The Faculty of Business & Law
In, of and for the CITY OF LONDON

LONDON metropolitan university

Centre for International Capital Markets

Discussion Papers

ISSN 1749-3412

Long Memory In The Ukrainian Stock Market And Financial Crises

Guglielmo Maria Caporale, Luis Gil-Alana, Alex Plastun, Inna Makarenko

No 2014-03

LONG MEMORY IN THE UKRAINIAN STOCK MARKET AND FINANCIAL CRISES

Guglielmo Maria Caporale*
Brunel University, London, CESifo and DIW Berlin

Luis Gil-Alana**
University of Navarra

Alex Plastun
Ukrainian Academy of Banking

Inna Makarenko
Ukrainian Academy of Banking

October 2013

Abstract

This paper examines persistence in the Ukrainian stock market during the recent financial crisis. Using two different long memory approaches (R/S analysis and fractional integration) we show that this market is inefficient and the degree of persistence is not the same in different stages of the financial crisis. Therefore trading strategies might have to be modified. We also show that data smoothing is not advisable in the context of R/S analysis.

Keywords: *Persistence, Long Memory, R/S Analysis, Fractional Integration*

JEL Classification: *C22, G12*

*Corresponding author. Department of Economics and Finance, Brunel University, London, UB8 3PH.

Email: Guglielmo-Maria.Caporale@brunel.ac.uk

** The second named author acknowledges financial support from the Ministry of education of Spain (ECO2011-2014 ECON Y FINANZAS, Spain).

1. INTRODUCTION

As a result of the recent financial crisis the relevance of traditional models based on the efficient market hypothesis (EMH) has been questioned. An alternative paradigm is the so-called fractal market hypothesis (FMH – see Mandelbrot, 1972, and Peters, 1994), according to which stock prices are not linear and the normal distribution (a basic assumption of the EMH) cannot be used to explain their movements given the presence of "fat tails". Within this framework one of the key characteristics of financial time series is their persistence or long memory.

This paper uses two different approaches (i.e. R/S analysis and fractional integration) to estimate persistence in the Ukrainian stock market. In particular, we show that this feature is not the same at different stages of the financial crisis of 2007-2009. We also show that data smoothing does not improve the R/S method.

The layout of the paper is the following. Section 2 describes the data and outlines the Hurst exponent method as well as the I(d) techniques used. Section 3 presents the empirical results. Section 4 provides some concluding remarks.

2. DATA AND METHODOLOGY

The R/S method was originally applied by Hurst (1951) in hydrological research and improved by Mandelbrot (1972), Peters (1991, 1994) and others analysing the fractal nature of financial markets. Compared with other approaches it is relatively simple and suitable for programming as well as visual interpretation.

For each sub-period range R (the difference between the maximum and minimum index within the sub-period), the standard deviation S and their average ratio are calculated. The length of the sub-period is increased and the calculation repeated until the size of the sub-period is equal to that of the original series. As a result, each sub-period is determined by the average value of R/S. The least square method is applied to these values and a regression is run, obtaining an estimate of the angle of the regression line. This estimate is a measure of the Hurst exponent, which is an indicator of market persistence. More details are provided below.

1. We start with a time series of length M and transform it into one of length $N = M - 1$ using logs and converting stock prices into stock returns:

$$N_i = \log\left(\frac{Y_{t+1}}{Y_t}\right), \quad t = 1,2,3,\ldots (M-1) \qquad (1)$$

2. We divide this period into contiguous A sub-periods with length n, so that $A_n = N$, then we identify each sub-period as I_a, given the fact that a = 1, 2, 3. . . , A. Each element I_a is represented as N_k with k = 1, 2, 3. . . , N. For each I_a with length n the average e_a is defined as:

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a}, \quad k = 1,2,3,\ldots N, \quad a = 1,2,3\ldots, A \qquad (2).$$

3. Accumulated deviations $X_{k,a}$ from the average e_a for each sub-period I_a are defined as:

$$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - e_a). \qquad (3)$$

The range is defined as the maximum index $X_{k,a}$ minus the minimum $X_{k,a}$, within each sub-period (I_a):

$$R_{Ia} = \max(X_{k,a}) - \min(X_{k,a}), \quad 1 \le k \le n. \qquad (4)$$

4. The standard deviation S_{Ia} is calculated for each sub-period I_a:

$$S_{Ia} = \left((\frac{1}{n})\sum_{k=1}^{n}(N_{k,a} - e_a)^2\right)^{0,5}. \qquad (5)$$

5. Each range R_{Ia} is normalized by dividing by the corresponding S_{Ia}. Therefore, the re-normalized scale during each sub-period I_a is R_{Ia}/ S_{Ia}. In the step 2 above, we

obtained adjacent sub-periods of length n. Thus, the average R/S for length *n* is defined as:

$$(R/S)_n = (1/A)\sum_{i=1}^{A}(R_{Ia}/S_{Ia}). \qquad (6)$$

6. The length *n* is increased to the next higher level, *(M - 1)/n*, and must be an integer number. In this case, we use *n*-indexes that include the initial and ending points of the time series, and Steps 1 - 6 are repeated until *n = (M - 1)/2*.

7. Now we can use least square to estimate the equation *log (R / S) = log (c) + Hlog (n)*. The angle of the regression line is an estimate of the Hurst exponent *H*. This can be defined over the interval [0, 1], and is calculated within the boundaries specified in Table 1.

[Insert Table 1 about here]

An important step in the R/S analysis is the verification of the results by calculating the Hurst exponent for randomly mixed data. In theory, these should be a random time series with a Hurst exponent equal to 0.5. In this paper, we will carry out a number of additional checks, including:

- Generation of random data;
- Generation of an artificial trend (persistent series);
- Generation of an artificial anti-persistent series.

In order to analyse persistence, in addition to the Hurst exponent and the R/S analysis we also estimate parametric/semiparametric models based on fractional integration or I(d) models of the form:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \qquad (9)$$

where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0), defined for our purposes as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Note that H and d are related through the equality H = d – 0.5.

In the semiparametric model no specification is assumed for u_t, while the parametric one is fully specified. For the former, the most commonly employed specification is based on the log-periodogram (see Geweke and Porter-Hudak, GHP, 1983). This method was later extended and improved by many authors including Künsch (1986), Robinson (1995a), Hurvich and Ray (1995), Velasco (1999a, 2000) and Shimotsu and Phillips (2002). In this paper, however, we will employ another semiparametric method: it is essentially a local 'Whittle estimator' in the frequency domain, which uses a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{s=1}^{m} \log \lambda_s \right), \qquad (10)$$

$$\overline{C(d)} = \frac{1}{m}\sum_{s=1}^{m} I(\lambda_s)\,\lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{T}, \qquad \frac{m}{T} \to 0,$$

where m is a bandwidth parameter, and $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi T}\left| \sum_{t=1}^{T} x_t\, e^{i\lambda_s t} \right|^2,$$

and d $\in$ (-0.5, 0.5). Under finiteness of the fourth moment and other mild conditions, Robinson (1995b) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroscedasticity and is more efficient than other more recent semiparametric competitors. Other recent refinements of this procedure can be found in

Velasco, 1999b, Velasco and Robinson, 2000; Phillips and Shimotsu, 2004, 2005 and Abadir et al. (2007).

Estimating d parametrically along with the other model parameters can be done in the frequency domain or in the time domain. In the former, Sowell (1992) analysed the exact maximum likelihood estimator of the parameters of the ARFIMA model, using a recursive procedure that allows a quick evaluation of the likelihood function. Other parametric methods for estimating d based on the frequency domain were proposed, among others, by Fox and Taqqu (1986) and Dahlhaus (1989) (see also Robinson, 1994 and Lobato and Velasco, 2008 for Wald and LM parametric tests based on the Whittle function).

Two of the main Ukrainian stock market indexes, namely the PFTS and UX indices respectively, are used for the empirical analysis. The sample period goes from 2001 to 2013 for PFTS and from 2008 to 2013 for UX. For most of the calculations we used the UX index, which is most frequently used nowadays to analyse the Ukrainian stock market, since the PFTS series, only starting in 2008, is relatively short. The different periods considered include that of the inflation "bubble" and market overheating, which created the preconditions for the crisis in 2007, the peak of the crisis at the end of 2008 and in the early part of 2009, and its attenuation towards the end of 2009 and in 2010 (Figure 1).

[Insert Figure 1 about here]

The peak of the crisis is defined on the basis of the dynamics of the CBOE Volatility Index (VIX), which is calculated from 1993 using the S&P 500 prices of options in the Chicago Stock Exchange, one of the largest organized trading platforms. It should be noticed that peaks of market volatility at 89.53 and 81.48 were observed during the announcement of the bankruptcy of Lehman Brothers and AIG in September - October 2008 with an overall increase in volatility in the second half of 2008. At the same time the decision to restructure the AIG debt led to better investment expectations of market participants and to a fall of the VIX index to 39.33 (Figure 2).

Also important is the choice of the interval of the fluctuations to analyse, i.e. 5, 30, 60 minutes, one day, one week, one month. We decided to focus on the 1-day interval, because higher frequency data generates significant fluctuations of fractals, and lower frequency data lose their analytical potential.

We incorporate data smoothing into the R/S analysis and test the following hypothesis: data smoothing (filtration) lowers the level of "noise" in the data and reduces the influence of abnormal returns; smoothing makes the data closer to the real state of the market.

We use the following simple methods:

1) Smoothing with moving averages (simple moving average and weighted moving average with periods 2 and 5);
2) Smoothing with the Irwin criterion.

The analysis is conducted for the Ukrainian stock market index (UX) over the period 2008-2013. Overall we analysed 1300 daily returns. As a control group we chose daily closes of UX (unfiltered data) and a set of randomly generated data. The estimates of the Hurst exponent for the mixed data sets are used as a criterion for the adequacy of the results.

[Insert Figures 3 – 8 about here]

The first stage is the visual analysis of both unfiltered and filtered data. The results are presented in Figures 3 - 8. The behaviour of the series does not change dramatically after filtering (smoothing), but the level of "noise" decreases. In terms of fractal theory, visual inspection reveals a decrease of the fractal dimension.

To confirm that the properties of the time series are the same and we only neutralise the level of unnecessary "noise", we filtered randomly generated data sets for which the fractal dimension should remain the same. However, visual inspection (see Figures 6 - 8) shows that the fractal dimension of the randomly generated data set also changes after filtering.

To corroborate the visual analysis we calculate the Hurst exponent for each type of filter.

[Insert Table 2 about here]

As can be seen from Table 2, filtering the data leads to over-estimating the Hurst exponent. The longer the averaging period (the bigger the level of filtering) the higher the Hurst exponent is, indicating dependency of the latter on the former.

Irwin's method also generates overestimates of the Hurst exponent and therefore is inappropriate as well. Overall, it appears that data smoothing artificially increases the Hurst exponent, and therefore further calculations will be based on the original data sets.

One more possible modification of the R/S analysis is the use of aliquant numbers of groups, i.e. computing the Hurst exponent for all possible groups. The results are presented in Table 3.

[Insert Table 3 about here]

Both the real financial data and the randomly generated ones suggest that the use of aliquant numbers of groups leads to overestimates of the Hurst exponent. Nevertheless, using them might be appropriate in the case of small data sets, but a correction of 0.03 - 0.05 should be made depending on the value of the Hurst exponent (the bigger it is the bigger the correction should be). Given these results, the standard methodology will be used below to estimate the Hurst exponent.

3. EMPIRICAL RESULTS

As a first stage of the analysis we estimate persistence of two Ukrainian stock market indices over the full sample (UX: 2008-2013, PFTS: 2001-2013). The results in Table 4 provide evidence of persistence and long memory.

Next, we estimate persistence during the financial crisis. We checked different window sizes and found that 300 (close to one calendar year) is the most appropriate on the basis of the behaviour of the Hurst exponent: for narrower windows its volatility increases dramatically, whilst for wider ones it is almost constant, and therefore the dynamics are not apparent.

Having calculated the first value of the Hurst exponent (for example, that for the date 13.07.2007 corresponds to the period from 21.04.2005 till 13.07.2007), each of the following ones is obtained by shifting forward the “data window”. The chosen size of the shift is 10, which provides a sufficient number of estimates to analyse the behaviour of the Hurst exponent. Therefore the second value is calculated for 27.07.2006 and characterises the market over the period 10.05.2005 till 27.07.2006, and so on. As a result we obtain 170 control points (Hurst exponent estimates) for different sub-samples characterised by various degrees of persistence in the Ukrainian stock market over the period 2005-2013 (see Fig. 9).

[Insert Figure 9 about here]

It is apparent that market persistence is not constant over the sample, increasing during the crisis. Consequently, trading strategies might have to be revised.

Semiparametric/parametric methods for the UX index

Next we focus on the UX index. Figure 10 displays four time series plots corresponding to the original prices, the corresponding returns, and the squared and absolute returns.

Figures 11 and 12 display respectively the correlograms and periodograms of each series.

[Insert Figures 10 -12 about here]

They suggest that the UX index is non-stationary. This can also be inferred from the correlogram and periodogram of the series. Stock returns might be stationary but there is still some degree of dependence in the data. Finally, the correlograms of the absolute and the squared returns also indicate high time dependence in the series.

Table 5 reports the estimates of d based on a parametric approach. The model considered is the following:

$$y_t = \alpha + \beta t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, ...,$$

where y_t stands for the (logged) stock market prices, assuming that the disturbances u_t are in turn a) white noise, b) autoregressive (AR(1), and c) of the Bloomfield-type, the latter being a nonparametric approach that produces autocorrelations decaying exponentially as in the AR case.

[Insert Table 5 about here]

We consider the three standard cases of i) no regressors ($\alpha = \beta = 0$ above), ii) with an intercept (i.e., $\beta = 0$), and iii) with an intercept and a linear time trend. The most relevant case is the one with an intercept. The reason is that the t-values imply that the coefficients on the linear time trends are not statistically significant in all cases, unlike those on the intercept. We have used a Whittle estimator of d (Dahlhaus, 1989) along with the parametric testing procedure of Robinson (1994).

The results indicate that for the log UX series the estimated value of d is significantly higher than 1 independently of the way of modelling the I(0) disturbances. As for the absolute and squared returns, the estimates are all significantly positive, ranging between 0.251 and 0.313.

[Insert Figure 13 about here]

Figure 13 focuses on the semiparametric approach of Robinson (1995b), extended later by many authors, including Abadir et al. (2007). Given the nonstationary nature of the UX series, first-differenced data are used for the estimation, then adding 1 to the estimated values to obtain the orders of integration of the series. When using the Abadir et al.'s (2007) approach, which is an extension of Robinson's (1995) that does not impose stationarity, the estimates were almost identical to those reported in the paper, and similar results were obtained with log-periodogram type estimators. Along with the estimates we also present the 95% confidence bands corresponding to the I(1) hypothesis for the UX data and the I(0) hypothesis for the absolute/squared returns. We display the estimates for the whole range of values of the bandwidth parameter m = 1, .,, T/2. It can be seen that the values are above the I(1) interval in the majority of cases, which is consistent with the parametric results reported in Table 5. For the absolute and squared returns, the estimates are practically all significantly above the I(0) interval, implying long memory behaviour. Overall, these results confirm the parametric ones. The estimated value of d is slightly above 1 for the log stock market prices, and significantly above 0 for both squared and absolute returns.

[Insert Figure 14 about here]

Figure 14 presents the stability results. We computed the estimates of d with two different approaches: a recursive one, initially using a sample of 300 observations, and then adding ten more observations each time, and a rolling one with a moving window of 300 observations. Persistence appears to decrease over time, especially for the volatility series.

4. CONCLUSIONS

This paper uses both the Hurst exponent and parametric/semiparametric fractional integration methods to analyse the long-memory properties of two Ukrainian stock market indices, namely the PFTS and UX indices. The evidence suggests that this market is inefficient and that persistence was not constant over time; in particular, it increased during the recent financial crisis, when the market became less efficient/more predictable and more vulnerable to market anomalies. This created the opportunity for profitable trading strategies exploiting the January, day of the week, end of the month, holidays effects and other market anomalies, or, alternatively, based on following trends (these issues will be examined in future papers). Finally, our study also shows that data smoothing is not advisable in the context of R/S analysis.

References

Abadir, K.M., W. Distaso and L. Giraitis, 2007, Nonstationarity-extended local Whittle estimation, Journal of Econometrics 141, 1353-1384.

Dahlhaus, R., 1989, Efficient parameter estimation for self-similar process. Annals of Statistics 17, 1749-1766.

Fox, R. and Taqqu, M., 1986, Large sample properties of parameter estimates for strongly dependent stationary Gaussian time series. Annals of Statistics 14, 517-532.

Geweke, J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, Journal of Time Series Analysis 4, 2221-238.

Hurst H. E., 1951. Long-term Storage of Reservoirs. Transactions of the American Society of Civil Engineers, 799 p.

Hurvich, C.M. and B.K. Ray, 1995, Estimation of the memory parameter for nonstationary or noninvertible fractionally integrated processes. Journal of Time Series Analysis 16, 17-41.

Künsch, H., 1986, Discrimination between monotonic trends and long-range dependence, Journal of Applied Probability 23, 1025-1030.

Lobato, I.N. and C. Velasco, 2007, Efficient Wald tests for fractional unit root. Econometrica 75, 2, 575-589.

Mandelbrot B., 1972. Statistical Methodology For Nonperiodic Cycles: From The Covariance To Rs Analysis, Annals of Economic and Social Measurement 1, 259-290

Peters E. E., 1991, Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility, NY. : John Wiley and Sons, Inc, 228 p

Peters E. E., 1994, Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, NY. : John Wiley & Sons, 336 p

Phillips, P.C. and Shimotsu, K., 2004. Local Whittle estimation in nonstationary and unit root cases. Annals of Statistics 32, 656-692.

Robinson, P. M., 1994, Efficient tests of nonstationary hypotheses. Journal of the American Statistical Association, 89, 1420-1437.

Robinson, P.M., 1995a, Log-periodogram regression of time series with long range dependence. Annals of Statistics 23, 1048-1072.

Robinson, P.M., 1995b, Gaussian semi-parametric estimation of long range dependence, Annals of Statistics 23, 1630-1661.

Shimotsu, K. and P.C.B. Phillips, 2002, Pooled Log Periodogram Regression. Journal of Time Series Analysis 23, 57-93.

Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally integrated time series models. Journal of Econometrics 53, 165-188.

Velasco, C. and P.M. Robinson, 2000, Whittle pseudo maximum likelihood estimation for nonstationary time series. Journal of the American Statistical Association 95, 1229-1243.

Velasco, C., 1999a, Nonstationary log-periodogram regression. Journal of Econometrics 91, 299-323.

Velasco, C., 1999b. Gaussian semiparametric estimation of nonstationary time series.Journal of Time Series Analysis 20, 87-127.

Velasco, C., 2000, Non-Gaussian log-periodogram regression, Econometric Theory 16, 44-79.

Tables and Figures

Table 1: Hurst exponent interval characteristics

Interval	Hypothesis	Distribution	«Memory» of series	Type of process	Trading Strategies
$0 \leq H < 0{,}5$	Data is fractal, fractal market hypothesis is confirmed	"Heavy tails" of distribution	Antipersistent series, negative correlation in instruments value changes	Pink noise with frequent changes in direction of price movement	Trading in the market is more risky for an individual participant
$H = 0{,}5$	Data is random, Efficient market hypothesis is confirmed	Movement of asset prices is an example of the random Brownian motion (Wiener process), time series are normally distributed	Lack of correlation in changes in value of assets (memory of series)	White noise of independent random process	Traders cannot "beat" the market with the use of any trading strategy
$0{,}5 < H \leq 1$	Data is fractal, fractal market hypothesis is confirmed	"Heavy tails" of distribution	Persistent series, positive correlation within changes in the value of assets	Black noise	Trend is present in the market

Table 2: Hurst exponent estimation for different variants of data filtration

	Unfiltered	SMA (2)	SMA (5)	WMA (2)	WMA (5)	Irwin
UX (daily returns)	0.67	0.69	0.73	0.69	0.73	0.70
UX (mixed data)	0.54	0.53	0.54	0.52	0.53	0.49
Random data	0.51	0.56	0.63	0.55	0.61	0.52
Mixed random data	0.53	0.52	0.51	0.51	0.51	0.54

Table 3: Hurst exponent estimates with standard methodology (aliquot number of groups) and modified (aliquant number of groups) for different data sets

	UX (close)	Random	UX (SMA 5)	UX (WMA 5)	UX (Irving)
Standard	0.67	0.51	0.73	0.73	0.7
Modified	0.7	0.55	0.78	0.77	0.73

Table 4: Full-sample analysis of Ukrainian stock market persistence

	PFTS	UX
Hurst exponent	0,665	0,667
Hurst exponent (mixed data)	0,53	0,54

Table 5: Estimates of d and 95% confidence intervals

Series: UX.DAT	No regressors	An intercept	A linear time trend
White noise	0.999 (0.966, 1.036)	**1.124** **(1.091, 1.162)**	1.123 (1.089, 1.162)
AR (1)	1.371 (1.311, 1.452)	**1.100** **(1.049, 1.161)**	1.099 (1.048, 1.152)
Bloomfield-type	0.994 (0.944, 1.062)	**1.099** **(1.056, 1.151)**	1.098 (1.051, 1.151)
SQUARED returns	No regressors	An intercept	A linear time trend
White noise	0.278 (0.245, 0.316)	**0.276** **(0.243, 0.314)**	0.274 (0.241, 0.313)
AR (1)	0.266 (0.218, 0.322)	**0.261** **(0.209, 0.311)**	0.257 (0.203, 0.311)
Bloomfield-type	0.254 (0.211, 0.328)	**0.251** **(0.207, 0.334)**	0.249 (0.199, 0.334)
ABSOLUTE returns	No regressors	An intercept	A linear time trend
White noise	0.268 (0.239, 0.300)	**0.259** **(0.229, 0.292)**	0.258 (0.228, 0.291)
AR (1)	0.326 (0.281, 0.372)	**0.311** **(0.264, 0.363)**	0.309 (0.261, 0.362)
Bloomfield-type	0.334 (0.291, 0.424)	**0.313** **(0.261, 0.376)**	0.312 (0.261, 0.375)

Figure 1 – Periodisation of financial crisis 2007-2009

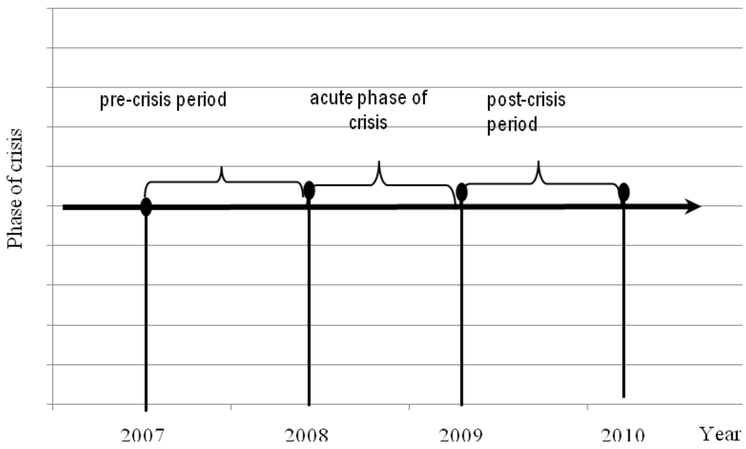


Figure 2 – Dynamics of the VIX Index in 2007-2010

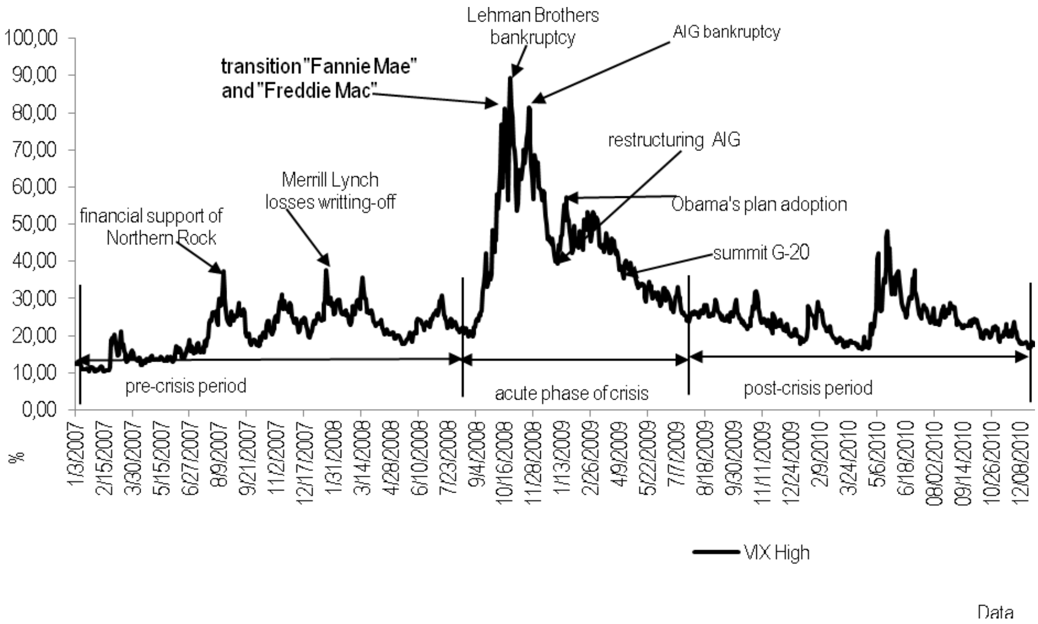
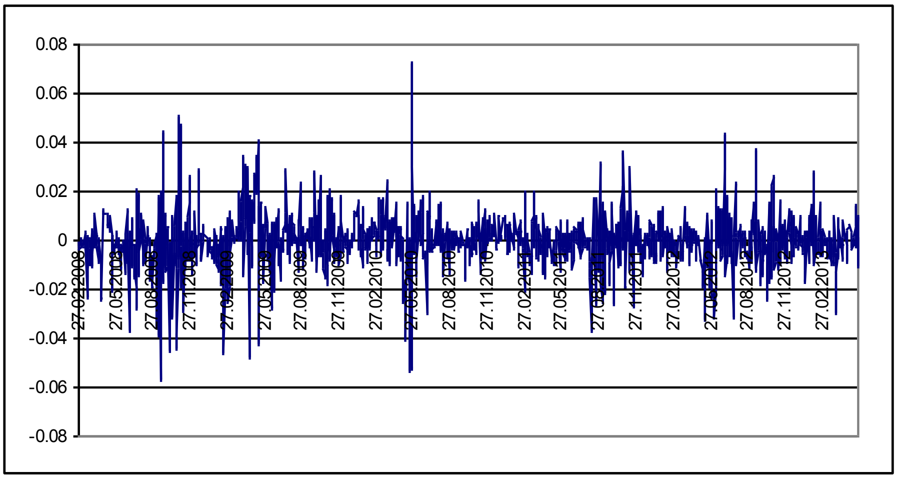


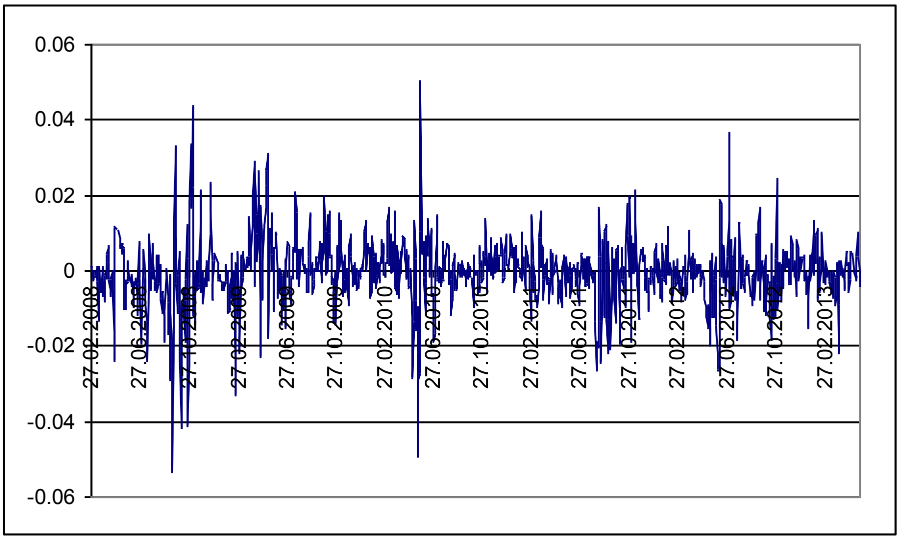
Figure 3

Visual interpretation of filtered and unfiltered UX data: SMA filtration

a) Unfiltered UX data



b) Filtered with SMA 2



c) Filtered with SMA 5

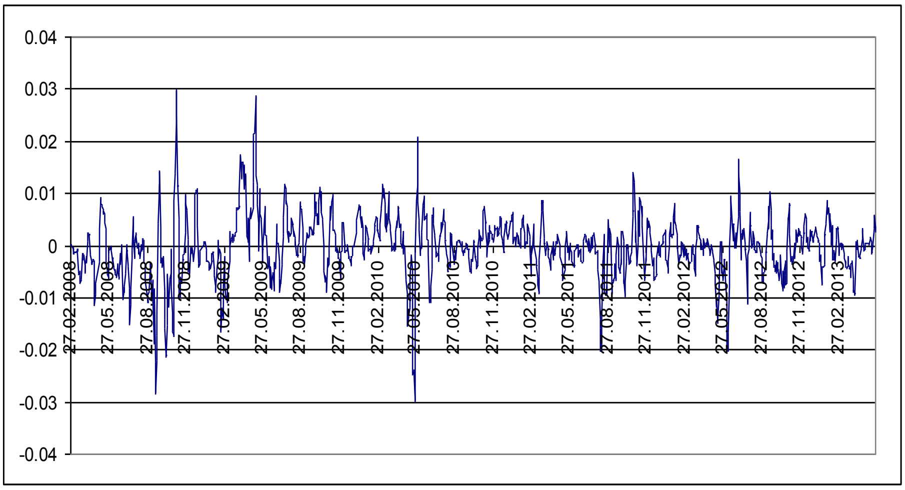
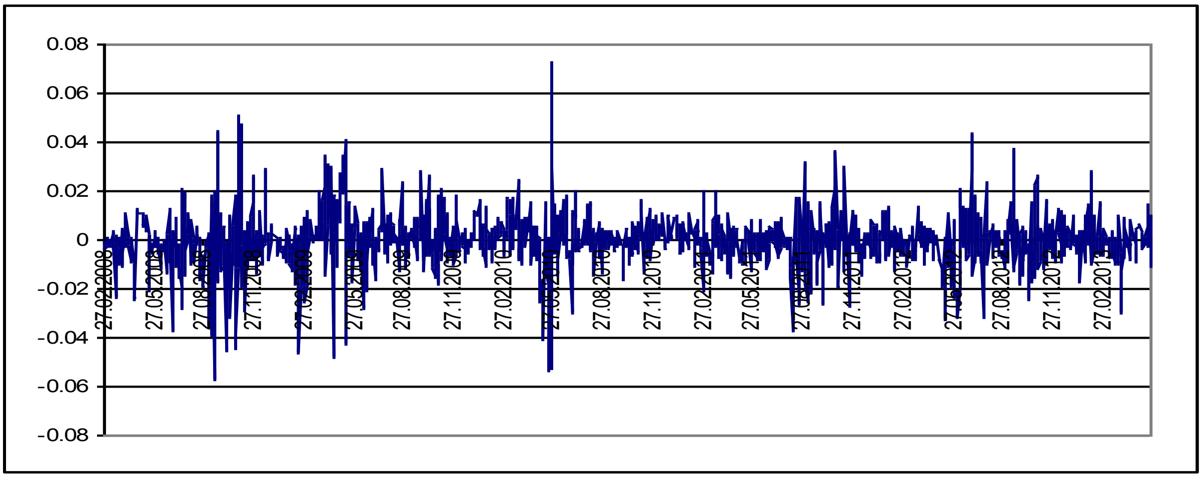


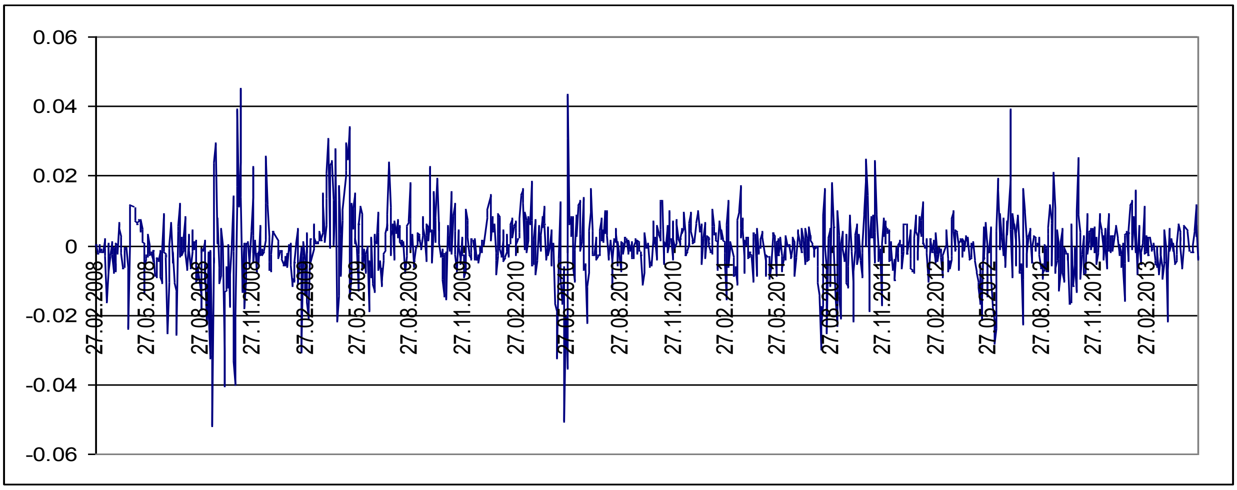
Figure 4

Visual interpretation of filtered and unfiltered UX data: WMA filtration

a) Unfiltered UX data



b) Filtered with WMA 2



c) Filtered with WMA 5

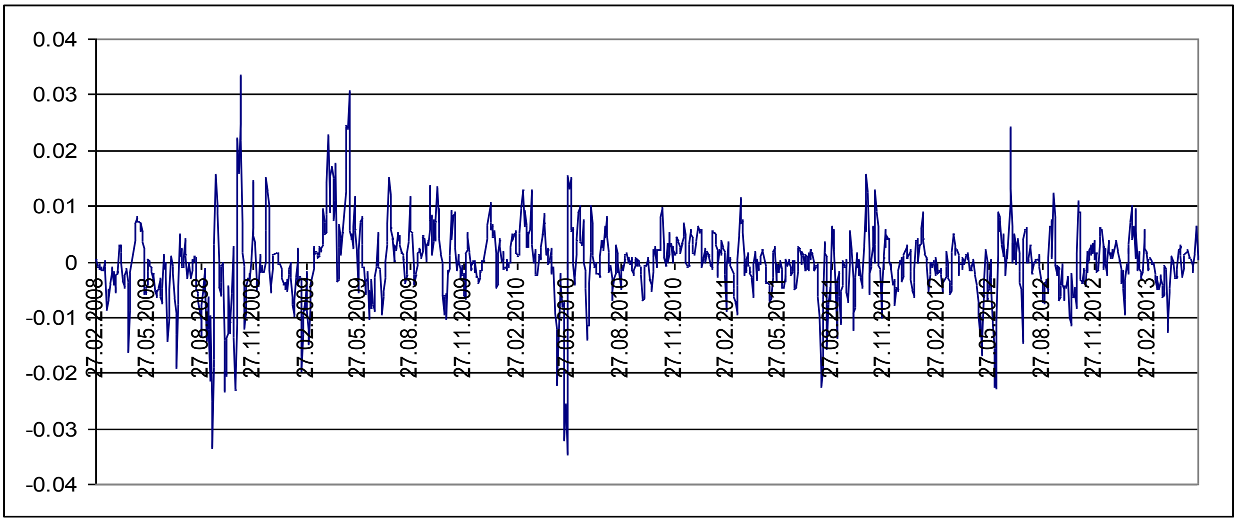
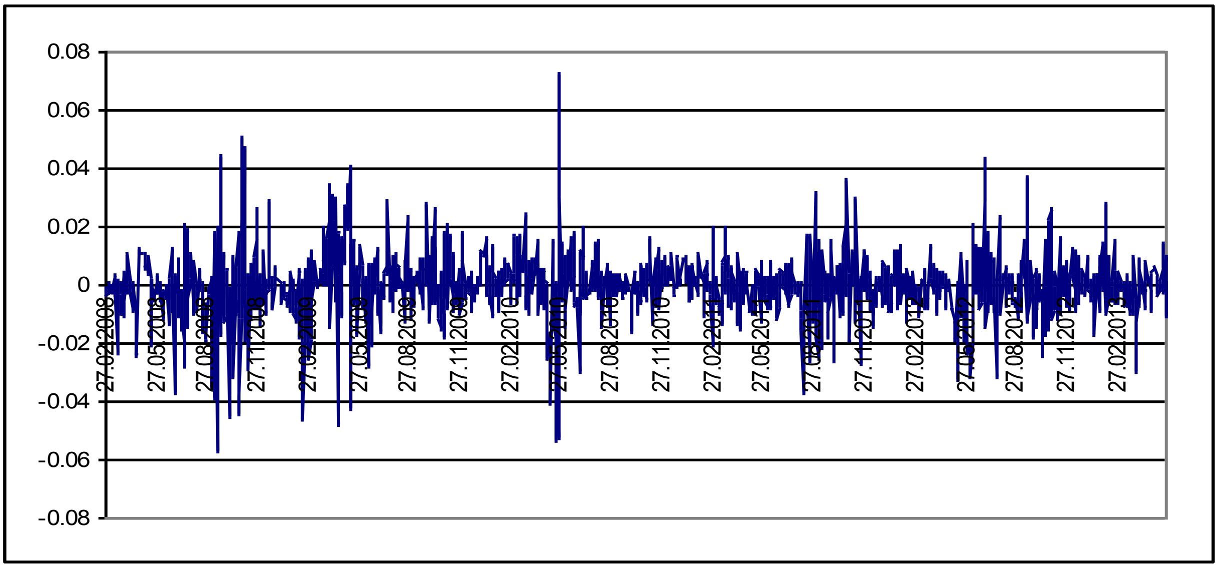


Figure 5

Visual interpretation of filtered and unfiltered UX data: Irwin filtration

a) Unfiltered UX data



b) Filtered with Irwin

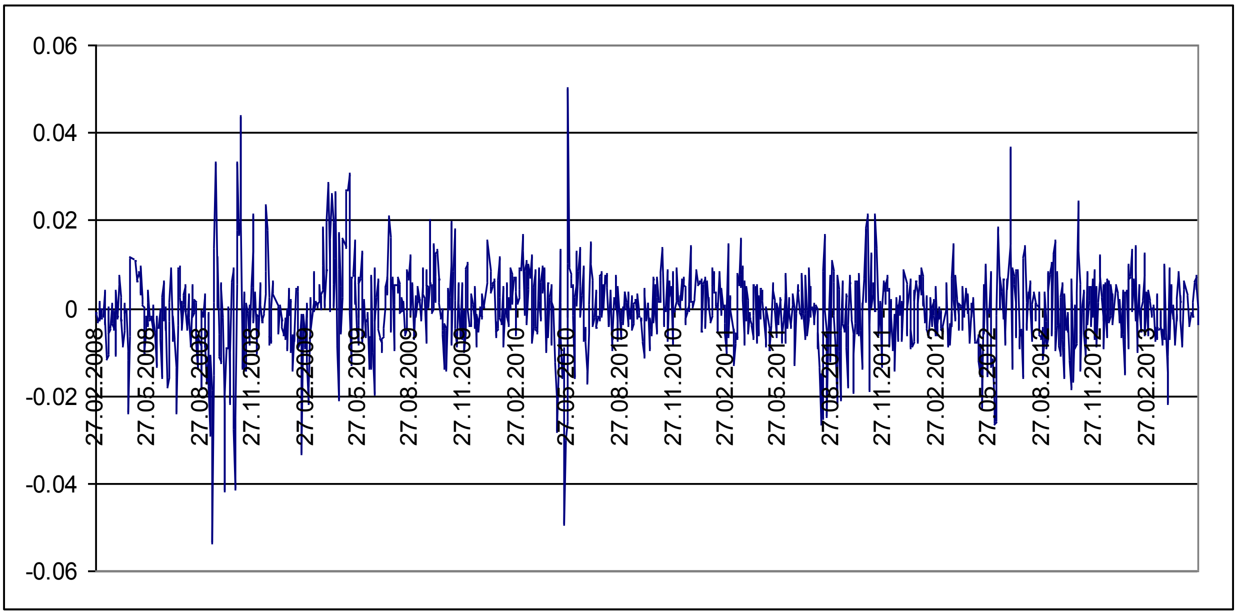
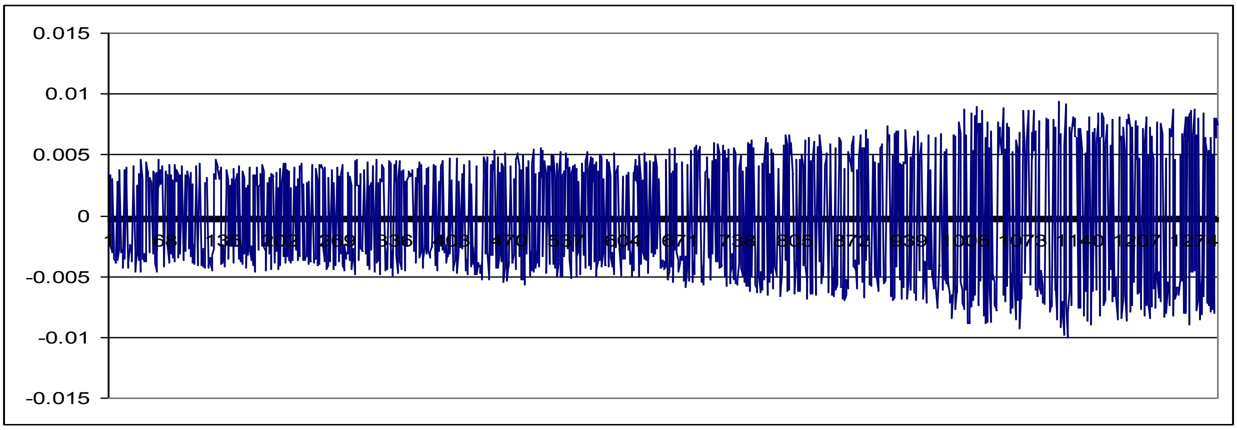


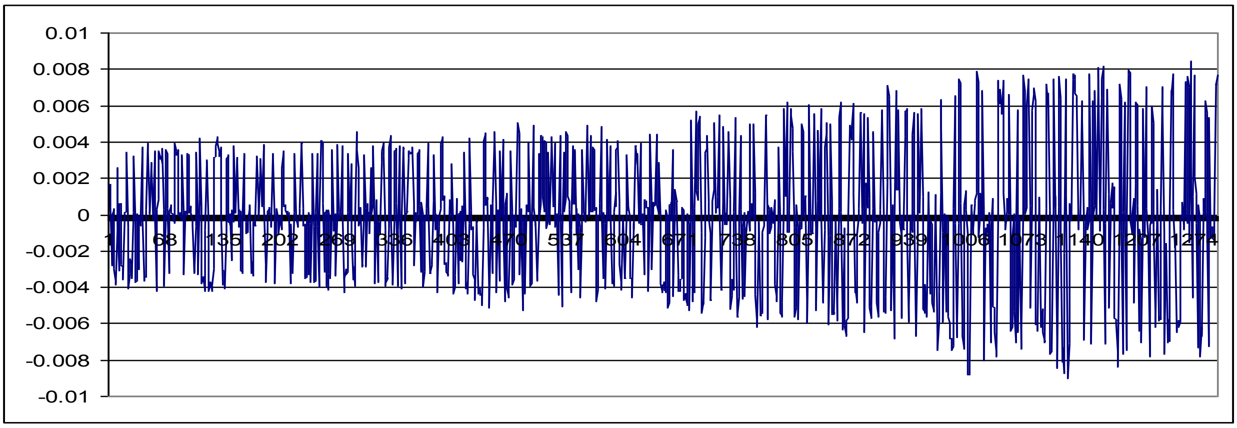
Figure 6

Visual interpretation of filtered and unfiltered randomly generated data: SMA filtration

a) Randomly generated data



b) Randomly generated data filtered with SMA 2



c) Randomly generated data filtered with SMA 5

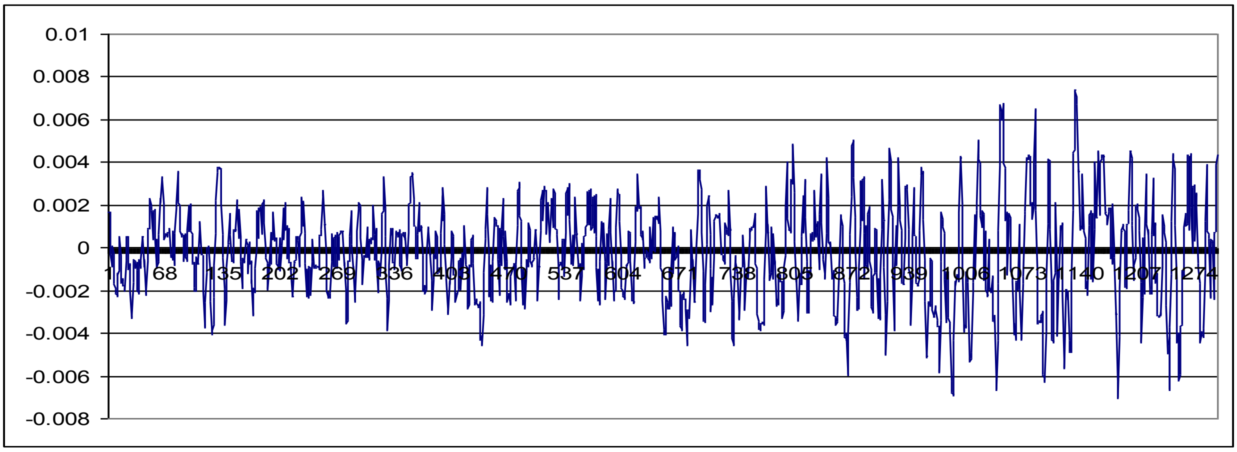
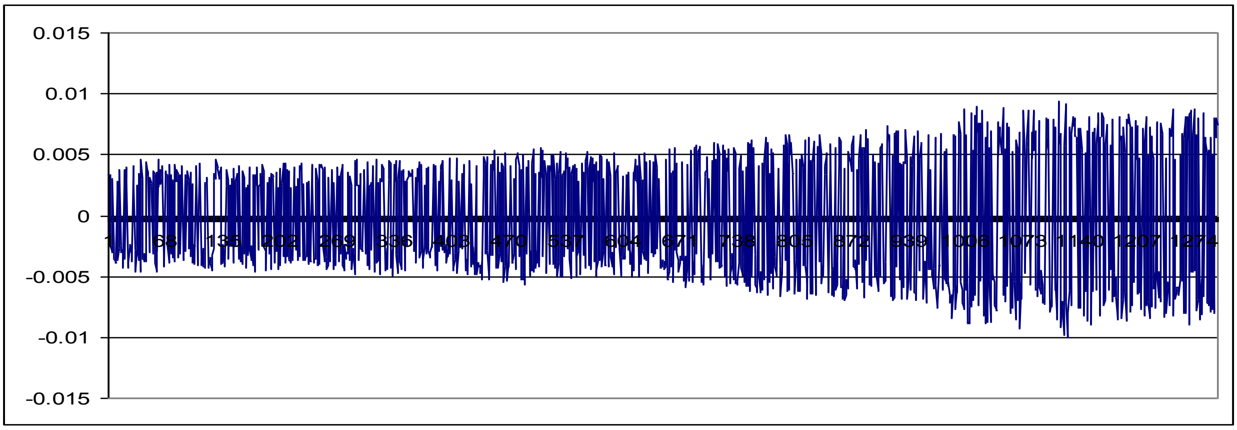


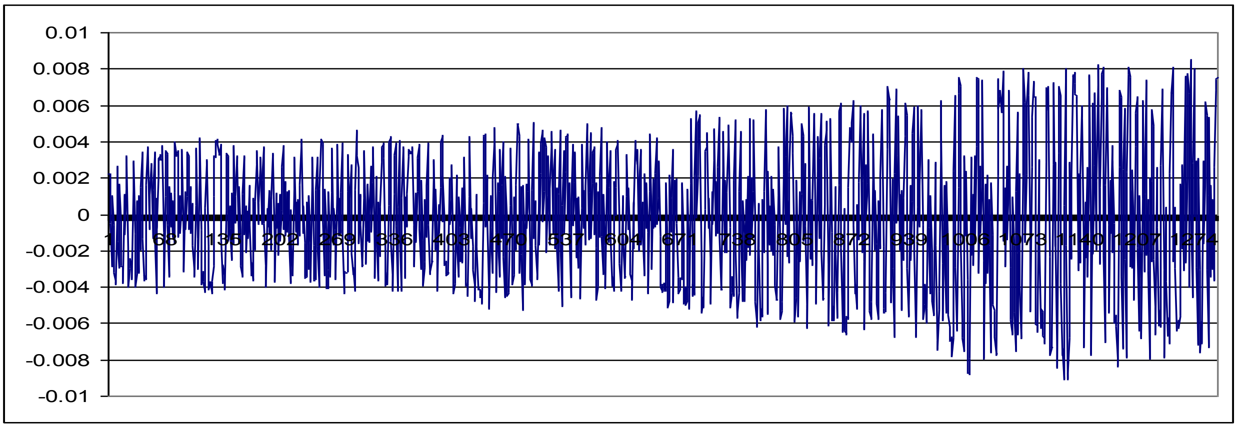
Figure 7

Visual interpretation of filtered and unfiltered randomly generated data: WMA filtration

a) Randomly generated data



b) Randomly generated data filtered with WMA 2



c) Randomly generated data filtered with WMA 5

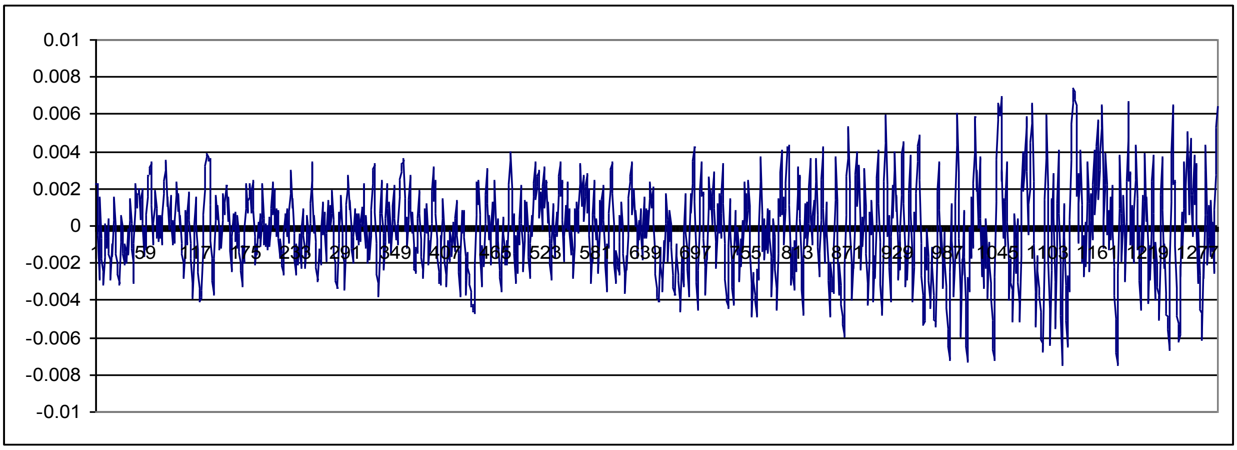
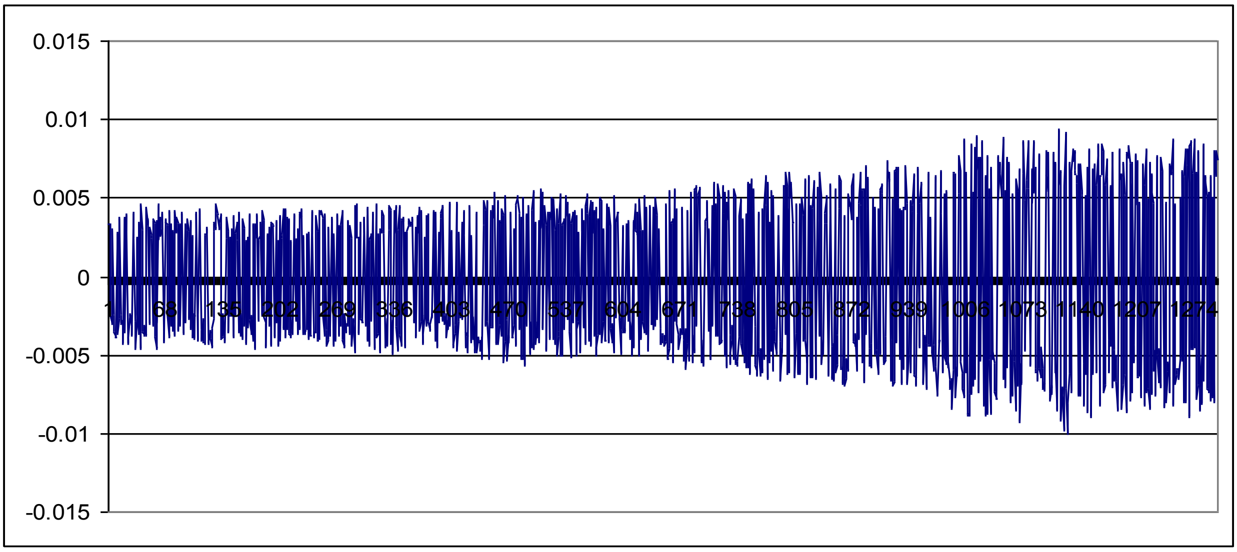


Figure 8

Visual interpretation of filtered and unfiltered randomly generated data: Irwin filtration

a) Randomly generated data



b) Randomly generated data filtered with Irwin

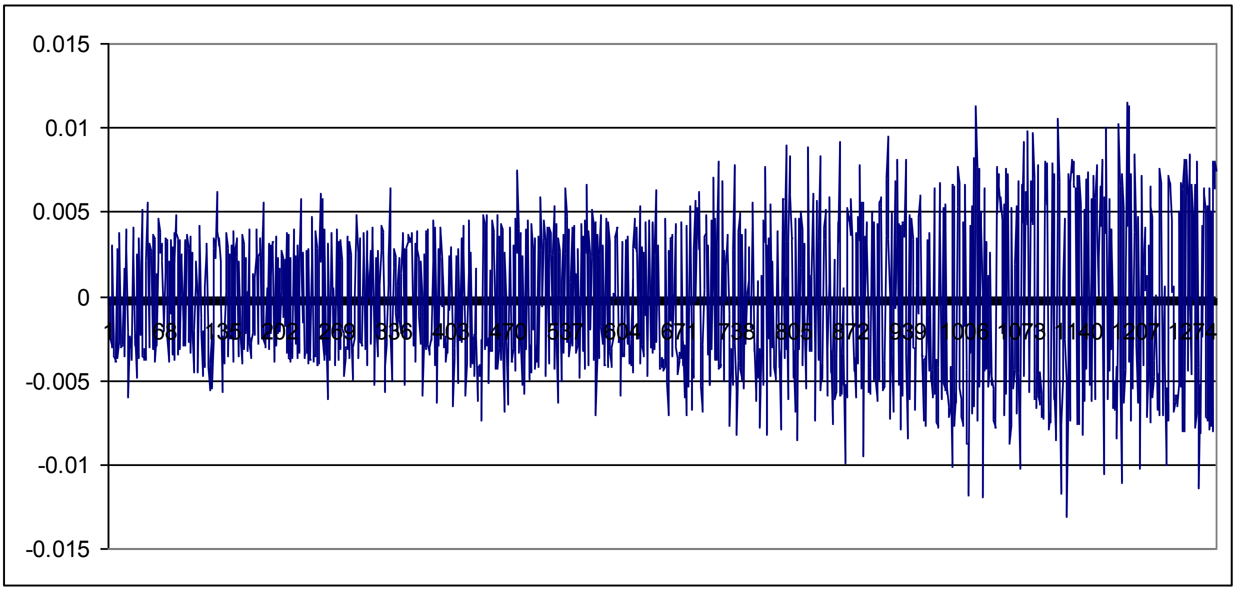


Figure 9: Dynamics of Hurst exponent during 2003-2013 (calculated on PFTS data with “data window” = 300, shift = 10)

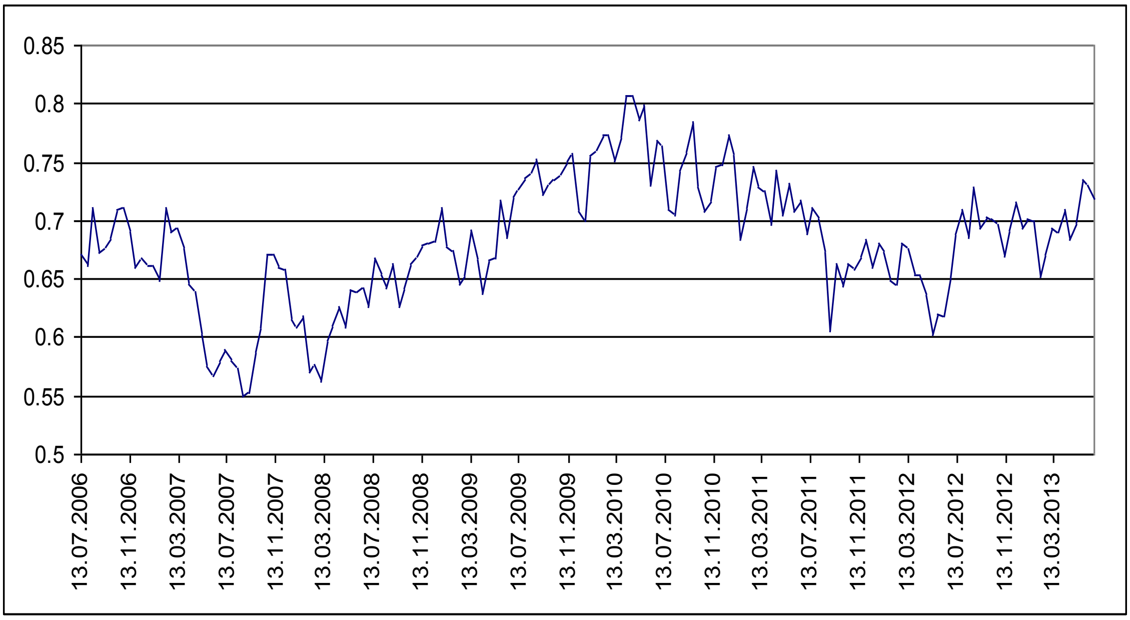
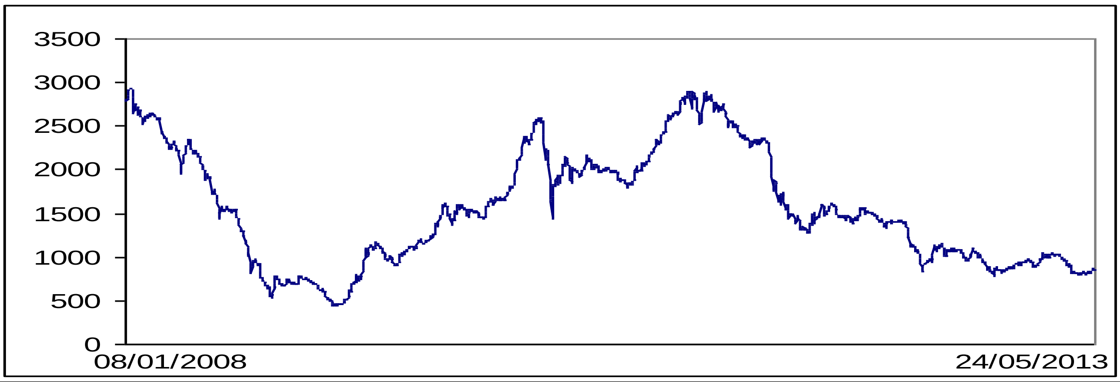
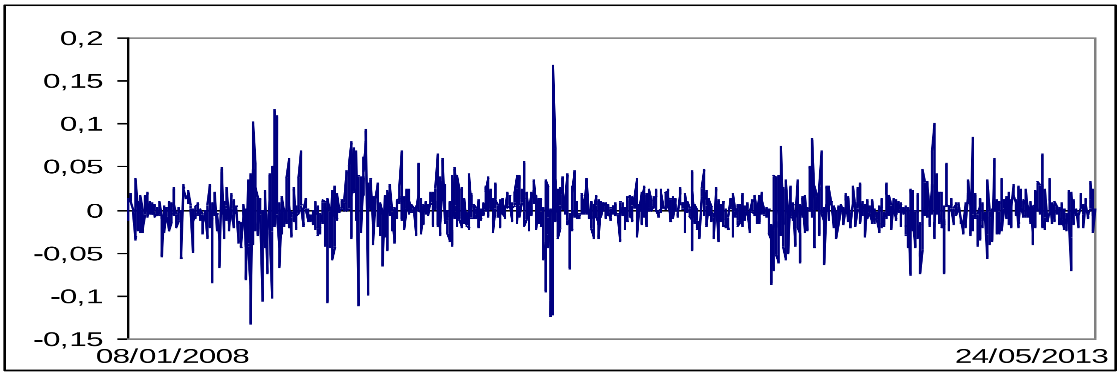


Figure 10: Time series UX data

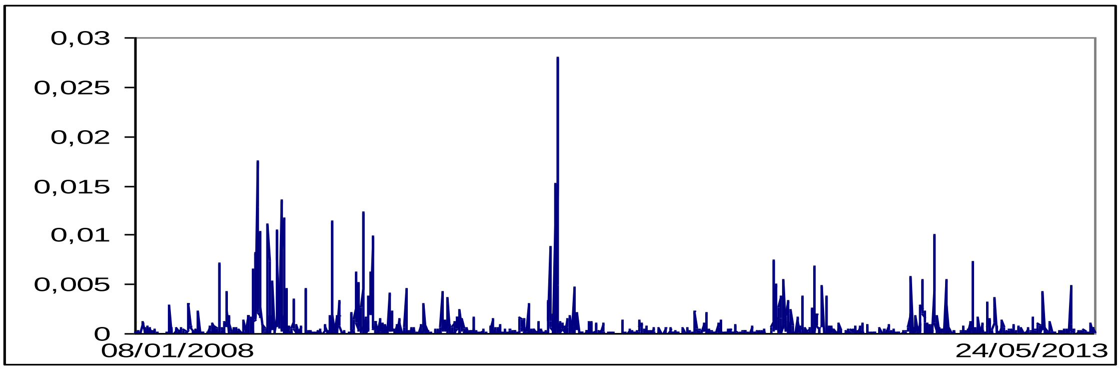
ai) Stock market prices



b) Stock market returns



c) Squared market returns



d) Absolute market returns

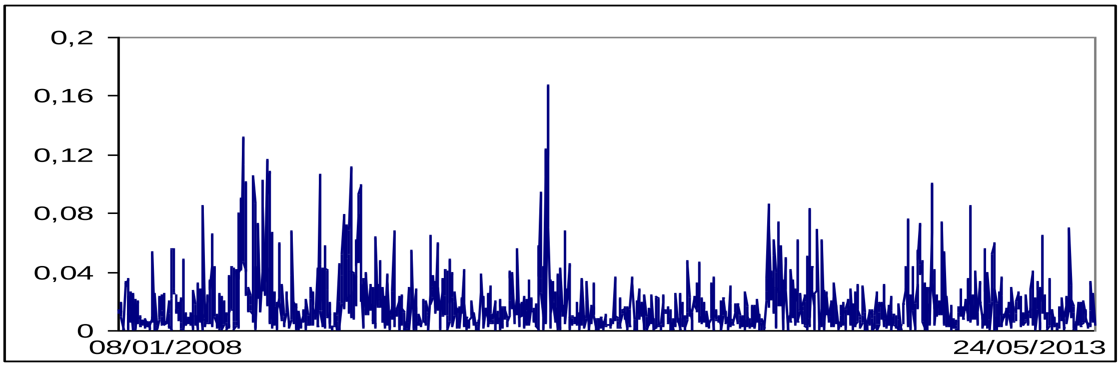
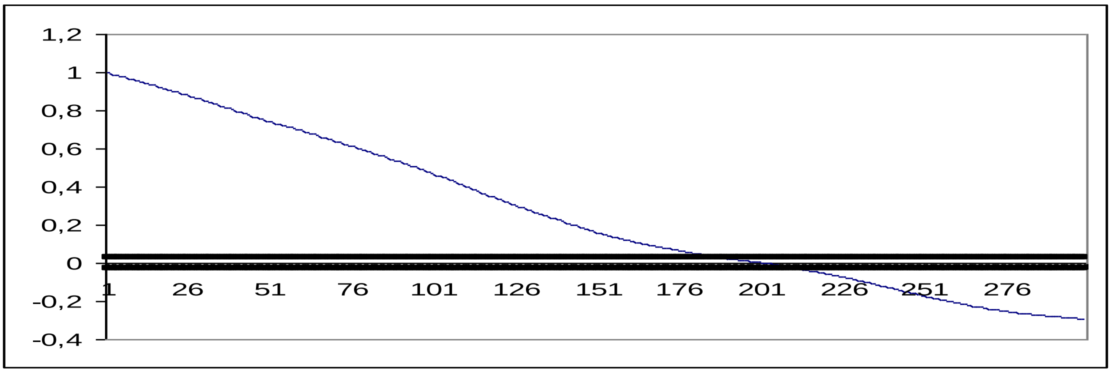
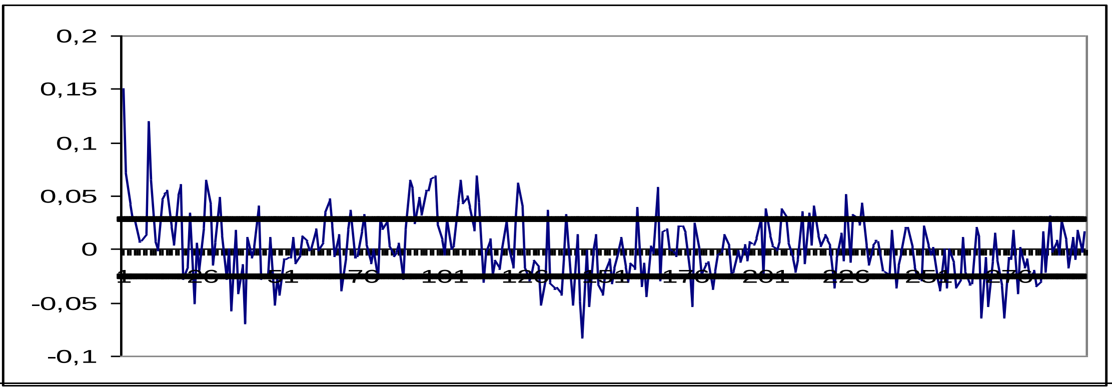


Figure 11: Correlograms

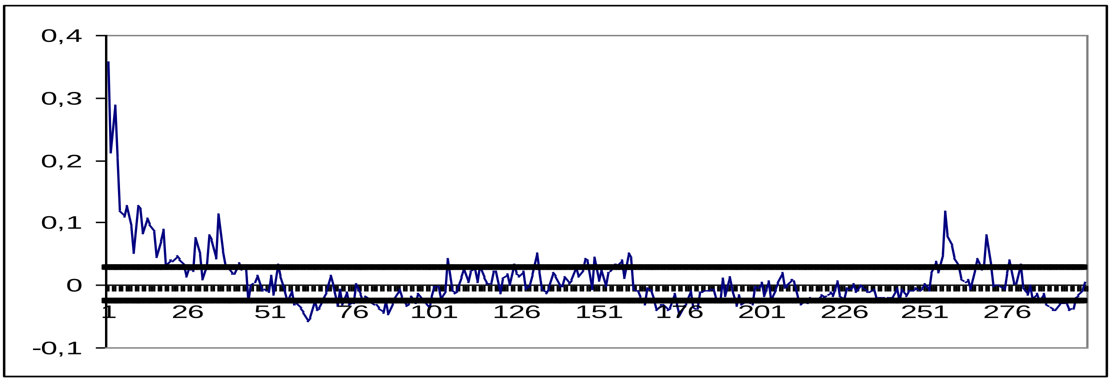
a) Stock market prices



b) Stock market returns



c) Squared market returns



d) Absolute market returns

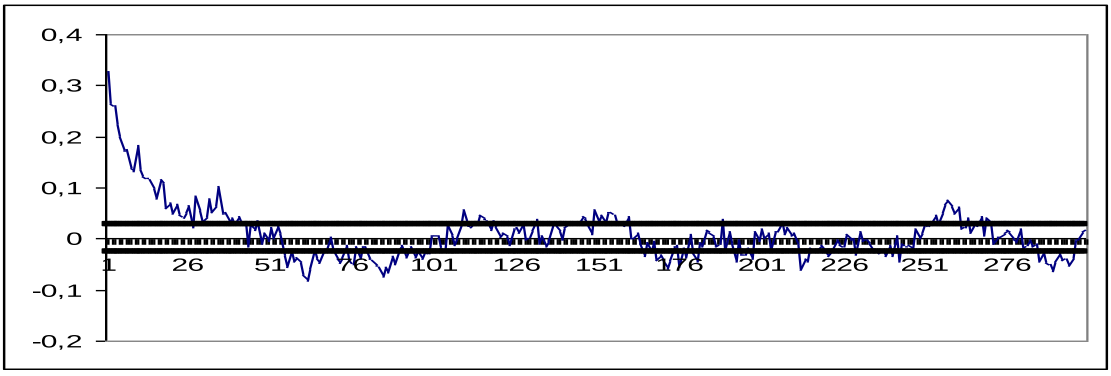


Figure 12: Periodograms

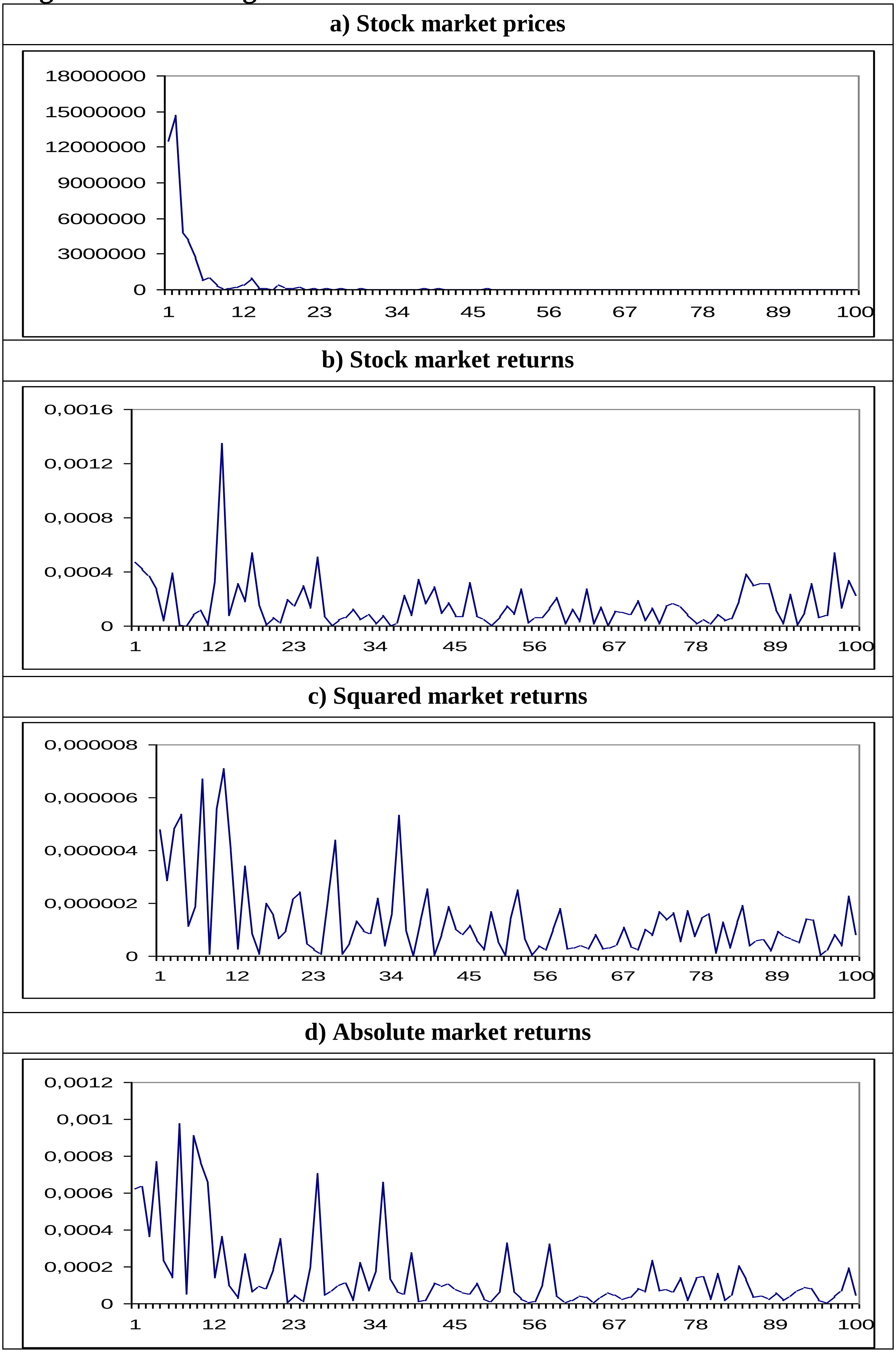
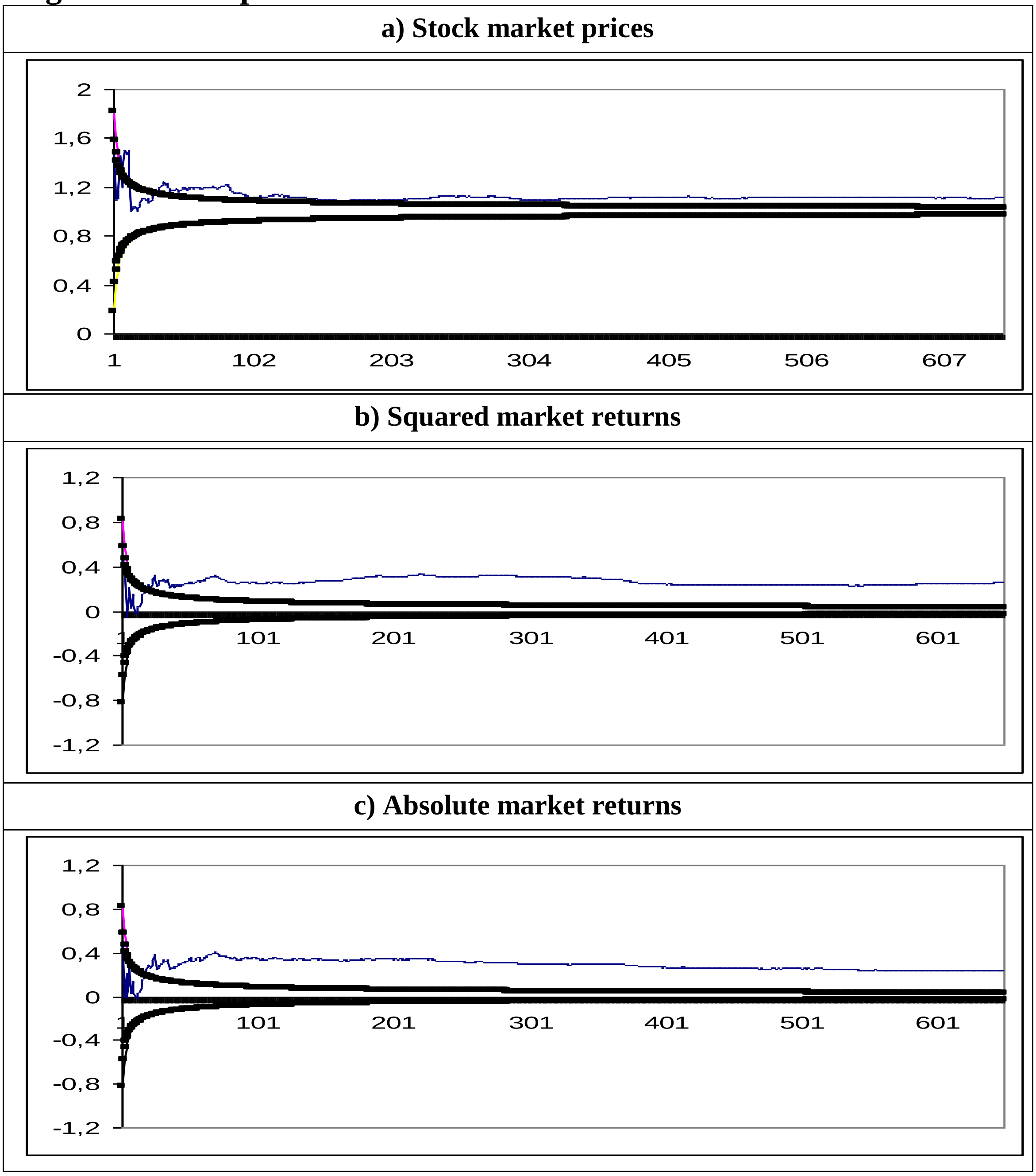


Figure 13: Semiparametric Whittle estimates of d



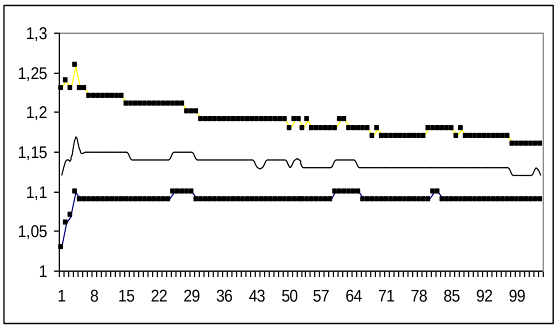
The horizontal axis concerns the bandwidth parameter while the vertical one refers to the estimated value of d.

The bold lines correspond to the 95% confidence interval corresponding to the I(1) (a) and the I(0) (b and c) hypotheses.

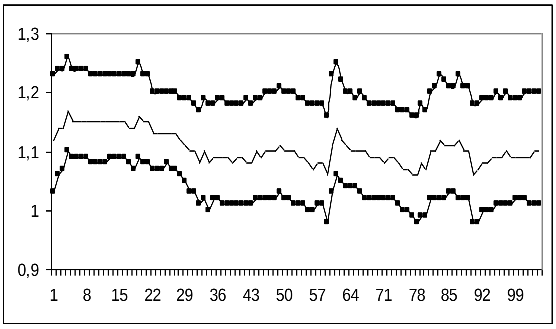
Figure 14: Stability results based on recursive estimates

a) Stock market prices

Adding 10 observation each time

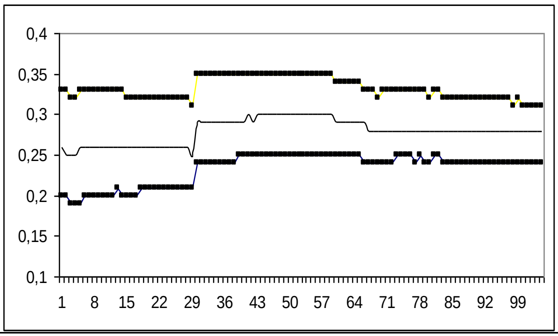


Moving windows of 300 observations

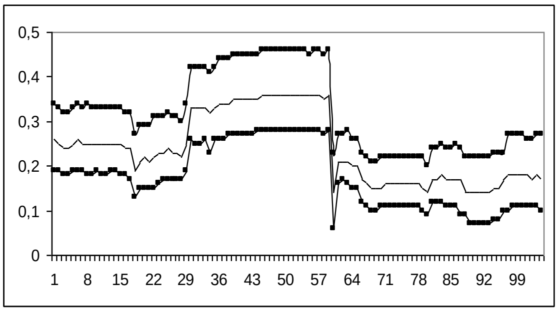


b) Squared market returns

Adding 10 observation each time

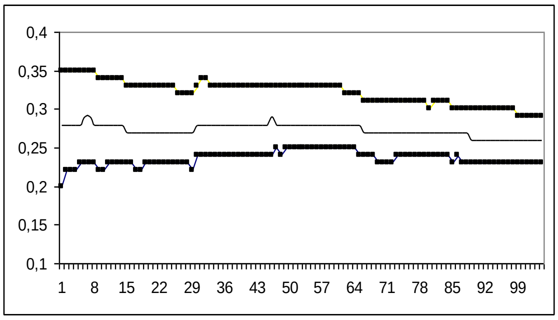


Moving windows of 300 observations



c) Absolute market returns

Adding 10 observation each time



Moving windows of 300 observations