disturbances. Employing higher AR orders leads essentially to the same results for both break dates and fractional differencing parameters. Starting with the real per capita GDP series (in

[10] This method is briefly described in Appendix B.

Table 3a) we observe that the break date is found to occur at 1978Q2 in the four cases examined, and the same break date is found for the growth rate series (see Table 3b) if the disturbances are autocorrelated. We report in the table in bold the estimates corresponding to the selected model for each series.[11] Thus, for real per capita GDP the selected model is

$$y_t \;=\; \underset{(19.437)}{0.01551} + \underset{(10.553)}{0.00012}\,t + x_t, \quad (1-L)^{0.490} x_t \;=\; u_t, \quad u_t = 0.774 u_{t-1} \;\varepsilon_t, \quad t = 1, \ldots, T^*$$

and

$$y_t \;=\; \underset{(0.749)}{0.00171} + \underset{(16.546)}{0.00020}\,t + x_t, \quad (1-L)^{0.314} x_t \;=\; u_t, \quad u_t = 0.875 u_{t-1} \;\varepsilon_t, \quad t = T^* + 1, \ldots, T$$

and for the growth rate series,

$$y_t^* \;=\; \underset{(9.168)}{0.00556} \;+\; x_t, \qquad (1-L)^{-0.416} x_t \;=\; u_t, \qquad u_t = 0.742 u_{t-1} \;\varepsilon_t, \qquad t = 1, \ldots, T^*$$

and

$$y_t^* \;=\; \underset{(16.793)}{0.00429} \;+\; x_t, \qquad (1-L)^{-0.666} x_t \;=\; u_t, \qquad u_t = 0.876 u_{t-1} \;\varepsilon_t, \qquad t = 1, \ldots, T^*$$

with y_t^* = (1 - L)log(y_t), and T^* = 1978Q2 for the two series. Thus, we observe a reduction in the degree of persistence (as measured by the fractional differencing parameters) in the two cases, from 0.490 to 0.314 in case of the raw series, and from 0.584 to 0.334 in the log of GDP.

[INSERT FIGURE 4 ABOUT HERE]

[11] As in the previous section the models were selected according to the t-values for the deterministic terms along with LR tests.

Figure 4 displays the impulse response functions and their corresponding 95% confidence bands for the two series in the two subsamples. The results are similar for the two series, a rapid decrease being observed in the post-1978 period.

[INSERT FIGURE 5 ABOUT HERE]

Figure 5 displays the estimated time trends for the two subsamples in the original data and the two estimated intercepts in case of the growth rate series. We observe that in the original series the estimated trends fit the data relatively well, and there is some persistence in the deviations of the raw data from the trends that is modelled through a stationary long-memory process.

6. Conclusions

This paper analyses the long memory properties of quarterly per capita real output in the US using non-parametric, semi-parametric and parametric techniques. The results vary substantially depending on the methodology employed. Evidence of fractional integration with mean-reverting behaviour is obtained in a parametric context if the underlying disturbances are weakly autocorrelated: shocks affecting the series revert to the original trends though at a very slow hyperbolic rate. We also examine the possibility of a structural break in the data and the results indicate that there is a slight reduction in the degree of persistence after the break that is found to occur in the second quarter of 1978.

Note that the approach employed in this article does not directly test the hypothesis of fractional integration versus structural breaks as instead in Mayoral (2006), who allows for breaks in the deterministic terms imposing the same degree of integration before and after the break(s). The same happens with other methods such as those used by Robinson (1994), Hidalgo and Robinson (1996) and Lazarova (2005), which allow breaks in the deterministic components with long-memory innovations. Unlike these approaches, we allow different orders of integration before and after the break date. Other methods such as Ohanissian et al. (2008) can also be employed, though this method exploits the invariance property of the long-memory parameter for temporal aggregation. However, the present paper does not deal with temporal aggregation since it focuses exclusively on quarterly data. Moreover, Ohanissian et al.'s (2008) method is semi-parametric (and based on Geweke and Porter-Hudak's 1983 approach) while we employ purely parametric specifications in the presence of a break. An interesting extension of our analysis would be to estimate a more flexible model for the break, for instance incorporating Markov switching along with fractional integration. However, at present the necessary theory is still missing and no procedure is available to jointly estimate the parameters in such a model. Work in this direction is now in progress.

Appendix A

The LM test of Robinson (1994) for testing H_o: d = d_o, in (6) and (7) is

$$\hat{r} \;=\; \frac{T^{1/2}}{\hat{\sigma}^2}\hat{A}^{-1/2}\,\hat{a}\,,$$

where T is the sample size and

$$\hat{a} \;=\; \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\,g(\lambda_j;\hat{\tau})^{-1}\,I(\lambda_j); \qquad \hat{\sigma}^2 \;=\; \sigma^2(\hat{\tau}) \;=\; \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\tau})^{-1}\,I(\lambda_j);$$

$$\hat{A} \;=\; \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 \;-\; \sum_{j=1}^{T-1}\psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\varepsilon}(\lambda_j) \;=\; \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \qquad \lambda_j \;=\; \frac{2\pi\ j}{T}; \qquad \hat{\tau} \;=\; \arg\min\sigma^2(\tau).$$

$\hat{a}$ and $\hat{A}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to d (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of u_t evaluated under the null, i.e.:

$$\hat{u}_t \;=\; (1 \,-\, L)^{d_o}\,y_t \;-\; \hat{\beta}'w_t; \;\; \hat{\beta} \;=\; \left(\sum_{t=1}^{T}w_t\,w_t'\right)^{-1}\sum_{t=1}^{T}w_t\,(1 \,-\, L)^{d_o}\,y_t; \qquad w_t \;=\; (1 \,-\, L)^{d_o}\,z_t,$$

$z_t = (1, t)^T$, and g is a known function related to the spectral density function of u_t:

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi}g(\lambda;\tau), \qquad -\pi \;<\; \lambda \;\le\; \pi.$$

Appendix B

We examine a model of the form:

$$y_t \;=\; \alpha_1 \,+\, \beta_1 t \;+\; x_t; \qquad (1 - L)^{d_1}x_t \;=\; u_t, \qquad t \;=\; 1,\ldots,T_b,$$

$$y_t \;=\; \alpha_2 \,+\, \beta_2 t \;+\; x_t; \qquad (1 - L)^{d_2}x_t \;=\; u_t, \qquad t \;=\; T_b + 1,\ldots,T,$$

where the α's and the β's are the coefficients corresponding respectively to the intercepts and the linear trends; d_1 and d_2 may be real values, u_t is I(0), and T_b is the time of a break that is supposed to be unknown. This model can also be written as:

$$(1-L)^{d_1} y_t \;=\; \alpha_1 \,\tilde{1}_t(d_1) \;+\; \beta_1 \,\tilde{t}_t(d_1) \;+\; u_t, \quad t \;=\; 1, \ldots, T_b,$$

$$(1-L)^{d_2} y_t \;=\; \alpha_2 \,\tilde{1}_t(d_2) \;+\; \beta_2 \,\tilde{t}_t(d_2) \;+\; u_t, \quad t \;=\; T_b+1, \ldots, T,$$

where $\tilde{1}_t(d_i) = (1-L)^{d_i} 1$, and $\tilde{t}_t(d_i) = (1-L)^{d_i} t$, $i = 1, 2$.

The procedure is based on the least square principle. First we choose a grid for the values of the fractionally differencing parameters d_1 and d_2, for example, d_{io} = 0, 0.01, 0.02, …, 1, i = 1, 2. Then, for a given partition $\{T_b\}$ and given initial d_1, d_2-values, $(d_{1o}^{(1)}, d_{2o}^{(1)})$, we estimate the α's and the β's by minimising the sum of squared residuals,

$$\min_{\text{w.r.t.}\{\alpha_1,\alpha_2,\beta_1,\beta_2\}} \sum_{t=1}^{T_b} \left[(1-L)^{d_{1o}^{(1)}} y_t - \alpha_1 \tilde{1}_t(d_{1o}^{(1)}) - \beta_1 \tilde{t}_t(d_{1o}^{(1)}) \right]^2 + \sum_{t=T_b+1}^{T} \left[(1-L)^{d_{2o}^{(1)}} y_t - \alpha_2 \tilde{1}_t(d_{2o}^{(1)}) - \beta_2 \tilde{t}_t(d_{2o}^{(1)}) \right]^2 .$$

Let $\hat{\alpha}(T_b;\, d_{1o}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b;\, d_{1o}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for partition $\{T_b\}$ and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values into the objective function, we have RSS(T_b; $d_{1o}^{(1)}$, $d_{2o}^{(1)}$), and minimising this expression for all values of d_{1o} and d_{2o} in the grid we obtain $RSS(T_b) = \arg\min_{\{i,j\}} RSS(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$. Then, the estimated break date, $\hat{T}_k$, is such that $\hat{T}_k = \arg\min_{i=1,\ldots,m} RSS(T_i)$, where the minimisation is done over all partitions T_1, T_2, …, T_m, such that $T_i - T_{i-1} \geq |\varepsilon T|$. Then, the regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$, for i = 1 and 2.

References

Adelman, I., 1965, Long cycles: Fact or artifacts. American Economic Review 55, 444-463.

Andrews, D.W.K., 1991. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica 59, 817-858.

Barro, J., 1991, Economic growth in a cross section of countries, Quarterly Journal of Economics, 2, 407-443.

Barro, J. and X. Sala-i-Martin, 1991, Convergence across states and regions, Brooking Papers on Economic Activity, 1, 107-182.

Barro, J. and X. Sala-i-Martin, 1992, Convergence, Journal of Political Economy, 100, 2, 223-251.

Barro, J. and X. Sala-i-Martin, 1995, Economic Growth, New York: McGraw-Hill, 383-401.

Beran, J., 1994, Statistics for Long-Memory Processes, New York: Chapman & Hall.

Beran, J., 1995, Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models, Journal of the Royal Statistical Society B, 57, 659-672.

Chambers, M., 1998, Long memory and aggregation in macroeconomic time series. International Economic Review 39, 1053-1072.

Cheung, Y.W., 1993, Tests for fractional integration. A Monte Carlo investigation, Journal of Time Series Analysis 14, 331-345.

Dahlhaus, R., 1989, Efficient parameter estimation for self-similar process. Annals of Statistics 17, 1749-1766.

Diebold, F.X. and Inoue, A., 2001, Long memory and regime switching. Journal of Econometrics 105, 131-159.

Dolado, J.J., J. Gonzalo and L. Mayoral, 2003, A fractional Dickey-Fuller test for unit roots, Econometrica 70, 1963-2006.

Fox, R. and Taqqu, M., 1986, Large sample properties of parameter estimates for strongly dependent stationary Gaussian time series. Annals of Statistics 14, 517-532.

Geweke, J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, Journal of Time Series Analysis 4, 221-238.

Gil-Alana, L.A., 2000. Fractional integration in the purchasing power parity. Economics Letters 69, 285-288.

Gil-Alana, L.A., 2005. Testing and forecasting the degree of integration in the US inflation rate. Journal of Forecasting 24, 173-187.

Gil-Alana, L.A., 2008, Fractional integration and structural breaks at unknown periods of time, Journal of Time Series Analysis 29, 163-185.

Gil-Alana, L.A. and Robinson, P.M., 1997. Testing of unit roots and other nonstationary hypotheses in macroeconomic time series. Journal of Econometrics 80, 241-268.

Giraitis, L., P. Kokoszka and R. Leipus, 2001, Testing for long memory in the presence of a general trend, Journal of Applied Probability 38, 1033-1054.

Giraitis, L., P. Kokoszka, R. Leipus and Teyssiere, G., 2003. Rescaled variance and related tests for long memory in volatiluty and levels. Journal of Econometrics 112, 265-294.

Granger, C.W.J., 1966, The typical spectral shape of an economic variable. Econometrica 37, 150-161.

Granger, C.W.J., 1980, Long memory relationships and the aggregation of dynamic models. Journal of Econometrics 14, 227-238.

Granger, C.W.J. and N. Hyung, 2004, Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. Journal of Empirical Finance 11, 399-421.

Harrison, M. and G. Treacy, 1997, On the small sample distribution of the R/S statistic, The Economic and Social Review 28, 357-380.

Haubrich, J.G. and Lo, A.W., 2001. The sources and nature of long term memory in aggregate output. Economic Review of the Federal Reserve Bank of Cleveland 37, 15-30.

Hauser, M. A., 1999, Maximum likelihood estimators for ARFIMA models: A Monte Carlo study. Journal of Statistical Planning and Inference 8, 223-255.

Hidalgo, F.J. and P.M. Robinson, 1996, Testing for structural change in a long memory environment, Journal of Econometrics 70, 1, 159-174.

Hurvich, C.M. and B.K. Ray, 1995, Estimation of the memory parameter for nonstationary or noninvertible fractionally integrated processes. Journal of Time Series Analysis 16, 17-41.

Izzeldin, M. and A. Murphy, 2000, Bootstrapping the small sample critical values of the rescaled range statistic, The Economic and Social Review 31, 351-359.

Jones, C., 1995, Time series tests of endogenous growth models, Quarterly Journal of Economics, 110, 495-525.

Künsch, H., 1986, Discrimination between monotonic trends and long-range dependence, Journal of Applied Probability 23, 1025-1030.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, and Y. Shin, 1992, Testing the null hypothesis of stationarity against the alternative of a unit root, Journal of Econometrics 54, 159-178.

Lazarova, S., 2005, Testing for structural change in regression with long memory processes, Journal of Econometrics 129, 329-372.

Lee, D. and P. Schmidt, 1996, On the Power of the KPSS Test of Stationarity Against Fractionally Integrated Alternatives, Journal of Econometrics, 73, 285-302.

Lo, A.W., 1991. Long term memory in stock market prices. Econometrica 59, 1279-1313.

Mandelbrot, B.B., 1977, Fractals, form, chance and dimension, Freeman, San Francisco.

Mankiw, N., Romer, D. and D. Weil, 1992, A contribution to the empirics of growth, Quarterly Journal of Economics, 107, 2, 407-437.

Mayoral, L., 2004, A new minimum distance estimator for ARFIMA processes, Working Paper, Univeristat Pompeu Fabra.

Mayoral, L., 2006, Further evidence on the statistical properties of real GNP, Oxford Bulletin of Economics and Statistics 68, 901-920.

McLeod, A.I. and K.W. Hipel, 1978 Preservation of the rescaled adjusted range. A reassessment of the Jurst phenomenon, Water Resources Research 14, 491-507.

Michelacci, C. and P. Zaffaroni, 2000, (Fractional) Beta convergence, Journal of Monetary Economics 45, 129-153.

Mikosch, T. and C. Starica, 2004, Nonstationarities in financial time series, the long range dependence and the IGARCH effects, Review of Economics and Statistics 86, 1, 378-390.

Nelson, C. and C. Plosser, 1982, Trends and random walks in macroeconomic time series: some evidence and implications, Journal of Monetary Economics, 10, 139-162.

Ohanissian, A. J.R. Russell and R.S. Tsay, 2008, True or spurious long memory? A new test, Journal of Business and Economic Statistics 26, 2, 161-175.

Parke, W.R., 1999, What is fractional integration?, The Review of Economics and Statistics 81, 632-638.

Phillips, P.C.B. and Shimotsu, K., 2004. Local Whittle estimation in nonstationary and unit root cases. Annals of Statistics 32, 656-692.

Phillips, P.C.B. and Shimotsu, K., 2005. Exact local Whittle estimation of fractional integration. Annals of Statistics 33, 1890-1933.

Phillips, P.C.B. and Z. Xiao, 1999, A primer on unit root testing, Journal of Economic Surverys 12, 423-470.

Quah, D., 1993, Empirical cross-section dynamics in economic growth, European Economic Review, 37, 2/3, 426-434.

Robinson, P.M., 1978, Statistical inference for a random coefficient autoregressive model. Scandinavian Journal of Statistics 5, 163-168.

Robinson, P.M., 1994. Efficient tests of nonstationary hypotheses. Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M., 1995a, Log-periodogram regression of time series with long range dependence. Annals of Statistics 23, 1048-1072.

Robinson, P.M., 1995b, Gaussian semi-parametric estimation of long range dependence, Annals of Statistics 23, 1630-1661.

Robinson, P.M. and M. Henry, 1999, Long and short memory conditional heteroskedasticity in estimating the memory in levels, Econometric Theory 15, 299-336.

Sala-i-Martin, X., 1996, The classical approach to convergence analysis, Economic Journal, 106, 1019-1036.

Shimotsu, K. and P.C.B. Phillips, 2002, Pooled Log Periodogram Regression. Journal of Time Series Analysis 23, 57-93.

Silverberg, G. and Verspagen, B., 2000, A note on Michelacci and Zaffaroni long memory and time series of economic growth, ECIS Working Paper 00 17 Eindhoven Centre for Innovation Studies, Eindhoven University of Technology.

Smith, J., Taylor, N. and Yadav, S., 1997, Comparing the bias and misspecification in ARFIMA models. Journal of Time Series Analysis 18, 507-527.

Solow, R.M., 1956, A contribution to the theory of economic growth, Quarterly Journal of Economics, 70, 65-94.

Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally integrated time series models. Journal of Econometrics 53, 165-188.

Tanaka, K., 1999. The nonstationary fractional unit root. Econometric Theory 15, 549-582.

Taqqu, M.S., W. Willinger and R. Sherman, 1997, Proof of a fundamental result in self-similar traffic modelling. Computer Communication Review 27, 5-23.

Teverovsky, V. and M.S. Taqqu, 1997, Testing for long range dependence in the presence of shifting means or a slowly declining trend using a variance-type estimator, Journal of Time Series Analysis 18, 3, 279-304.

Teverovsky, V., M.S. Taqqu and Willinger, W., 1999. A critical look at Lo's modified R/S statistic. Journal of Statistical Planning and Inference 80, 211-227.

Velasco, C., 1999a, Nonstationary log-periodogram regression. Journal of Econometrics 91, 299-323.

Velasco, C., 1999b. Gaussian semiparametric estimation of nonstationary time series. Journal of Time Series Analysis 20, 87-127.

Velasco, C., 2000, Non-Gaussian log-periodogram regression, Econometric Theory 16, 44-79.

Velasco, C. and P.M. Robinson, 2000, Whitle pseudo maximum likelihood estimation for nonstationary time series. Journal of the American Statistical Association 95, 1229-1243.

Willinger, W., M.S. Taqqu and Teverovsky, V., 1999. Stock market prices and long range dependence. Finance and Stochastics 3, 1-13.

Figures and Tables

Figure 1: Time series plots

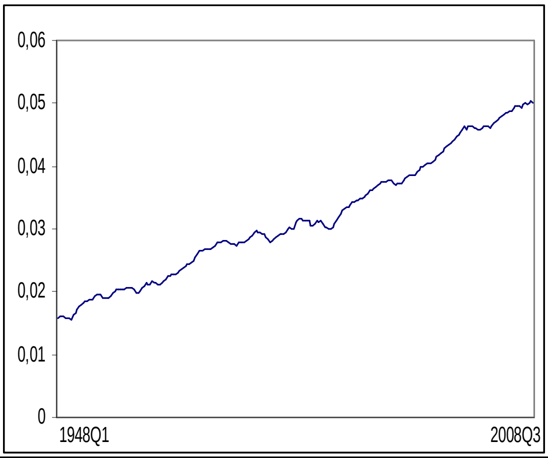
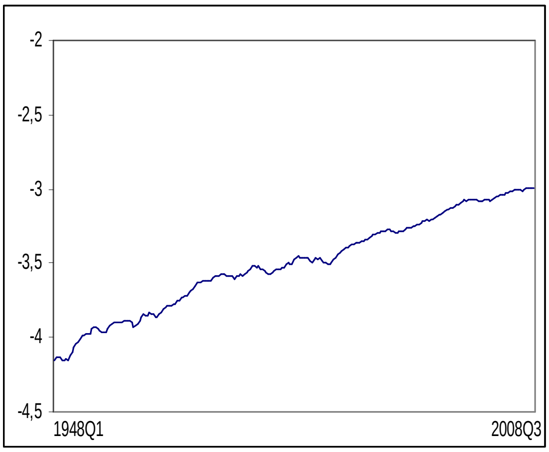
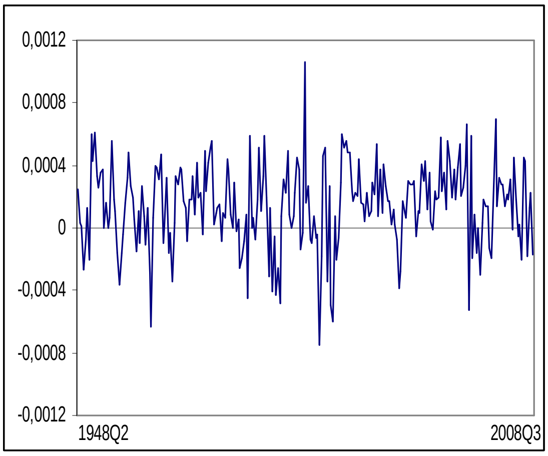
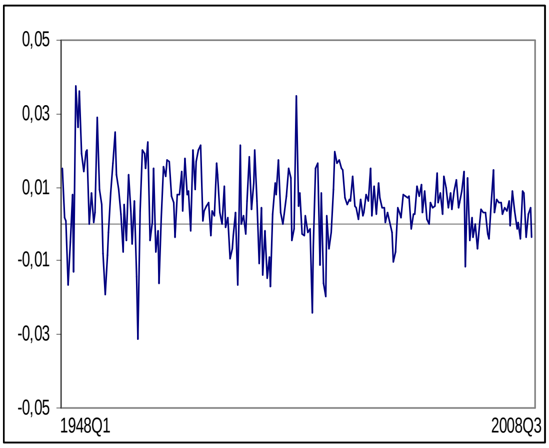
Real GDP per capita	Log of real GDP per capita
	
First differences of real GDP per capita	**Real GDP per capita growth**
	

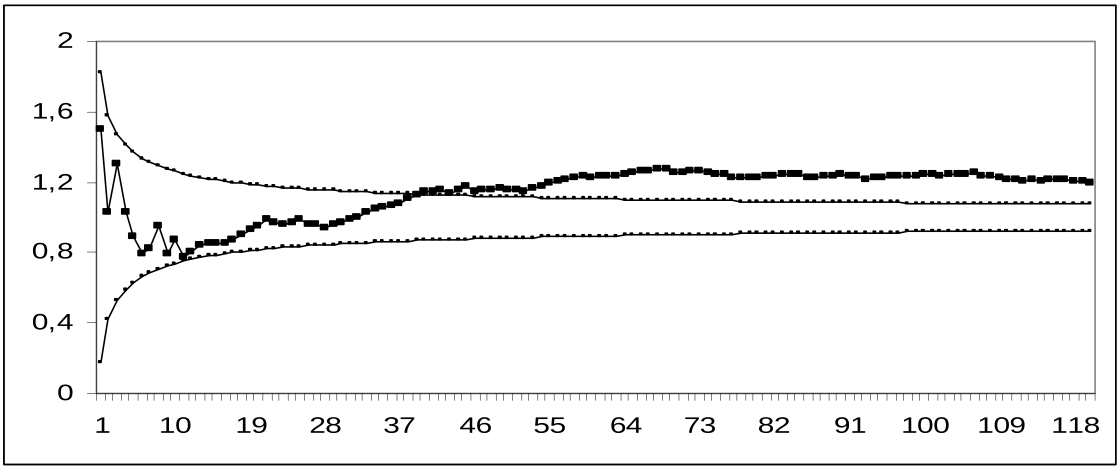
Table 1: Non-parametric statistics for long memory

a) Real per capita GDP (in first differences)										
	0	1	2	3	4	5	10	50	100	q^*
Lo's mod. (1991)	1.6124 (0.0870)	1.4362 (0.0659)	1.3233 (0.0510)	1.2632 (0.0425)	1.2300 (0.0377)	1.2182 (0.0870)	1.2273 (0.0373)	1.5494 (0.0797)	1.7049 (0.0972)	1.2182 (0.0870)
Giraitis et al. (2003)	0.1200	0.0952	0.0808	0.0736	0.0698	0.0685	0.0695	0.1108	0.1342	0.0685
a) Real per capita GDP growth										
	0	1	2	3	4	5	10	50	100	q^*
Lo's mod. (1991)	1.4351 (0.0651)	1.2457 (0.0400)	1.1421 (0.0242)	1.0933 (0.0162)	1.0764 (0.0134)	1.0779 (0.0136)	1.1154 (0.0199)	1.4420 (0.0669)	1.5130 (0.0754)	1.0851 (0.0148)
Giraitis et al. (2003)	0.0863	0.0650	0.0546	0.0501	0.0485	0.0487	0.0521	0.0876	0.0959	0.0493

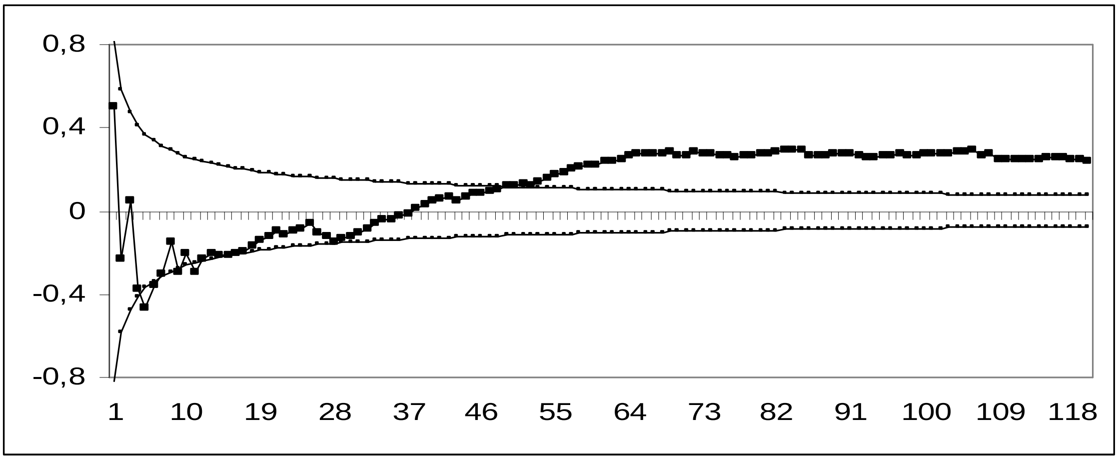
In Lo's (1991) modified R/S statistic, the 95% confidence interval with equal probabilities in both tails is [0.809, 1.862]. The values in parentheses refer to the estimates of the d's. In Giraitis et al. (2003), the critical value at the 5% level is 0.1869.

Figure 2: Whittle estimates of d based on a semiparametric method (Robinson, 1995)

a) Real per capita GDP



b) Real per capita GDP growth

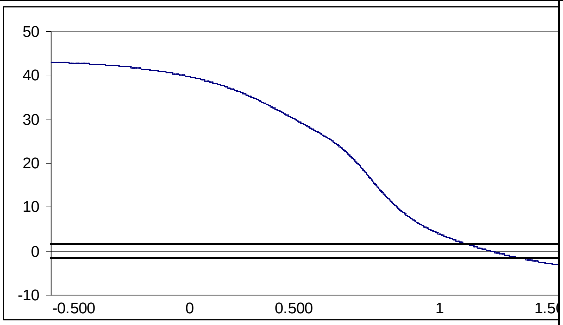


The horizontal axis refers to the bandwidth parameter while the vertical one displays the estimates of d.

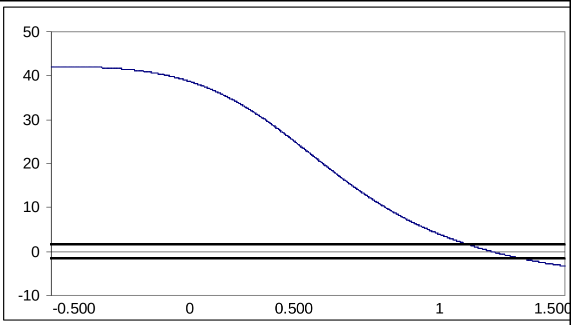
Figure 3: Estimates of d with Robinson (1994) for a range of values of d

a) Real per capita GDP

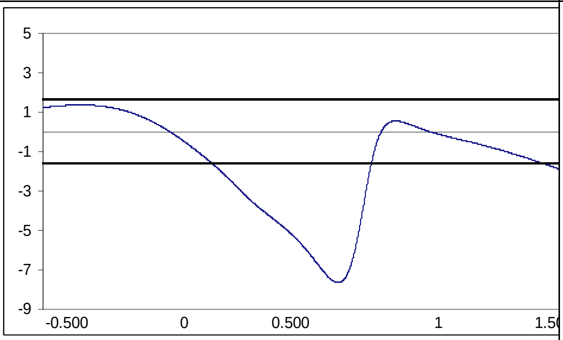
White noise with an intercept



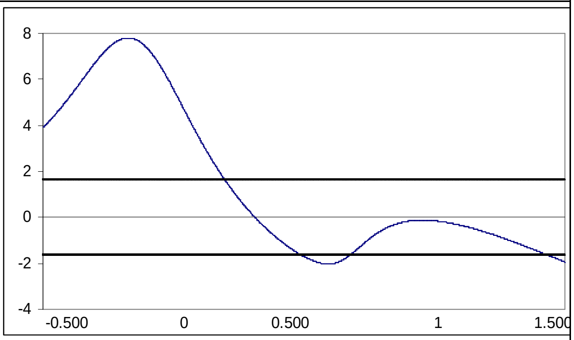
White noise with a linear time trend



AR(1) with an intercept

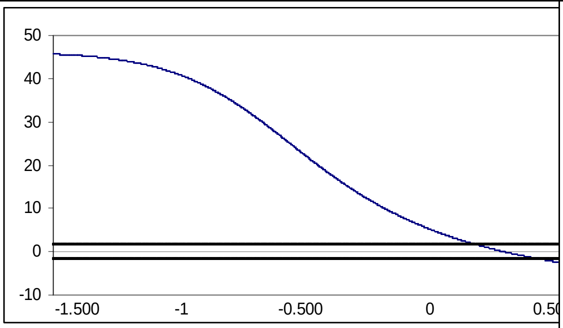


AR(1) with a linear time trend

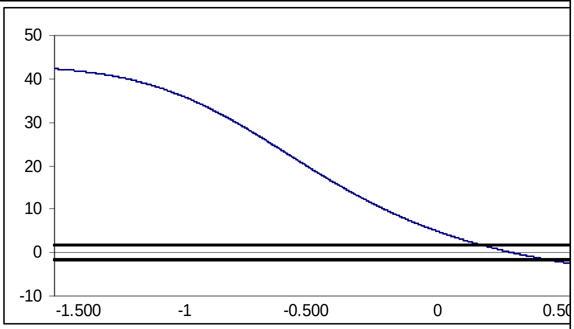


b) Real per capita GDP growth

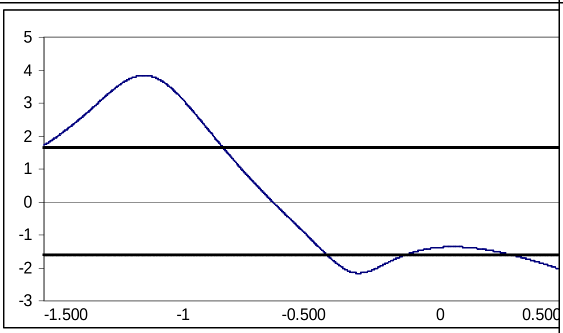
White noise with an intercept



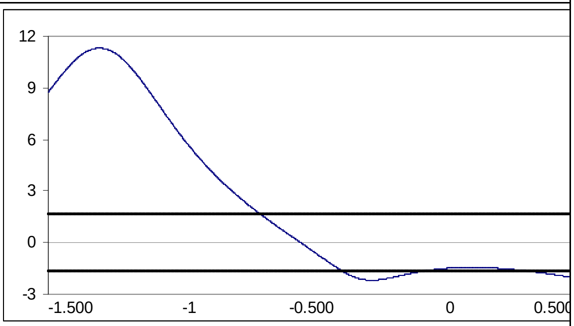
White noise with a linear time trend



AR(1) with an intercept



AR(1) with a linear time trend



The horizontal axis refers to the range of values of d under H_o. The vertical one displays the values of the test statistic, and the bold lines refer to the 95% non-rejection bands.

Table 2: Estimates of d based on a parametric approach (Robinson, 1994)

a) Real per capita GDP		
	Intercept	Linear time trend
White noise	1.212 (1.116, 1.333)	1.215 (1.119, 1.334)
AR (1)	-0.014 (-0.356, -0.156)	**0.312** **(0.196, 0.489)**
	0.794 (0.757, 1.423)	---
	0.985 (0.956, 1.423)	---
b) Real per capita GDP growth		
	Intercept	Linear time trend
White noise	0.260 (0.152, 0.394)	0.258 (0.146, 0.396)
AR (1)	**-0.622** **(-0.812, -0.409)**	-0.546 (-0.696, -0.390)

In parenthesis the 95% confidence band for the values of d. In bold the best model specification for each series.

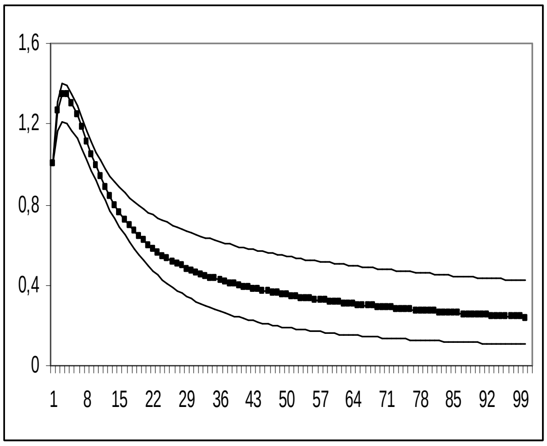
Table 3: Estimates of the coefficients in a model with a single break (Gil-Alana, 2008)

	Break date	First Subsample				Second subsample			
a) Real per capita GDP									
		d_1	α_1	β_1	ρ_1	d_2	α_2	β_2	ρ_2
Int + WN	1978Q2	1.328	0.01567 (66.689)	---	---	1.296	0.03108 (127.09)	---	---
Int + AR	1978Q2	1.236	0.01502 (21.787)	---	0.109	1.390	0.03050 (36.424)	---	-0.187
TT + WN	1978Q2	1.269	0.01562 (63.348)	0.00011 (1.668)	---	1.207	0.01372 (2.003)	0.00014 (2.537)	---
TT + AR	**1978Q2**	**0.490**	**0.01551 (19.437)**	**0.00012 (10.553)**	**0.774**	**0.314**	**0.00171 (0.749)**	**0.00020 (16.546)**	**0.875**
b) Real per capita GDP growth									
	Break date	First Subsample				Second subsample			
		d_1	α_1	β_1	ρ_1	d_2	α_2	β_2	ρ_2
Int + WN	1965Q4	0.340	0.00801 (1.617)	---	---	0.197	0.00358 (2.355)	---	---
Int + AR	**1978Q2**	**-0.416**	**0.00556 (9.168)**	**---**	**0.742**	**-0.666**	**0.00429 (16.793)**	**---**	**0.876**
TT + WN	1982Q4	0.238	0.00895 (2.091)	-0.00006 (-1.292)	---	0.219	0.01981 (3.190)	-0.00007 (-2.343)	---
TT + AR	1978Q2	-0.477	0.00827 (6.078)	-0.00004 (-2.063)	0.741	-0.666	0.00443 (1.715)	0.0000008 (-0.057)	0.875

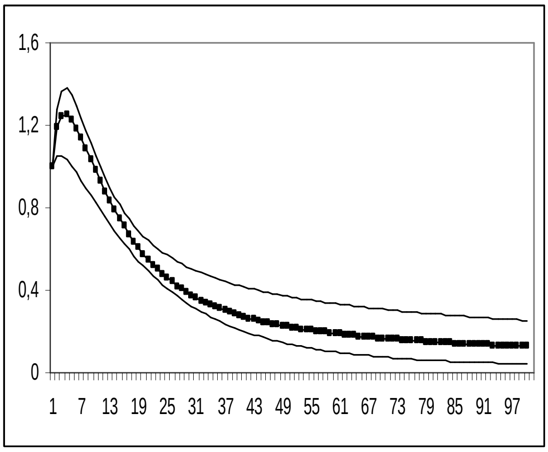
Int = intercept; TT = Time trend; WN = White noise and AR = Autoregression. In bold the selected model for each series. t-values in parentheses.

Figure 4: Impulse response functions and 95% confidence intervals

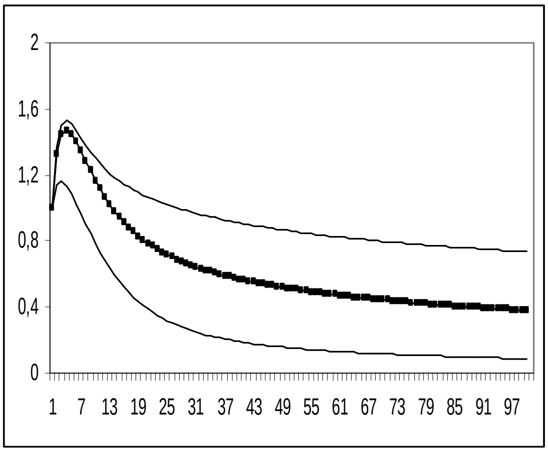
Real GDP per capita (1st subsample)



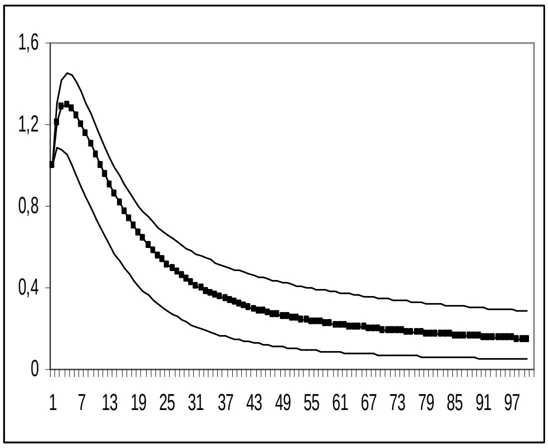
Real GDP per capita (2nd subsample)



Growth rate series (1st subsample)



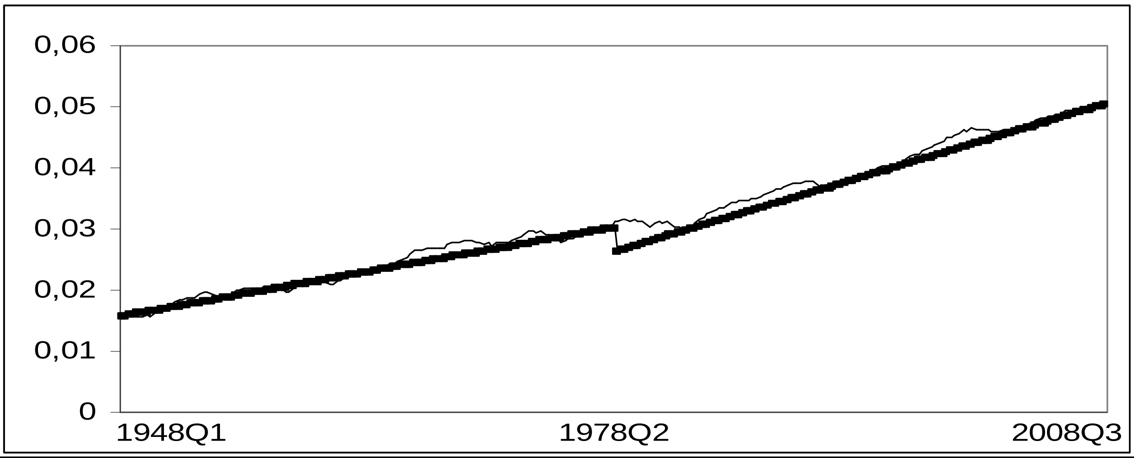
Growth rate series (2nd subsample)



The thin lines refer to the 95% confidence bands for the impulse responses.

Figure 5: Time series plots and their estimated deterministic terms

Real GDP per capita



Real GDP per capita growth