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No. 04/10

Working Paper Series

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Abstract

The sensitivity of the critical values of the F-test for cointegration discussed in Kanioura and Turner (2005) to the DGP is considered. Using simulation methods we find that these critical values are sensitive to the degree of autocorrelation and lag length in the DGP in finite samples. This sensitivity disappears asymptotically. Reference to the critical values reported here would be advised in finite sample applications of this test. Further, it may be advisable to produce critical values that more precisely reflect the autocorrelation properties of the variables being used in any particular application.

Keywords: Cointegration test, finite sample critical values, ADL model

JEL classification: C15

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1. Introduction

Kanioura and Turner (2005), hereafter KT, simulate critical values for an F-test of a single cointegrating vector in both a bivariate and multivariate autoregressive distributed lag (ADL) model assuming the regressors are weakly exogenous. They also compare the performance of this F-test to the Engle and Granger (1987) and Kremers *et al* (1992) cointegration tests. The former applies the Augmented Dickey-Fuller (ADF) test to the residuals of an Ordinary Least Squares (OLS) regression between the levels of variables that are postulated to cointegrate. The latter employs a single equation ADL specification and applies a t-test to the error-correction term that embodies a known cointegrating vector. An example of a known cointegrating vector that may naturally occur is the log of the consumption-income ratio in a model of the natural logarithms of consumption and income arising from the unit-income elasticity postulate – see Davidson *et al* (1978).

KT finds that, in finite samples, the F-test for cointegration has superior power to the Engle and Granger ADF test although the Kremers *et al* t-test has greater power than the F-test when the correct critical values of the t-test are known. However, Kremers *et al* find that the critical values of this t-test depends upon the specific parameters of the model being used to test cointegration. These parameters are typically unknown and KT demonstrate that the critical values can vary considerably with the specific parameters of the model which makes it impossible to appropriately apply the t-test with correct critical values in practice. Even if the parameters were known (which may be unlikely) it would be necessary to simulate critical values for the particular problem under study to obtain valid inference, hence complicating the application of the t-test in practice.² Hence, KT suggest that the F-test is the most powerful feasible test to apply in practice (it is also consistently more powerful than the alternative feasible test that they consider, being the Engle and Granger test). Indeed, a further advantage of the F-test over the t-test is that it is applicable in cases where the cointegrating vector is not known and needs to be estimated, which is normally the case.

The critical values produced by KT assume zero lags in both the DGP and estimated equation for the ADL model. We consider the sensitivity of KT's reported critical values of this F-test for cointegration to increasing the lag augmentation in the model (both in estimation of the test equation and in the DGP).³ In the language of Cheung and Lai (1995b) we assess the sensitivity of critical values to nuisance autoregressive (AR) roots in the DGP, given that KT assume that these are not present. Using Monte Carlo simulation Cheung and Lai (1995b)

² Ericsson and MacKinnon (2002, p. 311) suggest that the “... difficulty arises because one of the coefficients in the cointegrating vector ... is constrained. One solution is to estimate that coefficient unrestrictedly ...”. This involves adding the lagged levels of the variables that enter the cointegrating vector with *known* coefficients. Provided these added terms are insignificant the test can be appropriately applied with the critical values reported in Ericsson and MacKinnon (2002) assuming the cointegrating vector is known. However, if any of these terms are significant then the cointegrating vector is not known and the test is best applied as a t-test on the lagged level of the dependent variable in an ADL model: one can directly employ the critical values reported in Ericsson and MacKinnon (2002) in this case – see Ericsson and MacKinnon (2002, p. 310). Of course, this is likely to substantially lower the power of the t-test because it is the correct prespecification of the cointegrating vector that accords this test its considerable power gains – see, for example, Ericsson and MacKinnon (2002, p. 293, footnote 4). Since the t-test reported in KT assumes a correctly prespecified cointegrating vector the power advantages of this test over the F-test will not necessarily apply to the t-test when the cointegrating vector in the ADL needs to be estimated: when the cointegrating vector is not known the F-test is appropriate and the t-test is not.

³ KT (p. 267) find that the critical values of the F-test are not sensitive to the value of the coefficient on ΔX_t in the DGP for values ranging between plus and minus one. We set this coefficient equal to zero in the DGP in our simulations which, given KT's finding, should have no bearing on the critical values that we produce.

calculate the size distortion of the ADF test (with and without trend) for various values of the first-order non-unit AR roots in an ARIMA(1, 1, 0) DGP.⁴ For sample sizes between 50 and 250 observations they find that there is virtually no size distortion, except for when the autocorrelation coefficient has a magnitude close to one [almost an $I(2)$ process].⁵ Even in this case the size distortion is small, leading them to conclude that, for most applications, the presence of AR non-unit roots in the DGP do not invalidate inference from the ADF test based on finite sample critical values obtained from an ARIMA(0, 1, 0) DGP. We perform an analogous exercise (except we report critical values rather than the size distortion) for the F-test of cointegration examined by KT.

2. Simulation methodology

The bivariate dynamic linear regression model considered in KT is extended to allow for second-order autocorrelation thus:

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + u_t \quad (1)$$

Equation (1) can be reparameterised as:

$$\Delta Y_t = -\alpha_2 \Delta Y_{t-1} + \beta_0 \Delta X_t - \beta_2 \Delta X_{t-1} + (\alpha_1 + \alpha_2 - 1) Y_{t-1} + (\beta_0 + \beta_1 + \beta_2) X_{t-1} + u_t \quad (2)$$

The null of no cointegration can be tested in equation (2) using the F-statistic for the hypothesis that the coefficients on Y_{t-1} and X_{t-1} are jointly equal to zero. To obtain critical values the following general bivariate data generating process (DGP) is considered:

$$\Delta Y_t = \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta X_t + \gamma_3 \Delta X_{t-1} + \gamma_4 Y_{t-1} + \gamma_5 X_{t-1} + v_{1t} \quad (3)$$

$$\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t} \quad (4)$$

$$\begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} \sim IN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix} \right] \quad (5)$$

Kanioura and Turner (2005) produce critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$ against the alternative $H_1 : \gamma_4 \neq 0 \cup \gamma_5 \neq 0$ assuming the coefficients in the DGP satisfy $\gamma_i = 0$; $i = 1, 2, \dots, 6$. They estimated equation (3) by OLS with ΔX_{t-1} and ΔY_{t-1} excluded ($\gamma_1 = \gamma_3 = 0$). An intercept was included in the estimation of (3) because the constant may be non-zero in any particular sample even though the population intercept should be zero.

⁴ Cheung and Lai (1995a) suggest that temporal dependence in the DGP of a unit root process can be considered by using an error term with AR and MA components. They examined size distortion of standard critical values when the DGPs had such components, which is equivalent to producing new critical values for such DGPs and comparing them with standard critical values. In the case of the ADF test equation they find little size distortion is induced by DGPs with ARMA terms provided enough lags are employed in the test equation to accommodate them.

⁵ Similar conclusions are drawn from ADF tests based upon various ARIMA(0, 1, 1) DGPs, if there is a little more size distortion when the modulus of the moving average roots are close to unity.

We produce critical values for the same hypotheses except instead of assuming that both variables are generated as random walks we allow the first differences of both processes to be independently generated as first-order autoregressions, AR(1). That is, the coefficients in the DGP are assumed to satisfy $\gamma_i = 0$; $i = 2, 3, 4, 5$ with $|\gamma_i| \leq 1$; $i = 1, 6$. Critical values are produced for various combinations of the following values of γ_1 and γ_6 in the DGP:
 $\gamma_1 = -1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1$ and
 $\gamma_6 = -1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1$. Equation (3) is estimated by OLS including an intercept as well as ΔX_{t-1} and ΔY_{t-1} . It may be argued that the critical values that we produce may not be comparable to those reported in Kanioura and Turner (2005). This is because, although we use the same DGP as they do (for the case when $\gamma_1 = \gamma_6 = 0$) we estimate equation (3) with the addition of two redundant (they do not feature in the DGP) stationary variables, ΔX_{t-1} and ΔY_{t-1} . Because these two variables are redundant and stationary their presence should have little effect on the critical values.⁶ We consider this issue in more detail below.

Given that the estimated model, equation (3), encompasses the DGP there should be no differences in the asymptotic critical values for the various values of γ_1 and γ_6 provided $|\gamma_i| < 1$; $i = 1, 6$. This is because any autocorrelation in the DGP is modelled in the test equation. However, we consider whether the finite sample critical values are sensitive to the parameter values used in the DGP.

To consider variations in critical values for different lag lengths we also consider the following general bivariate DGP:

$$\Delta Y_t = \sum_{j=1}^p \gamma_{1j} \Delta Y_{t-j} + \gamma_2 \Delta X_t + \sum_{j=1}^p \gamma_{3j} \Delta X_{t-j} + \gamma_4 Y_{t-1} + \gamma_5 X_{t-1} + v_{1jt} \quad (6)$$

$$\Delta X_t = \sum_{j=1}^p \gamma_{6j} \Delta X_{t-j} + v_{2jt} \quad (7)$$

$$\begin{pmatrix} v_{1jt} \\ v_{2jt} \end{pmatrix} \sim IN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1j}^2 & 0 \\ 0 & \sigma_{1j}^2 \end{pmatrix} \right] \quad (8)$$

In this application we allow the first differences of both processes to be independently generated as p^{th} -order stationary autoregressions, AR(p). That is, the coefficients in the DGP are assumed to satisfy $\gamma_i = 0$; $i = 2, 3, 4, 5$, which yields (where L^j is the lag operator such that $L^j Z_t = Z_{t-j}$):

$$\Delta Y_t - \sum_{j=1}^p \gamma_{1j} \Delta Y_{t-j} = v_{1jt} \Rightarrow \left(1 - \sum_{j=1}^p \gamma_{1j} L^j \right) \Delta Y_t = v_{1jt} \quad (9)$$

⁶ The presence of such redundant stationary variables should only have an impact upon the efficiency of estimation. This impact is likely to be minor even in small samples.

$$\Delta X_t - \sum_{j=1}^p \gamma_{6j} \Delta X_{t-j} = v_{2jt} \Rightarrow \left(1 - \sum_{j=1}^p \gamma_{6j} L^j \right) \Delta X_t = v_{2jt} \quad (10)$$

To ensure that both ΔX_t and ΔY_t are consistent with stationarity (are non-explosive) all of their roots must lie outside of the unit circle or, equivalently, their inverted roots must all have a modulus below unity.⁷ The roots are obtained by solving the polynomial equations $\left(1 - \sum_{j=1}^p \gamma_{1j} L^j \right) = 0$ and $\left(1 - \sum_{j=1}^p \gamma_{6j} L^j \right) = 0$ for L .⁸

Kanioura and Turner (2005, p. 267) conduct sensitivity analyses to the specification of the DGPs. They demonstrate that the F-test critical values are invariant to the values of γ_2 in the DGP specified by (3) and the relative size of the residual variances used in the DGPs, see (5), given that, $\gamma_i = 0$; $i = 1, 3, 4, 5, 6$. However, if the DGPs are extended we show that the critical values are sensitive to the values of first-order autocorrelation coefficients (at least in

⁷ If the *inverted* root is a complex number (that is, it contains a real and an imaginary component) then the square root of the sum [and difference] of the squared real and imaginary component must be below one for the process to be stationary. For example, for the complex root $-0.239 + 0.337i$ the appropriate quantity to check is $\sqrt{(-0.239)^2 + (0.337)^2} \approx 0.413$ $\left[\sqrt{(-0.239)^2 - (0.337)^2} \approx 0.375 \right]$ which is less than unity in this case. Notice

that $i = \sqrt{-1}$, by definition.

⁸ For the non-seasonal DGPs of the AR(1) to AR(13) models we use the following coefficient values that ensure that the inverted roots all lie inside the unit circle. AR(1), $\gamma_{11} = \gamma_{61} = 0.8$; AR(2), $\gamma_{11} = \gamma_{61} = 0.5$ and $\gamma_{12} = \gamma_{62} = 0.3$; AR(3), $\gamma_{11} = \gamma_{61} = 0.4$, $\gamma_{12} = \gamma_{62} = 0.25$ and $\gamma_{13} = \gamma_{63} = 0.15$; AR(4), $\gamma_{11} = \gamma_{61} = 0.3$, $\gamma_{12} = \gamma_{62} = 0.22$, $\gamma_{13} = \gamma_{63} = 0.16$ and $\gamma_{14} = \gamma_{64} = 0.12$; AR(5), $\gamma_{11} = \gamma_{61} = 0.24$, $\gamma_{12} = \gamma_{62} = 0.20$, $\gamma_{13} = \gamma_{63} = 0.14$, $\gamma_{14} = \gamma_{64} = 0.12$ and $\gamma_{15} = \gamma_{65} = 0.1$; AR(6), $\gamma_{11} = \gamma_{61} = 0.18$, $\gamma_{12} = \gamma_{62} = 0.16$, $\gamma_{13} = \gamma_{63} = 0.13$, $\gamma_{14} = \gamma_{64} = 0.12$, $\gamma_{15} = \gamma_{65} = 0.11$ and $\gamma_{16} = \gamma_{66} = 0.10$; AR(7), $\gamma_{11} = \gamma_{61} = 0.15$, $\gamma_{12} = \gamma_{62} = 0.14$, $\gamma_{13} = \gamma_{63} = 0.13$, $\gamma_{14} = \gamma_{64} = 0.11$, $\gamma_{15} = \gamma_{65} = 0.10$, $\gamma_{16} = \gamma_{66} = 0.09$ and $\gamma_{17} = \gamma_{67} = 0.08$; AR(8), $\gamma_{11} = \gamma_{61} = 0.14$, $\gamma_{12} = \gamma_{62} = 0.13$, $\gamma_{13} = \gamma_{63} = 0.12$, $\gamma_{14} = \gamma_{64} = 0.11$, $\gamma_{15} = \gamma_{65} = 0.09$, $\gamma_{16} = \gamma_{66} = 0.08$, $\gamma_{17} = \gamma_{67} = 0.07$ and $\gamma_{18} = \gamma_{68} = 0.06$; AR(9), $\gamma_{11} = \gamma_{61} = 0.13$, $\gamma_{12} = \gamma_{62} = 0.12$, $\gamma_{13} = \gamma_{63} = 0.11$, $\gamma_{14} = \gamma_{64} = 0.10$, $\gamma_{15} = \gamma_{65} = 0.09$, $\gamma_{16} = \gamma_{66} = 0.08$, $\gamma_{17} = \gamma_{67} = 0.07$ and $\gamma_{18} = \gamma_{68} = 0.06$; AR(10), $\gamma_{11} = \gamma_{61} = 0.13$, $\gamma_{12} = \gamma_{62} = 0.12$, $\gamma_{13} = \gamma_{63} = 0.11$, $\gamma_{14} = \gamma_{64} = 0.10$, $\gamma_{15} = \gamma_{65} = 0.09$, $\gamma_{16} = \gamma_{66} = 0.07$, $\gamma_{17} = \gamma_{67} = 0.06$, $\gamma_{18} = \gamma_{68} = 0.055$, $\gamma_{19} = \gamma_{69} = 0.045$, $\gamma_{1,10} = \gamma_{6,10} = 0.04$ and $\gamma_{1,11} = \gamma_{6,11} = 0.03$; AR(12), $\gamma_{11} = \gamma_{61} = 0.12$, $\gamma_{12} = \gamma_{62} = 0.11$, $\gamma_{13} = \gamma_{63} = 0.10$, $\gamma_{14} = \gamma_{64} = 0.09$, $\gamma_{15} = \gamma_{65} = 0.08$, $\gamma_{16} = \gamma_{66} = 0.07$, $\gamma_{17} = \gamma_{67} = 0.06$, $\gamma_{18} = \gamma_{68} = 0.055$, $\gamma_{19} = \gamma_{69} = 0.045$, $\gamma_{1,10} = \gamma_{6,10} = 0.04$, $\gamma_{1,11} = \gamma_{6,11} = 0.035$ and $\gamma_{1,12} = \gamma_{6,12} = 0.03$; AR(13), $\gamma_{11} = \gamma_{61} = 0.095$, $\gamma_{12} = \gamma_{62} = 0.09$, $\gamma_{13} = \gamma_{63} = 0.085$, $\gamma_{14} = \gamma_{64} = 0.08$, $\gamma_{15} = \gamma_{65} = 0.07$, $\gamma_{16} = \gamma_{66} = 0.065$, $\gamma_{17} = \gamma_{67} = 0.06$, $\gamma_{18} = \gamma_{68} = 0.055$, $\gamma_{19} = \gamma_{69} = 0.05$, $\gamma_{1,10} = \gamma_{6,10} = 0.045$, $\gamma_{1,11} = \gamma_{6,11} = 0.04$, $\gamma_{1,12} = \gamma_{6,12} = 0.035$ and $\gamma_{1,13} = \gamma_{6,13} = 0.03$; For the seasonal DGPs the coefficient used are also defined to ensure that the inverted roots of the autoregressive, AR, process lie inside the unit circle. For the AR(1) and AR(2) process they are as follows: AR(1), $\gamma_{11} = \gamma_{61} = 0.4$; AR(2), $\gamma_{11} = \gamma_{61} = 0.4$ and $\gamma_{12} = \gamma_{62} = 0.3$; For the AR(3) to AR(13) process there are only three non-zero coefficients. The first and second autocorrelation coefficients are always 0.4 and 0.1, respectively. All other autocorrelation coefficients are zero except for the highest-order autocorrelation coefficient which takes the value 0.3.

finite samples). This dependency of critical values on parameter values in the DGP (equations (3), (4) and (5)) can be related to spurious regression in terms of size distortion – see Granger et al (2001). If one were to use critical values based on the DGPs' autocorrelation coefficients being zero then, if the true DGPs autocorrelation coefficients are both positive one obtains oversized tests (spurious regression) while if they are both negative one has undersized tests (whilst not a problem for size this may cause power issues). Related to this we also demonstrate that lag length affects the critical values (especially in small samples). This is illustrated using both seasonal and non-seasonal DGPs assuming particular sets of values for the coefficients in the DGPs.

3. Results

Table 1 reproduces our critical values reported in Tables, 2, 4, 5 and 8 for the case of $\gamma_1 = \gamma_6 = 0$ in the DGP (depicted by (3), (4) and (5)) and those reported by KT in their Table 1 for the two-variable ($k = 2$) case. The only difference in how these critical values are generated is that we include the redundant (because they do not feature in the DGP when $\gamma_1 = \gamma_6 = 0$) explanatory variables ΔX_{t-1} and ΔY_{t-1} in the estimation equation and KT do not. The critical values are broadly similar except there is a tendency for ours to be slightly smaller than KT's – our critical values are less than those reported by KT in nine out of the twelve cases reported below. To the extent that this represents a slight downward bias this may reflect the inefficiency of entering redundant variables in our estimating equation. With this slight effect in mind, we believe that our results are effectively the same as those produced by KT for the case considered and thus confirm that they have been generated in essentially the same way. Hence, our results provide a valid comparison to those of KT. Of course, the estimation equation that we use, including ΔX_{t-1} and ΔY_{t-1} , is appropriate for the case when $\gamma_1 \neq 0$ and $\gamma_6 \neq 0$ in the DGP and so will not suffer from such inefficiency. Therefore, our results are appropriate for the sensitivity analysis that we wish to conduct.

Table 2 provides the critical values for the DGP given by (3), (4) and (5) with all combinations of γ_1 and γ_6 , where these parameters take on the values in the DGP specified above, for regressions with a sample size of ($T = 50$). Table 3 to Table 11 report the corresponding critical values for $T = 75, 100, 200, 300, 400, 500, 1000, 5000$ and 10000 , respectively. From Table 2 the critical values associated with 99% of the way through the distribution values vary from 8.413 (with $\gamma_1 = -0.8$ and $\gamma_6 = -0.6$) to 10.225 (with $\gamma_1 = \gamma_6 = 0.8$) – this excludes the case of nonstationary DGPs.⁹ The corresponding 95% (90%) critical values are in the range of 5.869 (4.821) [$\gamma_1 = \gamma_6 = -0.8$] to 7.192 (5.892). Hence, there is notable variation in the critical values when $T = 50$. This range of critical values at the 95% level is 5.769 to 6.599 when $T = 100$, 5.765 to 6.116 when $T = 200$ and 5.685 to 5.938. This demonstrates that as the sample size increases the range falls. When $T = 10000$ this range is 5.676 to 5.739 which is very small and indicates that there is very little sensitivity to the DGP for large sample sizes. This small variation will contain sampling error and so the decline in range as the sample size increases may be taken as indicating that the critical values are asymptotically the same for all stationary DGPs.

Table 12 and 13 report the critical values of the DGPs given by (6), (7) and (8) for lag lengths from 0 to 13 and the sample sizes $T = 50, 75, 100, 200, 300, 400, 500, 1000, 5000$ and 10000 .

⁹ That is, where $\gamma_1 = -1$ and/or $\gamma_1 = 1$ and/or $\gamma_6 = -1$ and/or $\gamma_6 = 1$.

The particular values of the coefficients in the DGP are different in the sense that Table 12 is a nonseasonal DGP (the coefficients decline monotonically as the lag length increases) and Table 13 is seasonal (the coefficient on the last lag is larger than the majority of preceding lags). The coefficient values used in the DGPs are specified above.

The critical values vary with both the sample size and the number of lags. In Table 12 the 95% critical values range from 5.677 (with zero lags and $T = 10000$) to 7.081 (9 lags and $T = 75$) while in Table 13 the range is 5.656 (with zero lags and $T = 10000$) to 7.433 (3 lags and $T = 50$). Broadly speaking the critical values decline as the sample size increases, appearing to converge to an asymptotic value of about 5.7 to 5.8. Although there is some variation of the critical values with the number of lags, there is no continuous upward or downward bias in the trend as the number of lags increase. Any systematic variation in these critical values is likely due to the overall size of the autocorrelation coefficients, following the results of Tables 2 to 11.

4. Conclusion

The results indicate that the critical values of the F-test for cointegration (discussed in KT) are sensitive to the degree of autocorrelation and lag length in the DGP in finite samples. This sensitivity disappears asymptotically. Reference to the critical values reported here would be advised in finite sample applications of this test. If the F-test statistic is close to the critical values it may be worth producing critical values that more precisely reflect the autocorrelation properties of the variables being used in any particular application.

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Table 1

T↓	Table 2, 4, 5 & 8 with $\gamma_1 = \gamma_6 = 0$			KT (2005), k = 2		
	1%	5%	10%	1%	5%	10%
50	8.61	6.12	4.94	8.86	6.07	4.95
100	8.06	5.89	4.87	8.12	5.94	4.86
200	7.93	5.78	4.80	7.97	5.84	4.79
500	7.63	5.71	4.77	7.86	5.83	4.85

The section headed “Table 2, 4, 5 & 8 with $\gamma_1 = \gamma_6 = 0$ ” reproduces our critical values reported in Tables, 2, 4, 5 and 8 for the case of $\gamma_1 = \gamma_6 = 0$ in the DGP. The section headed “KT (2005), k = 2” are the corresponding critical values produced by KT in their Table 1 for the two-variable (k = 2) case. The only difference in how these two sets of critical values are generated is that we include the redundant explanatory variables ΔX_{t-1} and ΔY_{t-1} in the estimation equation and KT do not.

Table 2: critical values of F-test in ECM ADL regressions with sample size T = 50

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.778	4.743	4.689	4.832	4.815	4.828	4.879	4.970	5.177	5.517	6.320
	5.812	5.787	5.724	5.876	5.873	5.898	6.088	6.112	6.336	6.713	7.536
	8.154	8.145	8.113	8.312	8.342	8.322	8.715	8.553	8.965	9.357	10.298
-0.8	4.858	4.848	4.821	4.883	4.861	4.900	4.986	5.106	5.240	5.622	6.450
	5.931	5.909	5.869	5.953	5.887	5.979	6.095	6.213	6.377	6.842	7.723
	8.301	8.413	8.438	8.384	8.389	8.579	8.922	8.844	9.081	9.538	10.848
-0.6	4.833	4.853	4.856	4.887	4.880	4.899	4.998	5.010	5.255	5.627	6.539
	5.890	5.905	5.896	5.963	5.970	5.949	6.143	6.142	6.345	6.783	7.791
	8.414	8.381	8.507	8.406	8.435	8.568	8.821	8.702	8.964	9.677	10.696
-0.4	4.914	4.887	4.866	4.894	4.890	4.944	5.055	5.000	5.304	5.606	6.481
	6.022	5.959	5.987	6.003	6.010	6.009	6.185	6.086	6.414	6.774	7.714
	8.604	8.665	8.454	8.396	8.489	8.680	8.683	8.644	9.125	9.391	10.625
-0.2	4.896	4.903	4.883	4.869	4.909	4.968	5.034	5.116	5.299	5.620	6.529
	6.009	5.950	6.035	5.911	5.993	6.110	6.168	6.188	6.427	6.767	7.820
	8.523	8.560	8.559	8.394	8.479	8.816	8.763	8.778	9.119	9.537	10.627
0.0	4.951	4.914	4.887	4.961	4.928	4.943	5.051	5.140	5.264	5.635	6.525
	6.019	6.023	5.983	6.053	6.059	6.124	6.152	6.315	6.416	6.831	7.728
	8.475	8.587	8.367	8.699	8.548	8.614	8.697	8.980	9.176	9.820	10.290
0.2	4.958	4.910	4.957	5.031	5.032	4.986	5.056	5.176	5.358	5.633	6.604
	6.071	6.110	6.081	6.176	6.088	6.121	6.151	6.341	6.545	6.875	7.933
	8.698	8.761	8.679	8.863	8.522	8.657	8.715	9.024	9.335	9.627	10.889
0.4	4.970	5.026	5.016	5.055	5.069	5.087	5.145	5.235	5.365	5.756	6.640
	6.098	6.129	6.008	6.176	6.207	6.256	6.278	6.346	6.637	6.962	7.941
	8.528	8.595	8.474	8.763	8.572	8.961	8.911	9.118	9.339	9.628	10.996
0.6	5.152	5.050	5.114	5.151	5.110	5.205	5.243	5.325	5.488	5.786	6.733
	6.270	6.140	6.207	6.301	6.295	6.313	6.371	6.434	6.700	6.982	8.115
	8.697	8.682	8.924	8.939	8.950	8.926	9.047	9.040	9.367	9.919	11.298
0.8	5.212	5.326	5.312	5.311	5.320	5.349	5.442	5.507	5.640	5.892	6.881
	6.372	6.469	6.540	6.516	6.499	6.459	6.619	6.729	6.808	7.192	8.258
	9.015	9.117	9.393	9.092	8.991	9.018	9.580	9.557	9.634	10.225	11.316
1.0	5.333	5.315	5.391	5.492	5.443	5.472	5.638	5.726	5.816	6.153	6.945
	6.482	6.562	6.640	6.785	6.753	6.877	6.857	7.041	7.141	7.478	8.302
	9.507	9.230	9.355	9.873	9.757	9.887	9.681	10.044	10.288	10.567	11.663

Number of samples (replications) = 20000. Number of observations in a sample (T) = 50 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the following font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 3: critical values of F-test in ECM ADL regressions with sample size T = 75

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.753	4.763	4.756	4.806	4.859	4.806	4.868	4.931	5.053	5.379	6.288
	5.816	5.720	5.774	5.833	5.858	5.805	5.842	6.007	6.170	6.480	7.474
	8.115	7.957	8.067	8.213	8.303	8.211	8.171	8.414	8.503	8.967	10.018
-0.8	4.834	4.846	4.775	4.822	4.823	4.855	4.945	4.986	5.086	5.450	6.333
	5.835	5.885	5.788	5.865	5.855	5.884	5.991	6.006	6.120	6.492	7.462
	8.132	8.242	8.223	8.075	8.271	8.232	8.352	8.424	8.467	9.013	10.069
-0.6	4.857	4.792	4.829	4.820	4.896	4.799	4.940	4.976	5.169	5.378	6.356
	5.927	5.808	5.886	5.862	5.911	5.798	5.922	6.016	6.242	6.482	7.634
	8.163	8.250	8.244	8.232	8.102	8.231	8.226	8.391	8.649	8.966	10.155
-0.4	4.800	4.846	4.784	4.865	4.769	4.883	4.898	5.042	5.044	5.421	6.391
	5.832	5.874	5.839	5.910	5.753	5.895	5.934	6.070	6.105	6.561	7.560
	8.270	8.263	8.119	8.211	8.183	8.154	8.088	8.471	8.622	9.124	10.356
-0.2	4.836	4.854	4.780	4.835	4.859	4.949	4.947	4.930	5.129	5.395	6.381
	5.882	5.861	5.819	5.841	5.912	6.023	5.967	5.993	6.196	6.487	7.527
	8.072	8.276	8.019	8.269	8.248	8.315	8.457	8.370	8.758	9.020	10.167
0.0	4.886	4.858	4.872	4.928	4.942	4.925	4.957	5.021	5.179	5.413	6.428
	5.990	5.867	5.899	5.947	5.968	6.010	6.078	6.157	6.262	6.523	7.658
	8.346	8.255	8.269	8.341	8.376	8.255	8.510	8.468	8.569	9.067	10.307
0.2	4.933	4.945	4.891	4.826	4.879	4.929	5.001	4.994	5.144	5.415	6.521
	5.949	6.003	5.967	5.867	5.957	5.969	6.089	6.056	6.247	6.517	7.673
	8.380	8.315	8.341	8.126	8.162	8.567	8.310	8.427	8.732	9.149	10.406
0.4	4.870	4.889	4.921	4.937	4.935	4.934	5.015	5.061	5.184	5.457	6.476
	5.906	6.051	5.964	5.964	5.969	6.009	6.061	6.158	6.257	6.627	7.728
	8.246	8.450	8.311	8.436	8.360	8.427	8.364	8.624	8.767	9.168	10.390
0.6	4.950	4.956	4.939	4.947	5.034	5.024	5.055	5.149	5.226	5.473	6.535
	5.977	6.067	5.960	6.032	6.118	6.118	6.068	6.190	6.326	6.602	7.745
	8.372	8.416	8.328	8.386	8.701	8.530	8.583	8.579	8.579	9.054	10.399
0.8	5.122	5.129	5.135	5.127	5.125	5.176	5.170	5.278	5.366	5.626	6.656
	6.197	6.230	6.214	6.180	6.203	6.286	6.224	6.378	6.475	6.780	7.907
	8.746	8.796	8.772	8.867	8.532	8.630	8.740	8.817	8.857	9.350	10.508
1.0	5.267	5.339	5.325	5.371	5.420	5.427	5.460	5.436	5.731	5.973	6.823
	6.414	6.464	6.408	6.551	6.604	6.615	6.690	6.667	6.946	7.208	8.101
	8.849	9.115	9.078	9.169	9.287	9.207	9.184	9.446	9.651	10.053	10.918

Number of samples (replications) = 20000. Number of observations in a sample (T) = 75 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 4: critical values of F-test in ECM ADL regressions with sample size T = 100

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.759	4.730	4.800	4.806	4.803	4.813	4.827	4.943	4.967	5.194	6.273
	5.734	5.718	5.759	5.855	5.788	5.814	5.803	5.921	6.000	6.285	7.375
	7.872	8.056	8.030	8.185	7.998	8.009	8.037	8.175	8.317	8.667	9.865
-0.8	4.760	4.805	4.808	4.850	4.767	4.804	4.874	4.906	4.919	5.260	6.330
	5.739	5.769	5.785	5.846	5.805	5.829	5.885	5.880	5.979	6.322	7.482
	7.995	7.965	8.104	8.037	8.030	8.162	8.110	8.119	8.311	8.714	9.873
-0.6	4.779	4.775	4.811	4.733	4.792	4.791	4.907	4.886	5.021	5.252	6.307
	5.837	5.791	5.812	5.747	5.808	5.860	5.894	5.933	6.037	6.383	7.383
	8.035	8.167	8.066	7.935	8.219	8.156	8.198	8.207	8.513	8.739	9.944
-0.4	4.829	4.850	4.843	4.781	4.819	4.793	4.879	4.922	5.041	5.188	6.372
	5.798	5.824	5.854	5.809	5.861	5.859	5.889	5.890	6.106	6.246	7.498
	7.982	8.161	8.013	8.029	7.957	8.067	8.236	8.229	8.602	8.599	9.922
-0.2	4.826	4.822	4.857	4.843	4.851	4.848	4.880	4.891	5.018	5.247	6.306
	5.815	5.860	5.904	5.835	5.890	5.881	5.924	5.883	6.101	6.282	7.464
	8.004	8.117	8.191	8.049	8.178	8.173	8.169	8.099	8.351	8.654	10.247
0.0	4.851	4.841	4.866	4.794	4.801	4.871	4.923	4.888	5.045	5.344	6.331
	5.871	5.866	5.820	5.779	5.838	5.887	5.974	5.889	6.123	6.457	7.486
	8.058	8.113	8.200	8.033	8.198	8.060	8.264	8.179	8.474	8.948	9.990
0.2	4.841	4.784	4.888	4.861	4.879	4.885	4.922	4.947	5.069	5.317	6.348
	5.877	5.811	5.871	5.829	5.891	5.925	5.964	6.018	6.129	6.380	7.481
	7.994	8.148	8.219	8.005	8.236	8.381	8.127	8.436	8.362	8.839	9.980
0.4	4.883	4.897	4.894	4.859	4.868	4.865	4.986	4.955	5.101	5.278	6.399
	5.844	5.938	5.899	5.868	5.890	5.837	6.022	6.026	6.179	6.367	7.597
	8.126	8.118	8.081	8.060	8.145	8.286	8.352	8.419	8.379	8.632	10.125
0.6	4.962	4.928	4.981	4.959	4.956	5.019	4.964	5.025	5.121	5.292	6.433
	6.011	5.923	5.998	6.007	5.958	5.999	5.973	6.092	6.155	6.413	7.689
	8.361	8.174	8.397	8.413	8.253	8.411	8.273	8.409	8.545	8.758	10.110
0.8	5.053	5.026	5.041	5.047	5.056	5.026	5.077	5.139	5.226	5.404	6.591
	6.068	6.059	6.084	6.089	6.095	6.061	6.154	6.222	6.248	6.599	7.802
	8.331	8.323	8.575	8.479	8.408	8.497	8.701	8.683	8.844	9.049	10.538
1.0	5.339	5.276	5.301	5.349	5.322	5.313	5.426	5.394	5.539	5.836	6.709
	6.447	6.465	6.409	6.551	6.471	6.445	6.571	6.588	6.748	7.091	8.024
	9.174	9.177	8.990	9.051	8.913	9.062	9.172	9.313	9.212	9.616	10.861

Number of samples (replications) = 20000. Number of observations in a sample (T) = 100 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 5: critical values of F-test in ECM ADL regressions with sample size T = 200

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.748	4.721	4.811	4.735	4.768	4.750	4.817	4.832	4.896	5.017	6.168
	5.740	5.725	5.764	5.717	5.785	5.713	5.708	5.794	5.904	6.036	7.257
	8.015	7.815	7.868	7.810	8.070	7.927	7.782	7.928	8.058	8.333	9.728
-0.8	4.779	4.835	4.731	4.750	4.810	4.831	4.809	4.804	4.896	5.027	6.238
	5.779	5.765	5.699	5.727	5.799	5.760	5.779	5.806	5.886	6.039	7.341
	7.951	7.825	8.016	7.931	7.813	7.873	7.863	7.985	8.120	8.192	9.596
-0.6	4.792	4.770	4.812	4.769	4.837	4.800	4.797	4.829	4.860	5.014	6.362
	5.706	5.769	5.753	5.753	5.817	5.789	5.748	5.775	5.857	6.057	7.413
	7.802	7.824	7.867	7.859	7.854	7.889	7.899	7.913	7.871	8.249	9.911
-0.4	4.832	4.782	4.755	4.809	4.818	4.789	4.866	4.888	4.899	5.039	6.257
	5.745	5.798	5.702	5.735	5.754	5.772	5.891	5.902	5.853	6.082	7.347
	7.919	8.033	7.734	7.807	7.857	7.839	8.086	7.965	8.106	8.432	9.606
-0.2	4.797	4.863	4.784	4.796	4.767	4.771	4.813	4.823	4.888	5.055	6.373
	5.780	5.863	5.749	5.782	5.756	5.738	5.756	5.763	5.909	6.077	7.471
	7.897	7.932	7.738	7.863	7.901	7.775	7.832	7.835	8.058	8.156	9.664
0.0	4.850	4.826	4.828	4.792	4.813	4.801	4.824	4.892	4.914	4.982	6.246
	5.851	5.816	5.742	5.779	5.741	5.781	5.759	5.844	5.832	6.033	7.388
	7.943	7.941	7.895	7.954	7.936	7.925	7.852	8.128	8.005	8.249	9.839
0.2	4.799	4.830	4.773	4.852	4.845	4.786	4.775	4.850	4.896	5.013	6.240
	5.800	5.768	5.747	5.868	5.794	5.754	5.768	5.843	5.839	6.007	7.360
	8.068	7.834	7.764	7.919	8.036	7.859	7.919	7.803	8.094	8.281	9.639
0.4	4.844	4.810	4.839	4.854	4.860	4.795	4.843	4.847	4.906	5.060	6.301
	5.865	5.732	5.824	5.860	5.873	5.758	5.889	5.813	5.869	6.010	7.327
	7.962	7.895	7.967	7.908	7.949	8.111	7.951	7.896	8.025	8.193	9.801
0.6	4.771	4.843	4.886	4.818	4.876	4.930	4.842	4.826	4.916	4.980	6.352
	5.775	5.792	5.864	5.766	5.848	5.917	5.823	5.792	5.883	6.087	7.488
	7.895	7.998	8.042	8.010	8.042	8.093	7.889	7.966	7.998	8.419	9.752
0.8	4.918	4.862	4.939	4.924	4.899	4.972	4.920	5.004	4.978	5.097	6.394
	5.901	5.836	5.995	5.973	5.850	5.923	5.969	5.932	5.940	6.116	7.559
	8.200	8.020	8.278	8.136	8.060	8.041	8.175	7.953	8.081	8.353	9.997
1.0	5.203	5.254	5.251	5.263	5.277	5.328	5.323	5.339	5.369	5.509	6.761
	6.353	6.360	6.337	6.400	6.357	6.474	6.378	6.450	6.511	6.680	7.952
	8.658	8.742	8.879	8.925	8.733	8.932	8.779	8.914	8.721	9.051	10.539

Number of samples (replications) = 20000. Number of observations in a sample (T) = 200 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 6: critical values of F-test in ECM ADL regressions with sample size T = 300

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.790	4.795	4.791	4.810	4.759	4.781	4.745	4.801	4.841	4.907	6.246
	5.796	5.772	5.718	5.765	5.703	5.726	5.776	5.762	5.829	5.862	7.323
	7.899	7.805	7.820	7.774	7.661	7.860	7.858	7.872	7.941	8.029	9.696
-0.8	4.791	4.821	4.807	4.813	4.792	4.848	4.831	4.831	4.869	4.931	6.241
	5.797	5.735	5.772	5.716	5.768	5.833	5.786	5.774	5.811	5.877	7.242
	7.918	7.865	7.792	7.867	7.845	7.907	8.072	7.880	7.929	7.984	9.568
-0.6	4.753	4.751	4.778	4.778	4.776	4.773	4.752	4.819	4.848	4.966	6.194
	5.734	5.666	5.701	5.719	5.756	5.732	5.774	5.808	5.816	5.952	7.232
	7.756	7.737	7.717	7.919	7.710	7.827	7.880	8.012	8.052	8.147	9.706
-0.4	4.757	4.770	4.744	4.827	4.782	4.845	4.832	4.795	4.846	4.944	6.233
	5.701	5.722	5.685	5.764	5.726	5.874	5.767	5.766	5.807	5.973	7.277
	7.735	7.797	7.753	7.833	7.943	7.946	7.853	7.891	8.047	8.005	9.730
-0.2	4.816	4.773	4.728	4.800	4.785	4.845	4.831	4.803	4.902	4.926	6.257
	5.797	5.726	5.679	5.752	5.788	5.849	5.739	5.801	5.837	5.891	7.347
	7.772	7.782	7.741	7.905	7.902	8.089	7.921	7.794	7.994	8.180	9.578
0.0	4.827	4.803	4.793	4.781	4.802	4.754	4.855	4.872	4.846	4.969	6.223
	5.784	5.735	5.750	5.754	5.779	5.689	5.811	5.836	5.800	5.918	7.298
	7.791	7.604	7.930	7.859	7.926	7.693	7.908	7.887	7.972	8.109	9.735
0.2	4.798	4.767	4.825	4.819	4.798	4.840	4.770	4.798	4.851	4.870	6.228
	5.707	5.770	5.809	5.765	5.781	5.835	5.740	5.819	5.851	5.861	7.316
	7.735	7.734	7.871	8.081	7.868	7.956	7.827	7.863	8.103	8.097	9.581
0.4	4.799	4.805	4.768	4.766	4.819	4.840	4.809	4.889	4.870	4.918	6.270
	5.698	5.780	5.695	5.719	5.796	5.755	5.756	5.853	5.888	5.926	7.389
	7.736	7.868	7.849	7.769	7.972	7.829	7.869	7.925	8.050	8.090	9.775
0.6	4.838	4.822	4.843	4.890	4.831	4.815	4.830	4.851	4.859	4.981	6.233
	5.829	5.801	5.865	5.838	5.741	5.780	5.838	5.794	5.830	5.969	7.341
	8.007	8.050	7.947	7.955	7.918	7.933	8.210	7.898	8.001	8.053	9.707
0.8	4.837	4.879	4.863	4.842	4.854	4.870	4.821	4.879	4.896	4.994	6.264
	5.787	5.906	5.794	5.824	5.820	5.804	5.825	5.834	5.883	6.004	7.332
	7.921	7.904	7.892	8.054	7.888	8.080	8.032	7.925	7.958	8.032	9.750
1.0	5.249	5.195	5.188	5.187	5.235	5.252	5.219	5.289	5.308	5.403	6.668
	6.355	6.309	6.347	6.288	6.367	6.283	6.275	6.356	6.398	6.564	7.850
	8.750	8.515	8.852	8.608	8.958	8.623	8.535	8.657	8.787	8.871	10.329

Number of samples (replications) = 20000. Number of observations in a sample (T) = 300 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 7: critical values of F-test in ECM ADL regressions with sample size T = 400

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.717	4.826	4.776	4.727	4.774	4.787	4.785	4.802	4.796	4.885	6.197
	5.640	5.766	5.749	5.660	5.751	5.738	5.764	5.744	5.808	5.798	7.227
	7.684	7.809	7.766	7.852	8.016	7.719	7.836	7.896	7.832	7.985	9.380
-0.8	4.798	4.771	4.745	4.783	4.727	4.808	4.742	4.799	4.849	4.889	6.262
	5.778	5.690	5.682	5.739	5.696	5.758	5.660	5.767	5.841	5.848	7.324
	7.859	7.841	7.832	7.691	7.860	7.762	7.786	7.951	8.090	8.203	9.773
-0.6	4.728	4.760	4.786	4.769	4.739	4.755	4.734	4.833	4.783	4.876	6.242
	5.734	5.715	5.717	5.717	5.700	5.718	5.641	5.779	5.732	5.897	7.282
	7.937	7.806	7.705	7.844	7.611	7.877	7.553	7.841	7.996	8.130	9.459
-0.4	4.800	4.762	4.796	4.841	4.834	4.735	4.802	4.795	4.801	4.946	6.199
	5.729	5.692	5.741	5.817	5.854	5.694	5.753	5.744	5.781	5.889	7.212
	7.798	7.962	7.741	7.948	7.927	7.898	7.754	7.884	8.047	8.046	9.475
-0.2	4.761	4.743	4.710	4.792	4.778	4.813	4.792	4.806	4.815	4.955	6.187
	5.728	5.684	5.627	5.761	5.741	5.732	5.713	5.759	5.784	5.896	7.239
	7.829	7.915	7.698	7.840	7.839	7.862	7.843	7.824	7.996	8.039	9.534
0.0	4.772	4.783	4.808	4.784	4.777	4.763	4.836	4.831	4.805	4.909	6.172
	5.698	5.772	5.740	5.762	5.719	5.725	5.735	5.782	5.717	5.900	7.261
	7.740	7.776	7.807	7.762	7.839	7.785	7.843	7.927	7.904	7.922	9.606
0.2	4.715	4.787	4.801	4.733	4.802	4.855	4.753	4.805	4.858	4.924	6.192
	5.695	5.713	5.802	5.700	5.735	5.807	5.727	5.789	5.835	5.883	7.243
	7.685	7.912	7.876	7.751	7.851	7.993	7.829	7.890	7.954	7.945	9.500
0.4	4.839	4.756	4.843	4.847	4.805	4.828	4.799	4.804	4.807	4.977	6.206
	5.748	5.711	5.798	5.841	5.753	5.795	5.761	5.741	5.775	5.966	7.267
	8.030	7.758	7.878	8.052	7.848	7.969	7.957	7.820	7.830	7.935	9.482
0.6	4.831	4.773	4.880	4.843	4.794	4.816	4.803	4.848	4.876	4.923	6.279
	5.855	5.735	5.866	5.791	5.817	5.784	5.746	5.808	5.856	5.893	7.303
	7.888	7.834	7.815	7.904	7.963	7.851	7.878	7.966	7.914	8.051	9.464
0.8	4.863	4.857	4.864	4.792	4.879	4.892	4.867	4.852	4.864	4.945	6.250
	5.832	5.801	5.803	5.702	5.829	5.862	5.818	5.793	5.827	5.912	7.358
	8.129	7.978	8.006	7.852	7.850	8.104	8.040	7.928	8.066	8.075	9.795
1.0	5.140	5.204	5.183	5.221	5.150	5.187	5.224	5.233	5.346	5.317	6.685
	6.219	6.242	6.271	6.279	6.164	6.264	6.309	6.280	6.378	6.395	7.879
	8.668	8.608	8.686	8.514	8.497	8.608	8.524	8.771	8.826	8.768	10.395

Number of samples (replications) = 20000. Number of observations in a sample (T) = 400 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0: \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 8: critical values of F-test in ECM ADL regressions with sample size T = 500

		$\leftarrow \gamma_6 \rightarrow$										
$\downarrow \gamma_1$		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.800	4.701	4.752	4.762	4.729	4.829	4.813	4.818	4.815	4.900	6.141	
	5.745	5.657	5.691	5.702	5.655	5.826	5.724	5.764	5.799	5.893	7.136	
	7.802	7.766	7.739	7.859	7.786	7.882	7.839	7.726	7.944	7.909	9.323	
-0.8	4.748	4.766	4.759	4.744	4.754	4.810	4.799	4.837	4.818	4.867	6.168	
	5.660	5.741	5.714	5.748	5.696	5.773	5.757	5.773	5.795	5.789	7.213	
	7.808	7.865	7.649	7.911	7.857	7.849	7.866	7.710	7.844	7.878	9.520	
-0.6	4.790	4.777	4.758	4.720	4.773	4.797	4.788	4.758	4.900	4.898	6.210	
	5.714	5.685	5.730	5.690	5.752	5.741	5.806	5.725	5.825	5.876	7.261	
	7.841	7.807	7.752	7.784	7.871	7.794	7.783	7.792	8.033	7.848	9.588	
-0.4	4.759	4.830	4.770	4.795	4.843	4.814	4.796	4.737	4.844	4.908	6.171	
	5.701	5.815	5.712	5.783	5.825	5.775	5.740	5.693	5.833	5.870	7.244	
	7.872	7.848	7.776	7.916	7.867	7.884	7.861	7.852	7.915	8.123	9.584	
-0.2	4.771	4.760	4.744	4.781	4.784	4.767	4.822	4.854	4.861	4.845	6.229	
	5.697	5.684	5.692	5.712	5.790	5.684	5.712	5.802	5.822	5.847	7.272	
	7.911	7.770	7.783	7.764	7.907	7.702	7.884	7.850	7.780	7.837	9.502	
0.0	4.788	4.786	4.792	4.782	4.786	4.768	4.777	4.773	4.785	4.808	6.179	
	5.694	5.709	5.733	5.783	5.727	5.709	5.740	5.740	5.758	5.762	7.188	
	7.735	7.779	7.828	7.805	7.730	7.628	7.805	7.706	7.798	7.919	9.557	
0.2	4.780	4.805	4.746	4.758	4.767	4.794	4.765	4.826	4.847	4.849	6.192	
	5.715	5.756	5.738	5.691	5.722	5.726	5.681	5.765	5.790	5.812	7.233	
	7.791	7.885	7.899	7.789	7.832	7.846	7.824	7.905	7.939	7.990	9.470	
0.4	4.770	4.850	4.795	4.817	4.788	4.811	4.804	4.779	4.834	4.845	6.180	
	5.715	5.808	5.726	5.735	5.739	5.761	5.735	5.770	5.758	5.842	7.219	
	7.772	7.873	7.809	7.778	7.744	7.918	7.877	7.813	7.872	7.859	9.448	
0.6	4.816	4.818	4.817	4.790	4.864	4.766	4.781	4.810	4.887	4.889	6.209	
	5.759	5.767	5.721	5.774	5.817	5.741	5.717	5.750	5.881	5.938	7.297	
	7.909	7.818	7.752	7.817	7.932	7.860	7.820	7.909	7.967	7.950	9.505	
0.8	4.839	4.801	4.843	4.827	4.860	4.852	4.829	4.896	4.842	4.959	6.217	
	5.747	5.723	5.763	5.792	5.788	5.798	5.799	5.887	5.889	5.938	7.277	
	7.783	7.756	7.709	7.824	7.869	8.006	7.819	7.869	7.918	7.994	9.739	
1.0	5.185	5.169	5.202	5.109	5.174	5.227	5.203	5.170	5.242	5.267	6.610	
	6.250	6.269	6.339	6.201	6.300	6.305	6.275	6.281	6.320	6.391	7.770	
	8.686	8.632	8.562	8.390	8.569	8.643	8.718	8.605	8.663	8.749	10.280	

Number of samples (replications) = 20000. Number of observations in a sample (T) = 500 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0: \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 9: critical values of F-test in ECM ADL regressions with sample size T = 1000

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.768	4.739	4.861	4.796	4.787	4.778	4.749	4.795	4.765	4.835	6.227
	5.709	5.645	5.821	5.788	5.732	5.707	5.714	5.762	5.696	5.759	7.249
	7.732	7.769	7.868	7.821	7.762	7.618	7.785	7.764	7.862	7.993	9.524
-0.8	4.811	4.781	4.863	4.786	4.703	4.760	4.713	4.769	4.815	4.828	6.122
	5.747	5.701	5.748	5.714	5.631	5.706	5.708	5.685	5.762	5.772	7.108
	7.816	7.713	7.761	7.585	7.762	7.892	7.840	7.816	7.797	7.749	9.483
-0.6	4.750	4.787	4.761	4.805	4.769	4.776	4.719	4.792	4.715	4.829	6.176
	5.684	5.744	5.736	5.745	5.681	5.728	5.659	5.685	5.648	5.789	7.208
	7.742	7.757	7.776	7.811	7.580	7.779	7.704	7.706	7.589	7.921	9.484
-0.4	4.821	4.838	4.732	4.778	4.786	4.762	4.795	4.802	4.851	4.793	6.152
	5.731	5.789	5.694	5.694	5.715	5.677	5.763	5.774	5.774	5.751	7.194
	7.799	7.903	7.745	7.768	7.661	7.768	7.789	7.847	7.815	7.737	9.448
-0.2	4.708	4.795	4.760	4.780	4.804	4.782	4.753	4.850	4.752	4.801	6.142
	5.677	5.772	5.701	5.701	5.749	5.778	5.642	5.782	5.725	5.838	7.179
	7.727	7.822	7.829	7.733	7.840	7.764	7.619	7.800	7.732	7.938	9.576
0.0	4.798	4.770	4.783	4.770	4.757	4.789	4.801	4.806	4.829	4.833	6.154
	5.722	5.684	5.717	5.668	5.704	5.729	5.727	5.774	5.768	5.796	7.194
	7.682	7.770	7.817	7.681	7.858	7.990	7.675	7.808	7.841	7.992	9.485
0.2	4.751	4.798	4.825	4.775	4.745	4.812	4.767	4.789	4.814	4.900	6.186
	5.669	5.778	5.729	5.721	5.688	5.744	5.701	5.786	5.771	5.816	7.283
	7.705	7.831	7.819	7.813	7.652	7.709	7.646	7.984	7.812	7.947	9.582
0.4	4.824	4.787	4.819	4.774	4.764	4.747	4.802	4.791	4.807	4.865	6.139
	5.749	5.719	5.745	5.739	5.734	5.714	5.736	5.743	5.787	5.835	7.218
	7.915	7.651	7.863	7.821	7.778	7.680	7.780	7.857	7.855	7.775	9.601
0.6	4.759	4.732	4.819	4.766	4.780	4.754	4.778	4.805	4.850	4.804	6.181
	5.709	5.688	5.764	5.749	5.787	5.698	5.735	5.770	5.793	5.760	7.198
	7.823	7.731	7.755	7.904	8.043	7.725	7.851	7.875	7.821	7.816	9.318
0.8	4.787	4.868	4.761	4.801	4.786	4.748	4.817	4.810	4.755	4.835	6.210
	5.743	5.761	5.699	5.807	5.731	5.687	5.786	5.769	5.693	5.735	7.283
	7.847	7.749	7.789	7.827	7.899	7.811	8.011	7.825	7.598	7.851	9.388
1.0	5.153	5.143	5.132	5.090	5.094	5.120	5.149	5.116	5.103	5.230	6.618
	6.242	6.161	6.256	6.139	6.245	6.201	6.231	6.210	6.166	6.351	7.828
	8.604	8.419	8.459	8.424	8.415	8.756	8.567	8.538	8.429	8.854	10.384

Number of samples (replications) = 20000. Number of observations in a sample (T) = 1000 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 10: critical values of F-test in ECM ADL regressions with sample size T = 5000

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.756	4.779	4.761	4.810	4.783	4.757	4.753	4.752	4.778	4.718	6.200
	5.681	5.706	5.750	5.800	5.734	5.675	5.656	5.666	5.678	5.681	7.208
	7.756	7.835	7.829	7.877	7.816	7.743	7.672	7.931	7.601	7.631	9.409
-0.8	4.759	4.814	4.788	4.779	4.764	4.798	4.776	4.745	4.776	4.830	6.116
	5.695	5.744	5.736	5.712	5.697	5.737	5.737	5.676	5.732	5.787	7.173
	7.739	7.784	7.872	7.738	7.683	7.728	7.680	7.684	7.696	7.869	9.406
-0.6	4.745	4.797	4.778	4.748	4.771	4.788	4.796	4.755	4.805	4.790	6.127
	5.696	5.724	5.720	5.701	5.723	5.723	5.734	5.674	5.731	5.762	7.139
	7.688	7.646	7.674	7.696	7.784	7.712	7.615	7.824	7.751	7.767	9.315
-0.4	4.746	4.780	4.794	4.790	4.777	4.803	4.766	4.749	4.788	4.768	6.135
	5.656	5.678	5.700	5.734	5.715	5.760	5.659	5.710	5.691	5.724	7.161
	7.695	7.651	7.783	7.815	7.730	7.789	7.668	7.701	7.686	7.838	9.395
-0.2	4.770	4.755	4.729	4.791	4.817	4.775	4.773	4.720	4.742	4.813	6.204
	5.738	5.709	5.652	5.713	5.787	5.708	5.706	5.613	5.741	5.779	7.235
	7.875	7.780	7.630	7.710	7.792	7.648	7.794	7.658	7.717	7.786	9.375
0.0	4.760	4.747	4.794	4.760	4.780	4.773	4.791	4.761	4.789	4.824	6.057
	5.703	5.702	5.718	5.674	5.696	5.718	5.758	5.684	5.712	5.774	7.100
	7.671	7.702	7.797	7.700	7.675	7.815	7.926	7.763	7.745	7.812	9.344
0.2	4.831	4.801	4.776	4.807	4.801	4.783	4.802	4.740	4.792	4.781	6.116
	5.713	5.714	5.697	5.784	5.760	5.745	5.742	5.678	5.724	5.724	7.121
	7.782	7.686	7.733	7.728	7.842	7.690	7.830	7.688	7.836	7.803	9.544
0.4	4.755	4.739	4.743	4.786	4.694	4.783	4.802	4.809	4.728	4.786	6.158
	5.664	5.688	5.649	5.728	5.610	5.757	5.712	5.701	5.643	5.736	7.217
	7.811	7.687	7.800	7.789	7.678	7.879	7.891	7.700	7.590	7.642	9.332
0.6	4.756	4.787	4.788	4.849	4.778	4.754	4.736	4.794	4.742	4.759	6.152
	5.696	5.770	5.720	5.721	5.677	5.713	5.705	5.678	5.716	5.706	7.186
	7.817	7.818	7.811	7.851	7.808	7.795	7.568	7.777	7.730	7.925	9.304
0.8	4.812	4.711	4.766	4.814	4.782	4.793	4.755	4.795	4.806	4.793	6.119
	5.733	5.666	5.648	5.784	5.718	5.725	5.692	5.722	5.775	5.731	7.179
	7.757	7.663	7.661	7.775	7.713	7.800	7.649	7.816	7.881	7.712	9.391
1.0	5.015	5.016	5.033	4.984	5.030	4.966	5.036	5.048	4.992	4.999	6.499
	6.041	6.082	6.090	6.028	6.074	6.006	6.089	6.113	6.002	6.090	7.680
	8.256	8.379	8.599	8.363	8.306	8.292	8.302	8.345	8.322	8.507	10.001

Number of samples (replications) = 20000. Number of observations in a sample (T) = 5000 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 11: critical values of F-test in ECM ADL regressions with sample size T = 10000

$\downarrow \gamma_1$	$\leftarrow \gamma_6 \rightarrow$										
	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
-1.0	4.812	4.711	4.751	4.827	4.797	4.744	4.738	4.804	4.713	4.738	6.147
	5.737	5.709	5.655	5.777	5.698	5.662	5.678	5.716	5.630	5.679	7.190
	7.825	7.663	7.896	7.822	7.861	7.822	7.749	7.770	7.560	7.707	9.325
-0.8	4.767	4.755	4.790	4.787	4.770	4.760	4.776	4.782	4.791	4.741	6.145
	5.665	5.725	5.691	5.726	5.784	5.686	5.683	5.737	5.788	5.682	7.164
	7.738	7.724	7.731	7.686	7.777	7.833	7.662	7.803	7.842	7.645	9.528
-0.6	4.800	4.785	4.730	4.767	4.767	4.738	4.814	4.762	4.773	4.767	6.021
	5.662	5.676	5.697	5.710	5.705	5.708	5.727	5.747	5.756	5.708	7.079
	7.843	7.651	7.776	7.811	7.714	7.661	7.839	7.850	7.758	7.527	9.236
-0.4	4.746	4.739	4.822	4.751	4.801	4.781	4.747	4.786	4.772	4.797	6.194
	5.673	5.718	5.748	5.703	5.722	5.726	5.716	5.744	5.661	5.715	7.218
	7.811	7.769	7.757	7.713	7.826	7.752	7.765	7.997	7.656	7.851	9.482
-0.2	4.745	4.727	4.755	4.792	4.758	4.814	4.762	4.780	4.763	4.779	6.101
	5.670	5.678	5.695	5.740	5.745	5.786	5.731	5.703	5.710	5.762	7.100
	7.577	7.715	7.629	7.803	7.672	7.772	7.741	7.689	7.766	7.810	9.166
0.0	4.772	4.751	4.822	4.767	4.788	4.784	4.714	4.816	4.724	4.753	6.122
	5.729	5.714	5.734	5.633	5.767	5.703	5.691	5.709	5.666	5.686	7.178
	7.863	7.599	7.767	7.640	7.751	7.839	7.755	7.637	7.752	7.796	9.467
0.2	4.738	4.791	4.783	4.719	4.787	4.735	4.748	4.787	4.706	4.744	6.130
	5.645	5.705	5.703	5.655	5.713	5.660	5.649	5.716	5.644	5.700	7.132
	7.724	7.693	7.769	7.614	7.703	7.783	7.674	7.718	7.765	7.822	9.205
0.4	4.796	4.714	4.732	4.763	4.799	4.788	4.781	4.794	4.762	4.789	6.089
	5.762	5.652	5.747	5.730	5.782	5.761	5.715	5.682	5.766	5.678	7.105
	7.760	7.815	7.820	7.724	7.871	7.720	7.784	7.704	7.813	7.778	9.326
0.6	4.782	4.790	4.791	4.717	4.775	4.797	4.746	4.761	4.751	4.768	6.106
	5.717	5.717	5.716	5.661	5.728	5.717	5.700	5.703	5.706	5.706	7.161
	7.764	7.690	7.768	7.769	7.715	7.740	7.623	7.776	7.738	7.776	9.408
0.8	4.770	4.750	4.749	4.783	4.731	4.778	4.726	4.735	4.771	4.740	6.134
	5.687	5.683	5.656	5.739	5.661	5.726	5.689	5.672	5.729	5.657	7.137
	7.675	7.675	7.893	7.723	7.814	7.759	7.810	7.744	7.656	7.562	9.278
1.0	4.971	4.916	4.974	4.946	5.030	5.003	5.035	4.975	4.992	4.964	6.555
	6.000	5.975	6.037	5.991	6.168	6.057	6.104	6.074	6.028	6.041	7.710
	8.226	8.307	8.285	8.166	8.395	8.247	8.233	8.352	8.332	8.388	10.024

Number of samples (replications) = 20000. Number of observations in a sample (T) = 10000 after 500 discarded initial observations.

DGPs: $\Delta Y_t = \gamma_1 \Delta Y_{t-1} + v_{1t}$, $v_{1t} \sim N(0, 1)$; $\Delta X_t = \gamma_6 \Delta X_{t-1} + v_{2t}$, $v_{2t} \sim N(0, 1)$.

Regression: $\Delta Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 \Delta Y_{t-1} + \hat{\gamma}_2 \Delta X_t + \hat{\gamma}_3 \Delta X_{t-1} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + \nu_t$

Critical values for an F-test of the null hypothesis $H_0 : \gamma_4 = \gamma_5 = 0$, of no cointegration.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 12: critical values of an F-test in ECM ADL regressions (Non-seasonal DGPs)

Lags (p)	Sample size (T)									
	50	75	100	200	300	400	500	1000	5000	10000
0	4.932	4.870	4.930	4.840	4.839	4.808	4.770	4.752	4.773	4.758
	6.030	5.878	5.912	5.779	5.812	5.781	5.696	5.690	5.726	5.677
	8.491	8.298	8.066	8.006	7.967	7.903	7.779	7.729	7.636	7.642
1	5.997	5.609	5.385	5.085	5.032	4.953	4.873	4.836	4.793	4.747
	7.325	6.805	6.538	6.132	6.069	5.968	5.807	5.796	5.714	5.678
	10.311	9.327	8.986	8.542	8.182	8.295	7.883	7.830	7.748	7.649
2	6.097	5.762	5.539	5.175	5.100	5.004	4.945	4.847	4.810	4.760
	7.475	6.908	6.636	6.205	6.148	6.017	5.888	5.822	5.733	5.696
	10.741	9.515	9.094	8.500	8.297	8.373	7.977	7.832	7.772	7.666
3	6.177	5.857	5.619	5.277	5.138	5.028	4.972	4.876	4.799	4.754
	7.472	7.007	6.771	6.299	6.145	6.113	6.001	5.815	5.719	5.688
	10.583	9.682	9.119	8.588	8.531	8.367	7.955	7.795	7.741	7.625
4	6.155	5.872	5.725	5.334	5.213	5.086	5.015	4.862	4.793	4.747
	7.409	7.058	6.889	6.363	6.243	6.166	6.033	5.827	5.730	5.696
	10.544	9.791	9.449	8.552	8.449	8.464	8.082	7.912	7.780	7.701
5	6.138	5.934	5.815	5.430	5.302	5.131	5.071	4.904	4.812	4.752
	7.409	7.159	6.956	6.462	6.392	6.229	6.100	5.858	5.757	5.698
	10.533	9.681	9.472	8.731	8.647	8.579	8.143	7.935	7.855	7.724
6	6.010	5.882	5.824	5.507	5.354	5.214	5.109	4.916	4.829	4.747
	7.263	7.058	6.980	6.552	6.445	6.280	6.125	5.870	5.758	5.693
	10.297	9.620	9.485	8.785	8.763	8.597	8.199	7.957	7.816	7.753
7	5.966	5.883	5.912	5.545	5.440	5.277	5.181	4.920	4.827	4.767
	7.308	7.067	6.985	6.616	6.480	6.375	6.171	5.954	5.749	5.691
	10.272	9.786	9.509	8.816	8.768	8.793	8.382	7.954	7.849	7.771
8	5.861	5.841	5.884	5.600	5.451	5.309	5.186	4.939	4.815	4.766
	7.052	7.057	7.055	6.663	6.479	6.390	6.178	5.922	5.747	5.697
	10.221	9.800	9.557	8.806	8.867	8.739	8.494	8.036	7.878	7.785
9	5.755	5.846	5.904	5.615	5.473	5.343	5.231	4.971	4.817	4.765
	7.038	7.081	7.018	6.678	6.565	6.413	6.238	5.941	5.756	5.715
	10.031	9.647	9.495	8.949	8.771	8.869	8.570	8.095	7.904	7.838
10	5.744	5.816	5.898	5.585	5.456	5.306	5.217	4.960	4.819	4.773
	7.060	6.990	7.021	6.674	6.542	6.384	6.216	5.947	5.764	5.699
	9.891	9.754	9.571	8.974	8.833	8.822	8.599	8.104	7.881	7.811
11	5.605	5.713	5.907	5.636	5.503	5.382	5.252	5.013	4.823	4.763
	6.899	6.844	7.036	6.703	6.582	6.474	6.268	5.984	5.762	5.683
	9.890	9.484	9.529	9.010	8.928	8.932	8.626	8.134	7.927	7.824
12	5.493	5.650	5.840	5.644	5.492	5.400	5.278	4.989	4.825	4.761
	6.909	6.775	6.954	6.712	6.565	6.442	6.255	6.002	5.782	5.679
	10.090	9.573	9.488	8.985	9.014	8.862	8.710	8.209	7.891	7.824
13	5.417	5.534	5.773	5.684	5.573	5.469	5.320	5.035	4.825	4.774
	6.761	6.671	6.851	6.776	6.607	6.496	6.348	6.042	5.796	5.684
	10.055	9.562	9.219	9.178	9.051	8.997	8.802	8.267	7.906	7.867

Number of samples (replications) = 20000. Number of observations in a sample (T) after 500 discarded initial observations.

DGPs: $\Delta Y_t = \sum_{j=1}^p \gamma_{1j} \Delta Y_{t-j} + \nu_{1jt}$, $\nu_{1jt} \sim N(0, 1)$; $\Delta X_t = \sum_{j=1}^p \gamma_{6j} \Delta X_{t-j} + \nu_{2jt}$, $\nu_{2jt} \sim N(0, 1)$.

$$\text{Regression: } \Delta Y_t = \hat{\gamma}_0 + \sum_{j=1}^p \hat{\gamma}_{1j} \Delta Y_{t-j} + \hat{\gamma}_2 \Delta X_t + \sum_{j=1}^p \hat{\gamma}_{3j} \Delta X_{t-j} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_{jt}$$

For lags denoted (p =) 0 this corresponds to the case considered by Kanioura and Turner (2005).

Critical values for an F-test of the null hypothesis of no cointegration $H_0: \gamma_4 = \gamma_5 = 0$ are reported in the table.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).

Table 13: critical values of F-test in ECM ADL regressions (Seasonal DGPs)

Lags (L)	Sample size (T)									
	50	75	100	200	300	400	500	1000	5000	10000
0	4.972	4.911	4.874	4.811	4.830	4.803	4.828	4.794	4.728	4.795
	6.056	6.007	5.938	5.752	5.778	5.715	5.762	5.760	5.665	5.787
	8.673	8.395	8.245	7.951	8.012	7.714	7.825	7.836	7.790	7.724
1	5.204	5.099	4.976	4.863	4.868	4.796	4.834	4.832	4.716	4.797
	6.318	6.170	6.005	5.834	5.821	5.791	5.752	5.772	5.656	5.780
	9.067	8.534	8.360	7.987	8.088	7.778	7.910	7.827	7.771	7.749
2	5.796	5.493	5.288	5.046	5.012	4.858	4.905	4.854	4.732	4.817
	7.031	6.681	6.341	6.065	5.986	5.853	5.845	5.836	5.684	5.790
	9.760	9.311	8.836	8.435	8.106	8.005	7.927	7.934	7.748	7.731
3	6.173	5.925	5.713	5.338	5.216	5.005	5.039	4.919	4.763	4.816
	7.433	7.126	6.786	6.393	6.191	5.992	5.977	5.953	5.698	5.788
	10.360	9.694	9.284	8.688	8.337	8.257	8.097	8.027	7.819	7.840
4	6.054	5.935	5.729	5.383	5.261	5.056	5.061	4.942	4.772	4.813
	7.301	7.131	6.830	6.455	6.218	6.050	6.036	5.941	5.692	5.804
	10.158	9.753	9.326	8.737	8.416	8.412	8.193	8.062	7.881	7.824
5	5.960	5.970	5.781	5.446	5.290	5.085	5.103	4.963	4.774	4.812
	7.234	7.120	6.892	6.562	6.288	6.116	6.073	5.971	5.730	5.802
	10.106	9.840	9.331	8.871	8.466	8.498	8.198	8.054	7.868	7.789
6	5.860	5.835	5.758	5.528	5.312	5.108	5.107	4.984	4.769	4.823
	7.094	7.020	6.832	6.614	6.353	6.169	6.102	5.994	5.737	5.794
	10.043	9.655	9.273	8.808	8.513	8.479	8.371	8.079	7.856	7.778
7	5.718	5.810	5.744	5.562	5.368	5.130	5.134	4.994	4.771	4.821
	6.955	6.965	6.801	6.649	6.300	6.234	6.161	5.994	5.738	5.802
	10.044	9.543	9.277	8.867	8.610	8.546	8.397	8.078	7.881	7.774
8	5.606	5.753	5.685	5.579	5.347	5.159	5.185	5.010	4.783	4.828
	6.860	6.914	6.820	6.698	6.351	6.248	6.144	6.033	5.728	5.791
	9.894	9.498	9.214	8.933	8.506	8.563	8.411	8.122	7.839	7.803
9	5.543	5.655	5.683	5.610	5.384	5.211	5.212	5.048	4.794	4.824
	6.799	6.860	6.795	6.702	6.398	6.275	6.180	6.048	5.737	5.785
	9.789	9.481	9.190	9.020	8.551	8.659	8.330	8.068	7.828	7.786
10	5.442	5.570	5.620	5.634	5.410	5.258	5.242	5.075	4.795	4.832
	6.713	6.710	6.802	6.726	6.421	6.274	6.207	6.062	5.733	5.774
	9.792	9.320	9.270	9.040	8.635	8.656	8.389	8.119	7.861	7.763
11	5.450	5.496	5.541	5.611	5.378	5.309	5.246	5.101	4.796	4.825
	6.818	6.656	6.641	6.719	6.444	6.365	6.249	6.072	5.735	5.776
	10.026	9.329	9.139	9.018	8.593	8.567	8.403	8.093	7.832	7.790
12	5.426	5.455	5.479	5.629	5.412	5.324	5.260	5.101	4.797	4.824
	6.716	6.564	6.574	6.658	6.416	6.348	6.279	6.090	5.752	5.791
	10.157	9.273	9.140	8.858	8.694	8.580	8.513	8.236	7.807	7.818
13	5.420	5.369	5.436	5.647	5.426	5.330	5.309	5.114	4.809	4.835
	6.904	6.535	6.496	6.718	6.444	6.365	6.297	6.151	5.764	5.785
	10.633	9.233	9.089	8.902	8.686	8.711	8.550	8.293	7.792	7.768

Number of samples (replications) = 20000. Number of observations in a sample (T) after 500 discarded initial observations.

$$\text{DGPs: } \Delta Y_t = \sum_{j=1}^p \gamma_{1j} \Delta Y_{t-j} + v_{1jt}, \quad v_{1jt} \sim N(0, 1); \quad \Delta X_t = \sum_{j=1}^p \gamma_{6j} \Delta X_{t-j} + v_{2jt}, \quad v_{2jt} \sim N(0, 1).$$

$$\text{Regression: } \Delta Y_t = \hat{\gamma}_0 + \sum_{j=1}^p \hat{\gamma}_{1j} \Delta Y_{t-j} + \hat{\gamma}_2 \Delta X_t + \sum_{j=1}^p \hat{\gamma}_{3j} \Delta X_{t-j} + \hat{\gamma}_4 Y_{t-1} + \hat{\gamma}_5 X_{t-1} + v_{jt}$$

Critical values for an F-test of the null hypothesis of no cointegration $H_0 : \gamma_4 = \gamma_5 = 0$ are reported in the table.

The following levels of significance are indicated by the font types: 90% (red), 95% (black, bold, italic) and 99% (blue, italic).
