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Static output-feedback control for uncertain systems under input saturation and persistent disturbance[☆]

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ABSTRACT

This paper addresses the static output-feedback (SOF) control problem for discrete-time systems subject simultaneously to input saturation, non-vanishing disturbances, and uncertainties described in a polytopic form. Level sets of the Lyapunov function are used to characterize the estimation of the domain of admissible initial states for the system and also to determine a region where the states of the closed-loop system will be confined in the presence of persistent disturbance. The proposed design conditions are formulated as Linear Matrix Inequalities (LMIs). Numerical experiments demonstrate the effectiveness of the method in providing SOF controllers that ensure the states of the closed-loop uncertain system are confined to a bounded region despite the presence of persistent disturbances.

1. Introduction

The static output-feedback (SOF) control problem has received considerable attention over recent decades (Sadabadi & Peaucelle, 2016). Some methods impose a specific structure on the output matrix and use equality constraints to derive design conditions, which may lead to conservative results. This becomes even more challenging when the system is uncertain, as the structure imposed on precisely known systems will be stronger in an uncertain domain. However, advances in robust control theory have enabled the design of SOF gains using linear matrix inequalities (LMIs) without using equality constraints and imposing a particular structure on the output matrix (Agulhari et al., 2010; Peixoto et al., 2021).

A persistent challenge in control is the presence of input saturation, which can induce undesired dynamics and even unstable behavior if not addressed properly in the design phase. The output-feedback control for discrete-time systems under input saturation was addressed in Peixoto et al. (2022) that did not account for external perturbations affecting the system. Some strategies for output-feedback control design for continuous-time systems under input saturation considered the presence of disturbances belonging to the class of $\mathcal{L}_2$ signals, and the presence of nonlinear rational systems (Lima et al., 2022). For discrete-time systems under input saturation, similar considerations for disturbances belonging to the class of $\mathcal{L}_2$ signals were addressed in Saifia et al. (2020) and Zheng and Wu (2008). However, the design of SOF gains that simultaneously account for input saturation, system uncertainties, and non-vanishing disturbances in discrete-time systems remains unexplored. These factors introduce strong, coupled nonlinearities that significantly complicate the synthesis problem, rendering conventional linear design approaches inadequate and making the search for feasible controller solutions particularly challenging. Even for state-feedback control, the method was recently discussed in Seuret and Tarbouriech (2024).

This paper considers the SOF control design problem for discrete-time systems with polytopic uncertainties, under input saturation and non-vanishing disturbances. Differently, from Seuret and Tarbouriech (2024), which designs state-feedback controllers, the output-feedback control is tackled, and a sector condition proposed in Nguyen et al. (2017) specially to tackle the SOF control problem is employed. Moreover, the control gain is recovered from a slack variable, instead of using the Lyapunov matrix. This is of particular importance since the use of the Lyapunov function to compute a robust gain for uncertain systems prevents us from employing parameter-dependent Lyapunov functions in the design conditions. The level sets of the Lyapunov function will be used to characterize the estimation of the domain of admissible initial states for the system. Furthermore, as the system is under the influence of a non-vanishing disturbance signal, the conditions will guarantee that the closed-loop trajectories converge to a region that is a subset of the domain of admissible initial states for the system. The

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objectives are to increase the set of admissible initial conditions and decrease the size of the attractor where the closed-loop states will be confined. The conditions are presented as parameter-dependent LMIs that can be readily cast into finite conditions. This work extends the state of the art by addressing, for the first time, the simultaneous presence of input saturation, uncertainties, and non-vanishing disturbances in discrete-time SOF control design. The proposed method provides tractable LMI conditions that ensure robustness and performance without imposing restrictive structural assumptions, thus broadening the applicability of SOF control in practical uncertain systems. Numerical experiments illustrate the efficiency of the SOF controller obtained with the proposed method. Both precisely known systems and systems under uncertainties will be considered in the examples.

Notation: $\mathbb{N}_{\leq m}$ is the set of natural numbers less than or equal to m. Capital letters denote matrices, while lowercase letters denote vectors. For a matrix X, X_j denotes its jth row and $X_{i,j}$ denotes the entry in the ith row and jth column. For a vector x (x_k), x_i ($x_{k,i}$) denotes its ith entry (at time k). The symbol $\star$ stands for symmetric blocks in matrices. trace(X) represents the sum of the elements on the main diagonal of the matrix X, $\mathrm{He}(X) = X + X^\top$, $\mathcal{E}(W,\beta) = \left\{x \in \mathbb{R}^n, \quad x^\top W x \leq \beta^{-1}\right\}$, blkdiag $(X_1, \ldots, X_N)$ is used to represent a block diagonal matrix composed by the matrices $X_1, \ldots, X_N$.

2. Problem formulation

Consider the following discrete-time uncertain system subject to input saturation and persistent perturbation

$$\begin{aligned} x_{k+1} &= A(\alpha)x_k + B(\alpha)\mathrm{sat}(u_k) + w_k, \\ y_k &= C(\alpha)x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, $w_k \in \mathbb{R}^{n_w}$, and $y_k \in \mathbb{R}^{n_y}$ represent respectively the state, the control input, the disturbance, and the output vectors of the system at a given time k.

The uncertain matrices belong to a polytopic domain parameterized by the vector of time-invariant parameters $\alpha \in \Lambda_N$, being defined as

$$\begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & 0 \end{bmatrix} = \sum_{i=1}^{N} \alpha_i \begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix}, \quad \alpha \in \Lambda_N,$$

where Λ_N denotes the unit simplex with N vertices, that is $\Lambda_N = \left\{\alpha \in \mathbb{R}^N : \sum_{i=1}^{N} \alpha_i = 1,\ \alpha_i \geq 0,\ i \in \mathbb{N}_{\leq N}\right\}$. The system is perturbed by the unknown signal $w_k \in \mathbb{R}^{n_x}$ assumed to be bounded but non-vanishing, leading to the following assumption, also considered in Seuret and Tarbouriech (2024).

Assumption 1. The norm of the disturbance satisfies $w_k^\top w_k < \lambda$, for a positive scalar λ, for all $k \geq 0$. The set $\Omega_\lambda := \left\{w_k \in \mathbb{R}^{n_x}, \quad w_k^\top w_k \leq \lambda\right\}$, describes the region where the persistent disturbance belongs.

Moreover, the control input is subject to the component-wise saturation map $\mathrm{sat}(\cdot) : \mathbb{R}^{n_u} \to \mathbb{R}^{n_u}$ defined as

$$\mathrm{sat}\left(u_{k,l}\right) = \mathrm{sign}\left(u_{k,l}\right)\min\left(\left|u_{k,l}\right|, \bar{u}_l\right), \ \forall l \in \mathbb{N}_{\leq n_u},$$

where $\bar{u}_l \in \mathbb{R}$, with $\bar{u}_l > 0$, is the maximum allowed bound of the lth control input component due to the actuator saturation.

2.1. Problem definition

Consider the following SOF controller

$$u_k = \mathcal{L}y_k, \tag{2}$$

where $\mathcal{L} \in \mathbb{R}^{n_u \times n_y}$ is the control gain to be designed. The control input is subject to a dead-zone nonlinearity, defined as $\psi(u_k) = u_k - \mathrm{sat}(u_k)$, which represents the difference between the applied control signal and its saturated value. Taking this into account, the resulting closed-loop system can be written as

$$x_{k+1} = \mathcal{A}(\alpha)x_k - B(\alpha)\psi(u_k) + w_k, \tag{3}$$

with $\mathcal{A}(\alpha) = A(\alpha) + B(\alpha)\mathcal{L}C(\alpha)$. The dead-zone nonlinearity can be handled by employing the following lemma.

Lemma 1 (Nguyen et al., 2017). *Consider the positive definite diagonal matrix $T \in \mathbb{R}^{n_u \times n_u}$ and the matrix $G \in \mathbb{R}^{n_u \times n_x}$. Let the set*

$$\mathcal{D}_u = \left\{x_k \in \mathbb{R}^{n_x} : \left|\left(T^{-1}G\right)_l x_k\right| \leq \bar{u}_l\right\}. \tag{4}$$

If $x_k \in \mathcal{D}_u$, then for any $u_k \in \mathbb{R}^{n_u}$, one has

$$\psi(u_k)^\top T\left[u_k - \psi(u_k) - T^{-1}Gx_k\right] \geq 0. \tag{5}$$

The positive definite diagonal matrix T plays a crucial role in the formulation. In particular, it appears both in the definition of the set $\mathcal{D}_u$ and in the cone-bounded sector condition (5), which is used to characterize the dead-zone nonlinearity. Its presence in both expressions is essential for enabling the linearization of the problem.

The following problem is addressed in this paper:

($\mathcal{P}$) - When $w \neq 0$, design a SOF controller as in (2) such that the trajectories of the closed-loop system (3) initiated within the set of admissible initial conditions given by the level set $\mathcal{E}(P(\alpha), 1) \subseteq \mathcal{D}_u$ will converge to the set $\mathcal{E}(P(\alpha), \varepsilon) \subseteq \mathcal{E}(P(\alpha), 1)$, where the matrix $P(\alpha) > 0$, and $\varepsilon > 1$, are design variables. In addition, optimization procedures will be introduced to increase the estimation of the set of admissible initial conditions $\mathcal{E}(P(\alpha), 1)$, and minimize the region $\mathcal{E}(P(\alpha), \varepsilon)$ where the closed-loop trajectories will be confined.

3. Main results

Theorem 1. *Given the scalars $\mu \in (0, 1)$, $\lambda > 0$, and ξ, if there exist parameter-dependent matrices $X_1(\alpha), X_2(\alpha) \in \mathbb{R}^{n_x \times n_x}$, $X_3(\alpha) \in \mathbb{R}^{n_y \times n_x}$, $X_4(\alpha) \in \mathbb{R}^{n_u \times n_x}$, $M_1(\alpha)$, $M_2(\alpha) \in \mathbb{R}^{n_x \times n_y}$, $M_3(\alpha) \in \mathbb{R}^{n_y \times n_y}$, $M_4(\alpha) \in \mathbb{R}^{n_u \times n_y}$, $X \in \mathbb{R}^{n_u \times n_u}$, $G \in \mathbb{R}^{n_u \times n_x}$, $L \in \mathbb{R}^{n_u \times n_y}$, positive definite matrix $P(\alpha) \in \mathbb{R}^{n_x \times n_x}$, a diagonal matrix $T \in \mathbb{R}^{n_u \times n_u}$, and a scalar $0 < \kappa < 1$ such that the following conditions hold for all $\alpha \in \Lambda_N$*

$$X + X^\top > 0, \tag{6}$$

$$\begin{bmatrix} P(\alpha) & G_l^\top \\ \star & \bar{u}_l^2\left(2T_{l,l} - 1\right) \end{bmatrix} \geq 0, \quad l \in \mathbb{N}_{\leq n_u}, \tag{7}$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & X_1(\alpha) \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} & -X_2(\alpha)B(\alpha) & X_2(\alpha) \\ \star & \star & \Phi_{33} & \Phi_{34} & -X_3(\alpha)B(\alpha) & X_3(\alpha) \\ \star & \star & \star & \Phi_{44} & \Phi_{45} & X_4(\alpha) \\ \star & \star & \star & \star & -2T & 0 \\ \star & \star & \star & \star & \star & -\frac{\kappa\mu}{\lambda}I \end{bmatrix} < 0, \tag{8}$$

with

$$\Phi_{11} = (\mu - 1)P(\alpha) + \mathrm{He}\left(M_1(\alpha)C(\alpha) + X_1(\alpha)A(\alpha)\right),$$

$$\Phi_{12} = -X_1(\alpha) + A(\alpha)^\top X_2(\alpha)^\top + C(\alpha)^\top M_2(\alpha)^\top,$$

$$\Phi_{13} = \xi B(\alpha)L - M_1(\alpha) + \left(X_3(\alpha)A(\alpha) + M_3(\alpha)C(\alpha)\right)^\top,$$

$$\Phi_{14} = -\xi B(\alpha)X + X_1(\alpha)B(\alpha) + A(\alpha)^\top X_4(\alpha)^\top + C(\alpha)^\top M_4(\alpha)^\top,$$

$$\Phi_{15} = -X_1(\alpha)B(\alpha) - G^\top,$$

$$\Phi_{22} = P(\alpha) - X_2(\alpha) - X_2(\alpha)^\top,$$

$$\Phi_{23} = B(\alpha)L - M_2(\alpha) - X_3(\alpha)^\top,$$
$$\Phi_{24} = -B(\alpha)X + X_2(\alpha)B(\alpha) - X_4(\alpha)^\top,$$
$$\Phi_{33} = -M_3(\alpha) - M_3(\alpha)^\top,$$
$$\Phi_{34} = \xi L^\top + X_3(\alpha)B(\alpha) - M_4(\alpha)^\top,$$
$$\Phi_{44} = \mathrm{He}\left(-\xi X + X_4(\alpha)B(\alpha)\right),$$
$$\Phi_{45} = T - X_4(\alpha)B(\alpha),$$

then, the SOF control gain that solves the problem $(\mathcal{P})$ *can be computed as* $\mathcal{L} = X^{-1}L$.

Proof. Since (8) is a strict inequality, if it holds, there exists a small $\gamma > 0$ such that

$$\Phi + \gamma\Psi < 0, \tag{9}$$

where $\Psi = \mathrm{blkdiag}\left(-P(\alpha), 0, 0, 0, 0, \frac{1}{\lambda}I\right)$. Moreover, if $0 < \mu_3 < \gamma$, then (9) also ensures that

$$\Phi + \tilde{\Psi} < 0, \tag{10}$$

where $\tilde{\Psi} = \mathrm{blkdiag}\left(-\mu_3 P(\alpha), 0, 0, 0, 0, \frac{\gamma}{\lambda}I\right)$. By substituting $\varepsilon = \kappa^{-1}$, the (6,6)-block of the matrix (10) becomes:

$$\frac{\gamma}{\lambda}I - \frac{\mu}{\lambda\varepsilon}I = -\frac{\mu}{\lambda\varepsilon}\left(1 - \frac{\gamma\varepsilon}{\mu}\right)I.$$

Define $\mu := \mu_1 \in (0, 1)$ and $\mu_2 := \frac{\mu}{\lambda\varepsilon}\left(1 - \frac{\gamma\varepsilon}{\mu}\right)$. Since $\gamma > 0$ is a small scalar, it will guarantee that $\mu_2 > 0$.

Note that (6) ensures that matrix X is nonsingular. Consider the following matrix (following Elimination Lemma arguments)

$$\mathcal{M} = \begin{bmatrix} I & 0 & 0 \\ A(\alpha) + B(\alpha)\mathcal{L}C(\alpha) & -B(\alpha) & I \\ C(\alpha) & 0 & 0 \\ \mathcal{L}C(\alpha) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Pre- and post-multiplying (10) by $\mathcal{M}^\top$ and $\mathcal{M}$, respectively, yields

$$\begin{bmatrix} \mathcal{R}(\alpha) & \star & \star \\ \mathcal{S}(\alpha) & B^\top(\alpha)P(\alpha)B(\alpha) - 2T & \star \\ P(\alpha)\mathcal{A}(\alpha) & -P(\alpha)B(\alpha) & P(\alpha) - \mu_2 I \end{bmatrix} < 0, \tag{11}$$

where $\mathcal{R}(\alpha) = (\mu_1 - \mu_3 - 1)P(\alpha) + \mathcal{A}(\alpha)^\top P(\alpha)\mathcal{A}(\alpha)$, $\mathcal{S}(\alpha) = -B(\alpha)^\top P(\alpha)\mathcal{A}(\alpha) - G + T\mathcal{L}C(\alpha)$. Pre- and post-multiplying (11) by $\begin{bmatrix} x_k^\top & \psi(u_k)^\top & w_k^\top \end{bmatrix}$ and its transpose, respectively, and using $V(x_k) = x_k^\top P(\alpha)x_k$ gives

$$V(x_{k+1}) - V(x_k) + (\mu_1 - \mu_3)\, x_k^\top P(\alpha)x_k - \mu_2 w_k^\top w_k$$
$$+2\psi(u_k)^\top T\left[u_k - \psi(u_k) - T^{-1}Gx_k\right] < 0.$$

Using Lemma 1, and $\Delta V = V(x_{k+1}) - V(x_k)$ one has

$$\Delta V + \mu_1 x_k^\top P(\alpha)x_k - \mu_2 w_k^\top w_k - \mu_3 x_k^\top P(\alpha)x_k < 0.$$

The choice for the parameters μ_1 and μ_2 ensures that $\mu_1\varepsilon^{-1} - \mu_2\lambda - \mu_3 > 0$. Note that

$$\mu\varepsilon^{-1} - \frac{\mu}{\varepsilon\lambda}\left(1 - \frac{\gamma\varepsilon}{\mu}\right)\lambda - \mu_3 > 0,$$

implies $\gamma - \mu_3 > 0$.

Thus, we can rewrite

$$\Delta V + \mu_1\left(x_k^\top P(\alpha)x_k - \varepsilon^{-1}\right) + \mu_2\left(\lambda - w_k^\top w_k\right)$$
$$+\mu_3\left(1 - x_k{}^\top P(\alpha)x_k\right) < 0. \tag{12}$$

Using the S-procedure arguments and the positivity of μ_1, μ_2, and μ_3, condition (12) guarantees that $\Delta V \leq 0$ for all (x_k, w_k) satisfying

$$\forall(x_k, w_k) \quad s.t. \begin{cases} x_k^\top P(\alpha)x_k \leq 1, & \to x \in \mathcal{E}(P(\alpha), 1), \\ x_k^\top P(\alpha)x_k \geq \varepsilon^{-1}, & \to x \notin \mathcal{E}(P(\alpha), \varepsilon), \\ w_k^\top w_k \leq \lambda. \end{cases}$$

In other words, inequality (12) ensures that the Lyapunov function decreases in the region between the two ellipsoids, i.e., when the state is inside $\mathcal{E}(P(\alpha), 1)$ but outside the attractor $\mathcal{E}(P(\alpha), \varepsilon)$, under bounded disturbances. This implies that trajectories starting in $\mathcal{E}(P(\alpha), 1)$ are driven toward $\mathcal{E}(P(\alpha), \varepsilon)$.

Since $\kappa < 1$, one has $\varepsilon > 1$, and consequently $\mathcal{E}(P(\alpha), \varepsilon) \subseteq \mathcal{E}(P(\alpha), 1)$. To show that $\mathcal{E}(P(\alpha), \varepsilon)$ is an invariant set, we now analyze the behavior of $V(x_k)$ once the state has reached this region. Using (12), we obtain

$$V(x_{k+1}) < V(x_k) - \mu_1\left(x_k^\top P(\alpha)x_k - \varepsilon^{-1}\right)$$
$$-\mu_2\left(\lambda - w_k^\top w_k\right) - \mu_3\left(1 - x_k{}^\top P(\alpha)x_k\right).$$

From Assumption 1, $\lambda - w_k^\top w_k \geq 0$, hence

$$V(x_{k+1}) \leq V(x_k) - \mu_1\left(V(x_k) - \varepsilon^{-1}\right) - \mu_3\left(1 - V(x_k)\right).$$

This simplifies to

$$V(x_{k+1}) \leq (1 - \mu_1 + \mu_3)V(x_k) + \mu_1\varepsilon^{-1} - \mu_3.$$

Considering the case where the state is already inside the attractor, i.e., $V(x_k) \leq \varepsilon^{-1}$, we obtain

$$V(x_{k+1}) \leq (1 - \mu_1 + \mu_3)\varepsilon^{-1} + \mu_1\varepsilon^{-1} - \mu_3,$$

or equivalently,

$$V(x_{k+1}) \leq \varepsilon^{-1} - \mu_3(1 - \varepsilon^{-1}).$$

As $\mu_3 > 0$ and $\varepsilon > 1$, one has $\mu_3(1 - \varepsilon^{-1}) > 0$, and it follows that

$$V(x_{k+1}) \leq \varepsilon^{-1}.$$

Hence, once the trajectories enter $\mathcal{E}(P(\alpha), \varepsilon)$, they remain inside it, establishing that this ellipsoid is invariant. Together with the previous argument showing $\Delta V < 0$ outside $\mathcal{E}(P(\alpha), \varepsilon)$, this proves convergence toward and invariance of the attractor set.

Finally, one can see that $(T_{l,l} - 1)^2 \geq 0$, therefore, it follows that $T_{l,l}^2 \geq (2T_{l,l} - 1)$. Based on this inequality, condition (7) implies

$$\begin{bmatrix} P(\alpha) & G_l^\top \\ \star & \bar{u}_l^2 T_{l,l}^2 \end{bmatrix} \geq 0, \quad l \in \mathbb{N}_{\leq n_u}, \ \alpha \in \Lambda_N. \tag{13}$$

Using the Schur complement Lemma in (13) yields

$$P(\alpha) - \frac{1}{\bar{u}_l^2 T_{l,l}^2} G_l^\top G_l \geq 0, \ \forall l \in \mathbb{N}_{\leq n_u}.$$

Pre- and post-multiplying the last equation by $x_k^\top$ and x_k, respectively, it follows that

$$\bar{u}_l^2 V(x_k) - x_k^\top T_{l,l}^{-2} G_l^\top G_l x_k \geq 0.$$

Since $V(x_k) \leq 1$, $\forall x_k \in \mathcal{E}(P(\alpha), 1)$, it follows that (4) holds and $\mathcal{E}(P(\alpha), 1) \subset \mathcal{D}_u$. ■

Remark 1. In this paper, the SOF control gain is recovered by using slack variables, instead of using the Lyapunov matrix. It allows the use of a parameter-dependent Lyapunov function that will help to increase the set of admissible initial conditions for the system. Moreover, unlike existing methods for SOF control under input saturation such as (Peixoto et al., 2022), this method addresses the presence of a bounded and persistent disturbance.

Remark 2. Once the scalars μ, λ and ξ are given, Theorem 1 presents a parameter-dependent LMI condition. In Section 4, the impact of these parameters on the condition is discussed. The parameter-dependent LMI condition presented in Theorem 1 must be tested for all $\alpha \in \Lambda_N$. By imposing a particular structure to the decision variables, for instance, affine or polynomial dependence on the uncertain parameter α, a set of finite LMI conditions can be obtained. Currently, existing computational tools such as ROLMIP (Agulhari et al., 2019) parser can handle the task of obtaining a finite set of LMIs by exploring the parameter-dependent structure of the decision variables. In this paper, we will focus on using affine structures for the parameter-dependent matrices. The precisely known case can be considered by simply removing the parameter dependency of all matrices in Theorem 1.

In addition to computing the SOF controller, two further objectives are pursued. The first seeks to maximize the set of admissible initial conditions, denoted as $\mathcal{E}(P(\alpha), 1)$, while the second aims to minimize the size of the attractor $\mathcal{E}(P(\alpha), \varepsilon)$, within which the closed-loop trajectories will be constrained. The parameters ρ_1 and ρ_2 are used to weight the two objectives according to the following optimization problem:

$$\mathcal{OP}_1 \equiv \begin{cases} \min & \rho_1\kappa + \rho_2\beta \\ \text{s.t.} & \text{Theorem 1}, \ \texttt{trace}(P(\alpha)) - \beta < 0. \end{cases} \tag{14}$$

The level set of the Lyapunov function given by $\mathcal{L}_V(\mu) = \{x_k \in \mathbb{R}^n : x^\top P(\alpha)x \leq \mu^{-1}\}$, can be characterized by the intersection of all ellipses in the uncertain domain: $\mathcal{L}_V(\mu) = \bigcap_{\forall\alpha\in\Lambda_N} \mathcal{E}(P(\alpha), \mu)$. It has been shown in Jungers and Castelan (2011) that for linear parameter-dependent matrices in the form of $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$ this problem can be equivalently addressed using a set of finite conditions $\mathcal{L}_V(\mu) = \bigcap_{i=1,\ldots,N} \mathcal{E}(P_i, \mu)$. This offline procedure can be tackled numerically or by using a second optimization problem. Given the matrices $P(\alpha)$ from the solution of $\mathcal{OP}_1$, find an ellipsoid $\mathcal{E}(H) \subseteq \mathcal{E}(P(\alpha))$ that satisfies:

$$\mathcal{OP}_2 \equiv \begin{cases} \min & \texttt{trace}(H) \\ \text{s.t.} & H - P(\alpha) \geq 0. \end{cases} \tag{15}$$

4. Numerical experiments

In the following examples, the purpose is to design SOF controllers for systems subject to persistent perturbation and input saturation. The precisely known case is considered in Example 1 whereas the uncertain case is explored in Example 2. In both cases the following procedure has been used to define the disturbance. Let $w_{0,k} = \begin{bmatrix}\sin(k) & \sin(k)\end{bmatrix}^\top$. Each vector $w_{0,k}$ is scaled to satisfy the norm constraint $\|w_k\|_2^2 \leq \lambda$:

$$w_k = \begin{cases} \dfrac{\sqrt{\lambda}}{\|w_{0,k}\|_2} w_{0,k}, & \text{if } \|w_{0,k}\|_2 > \sqrt{\lambda}, \\ w_{0,k}, & \text{otherwise}. \end{cases}$$

Example 1. Consider the following matrices for the discrete-time linear system (1) adapted from (Seuret & Tarbouriech, 2024).

$$A = \begin{bmatrix} 0.8 & 0.5 \\ -0.4 & 1.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

where $\bar{u} = 5$. The optimization problem (14) was solved using parameters $\rho_1 = 1$, $\rho_2 = 10^{-3}$, with λ set to 0.1 and 0.5, and μ values of 0.1, 0.3, and 0.6. A grid search over the parameter ξ was conducted within the range [0, 1000] at intervals of 0.5. The ξ value that minimized κ and at the same time maximized ε was then selected. Table 1 presents the values of ξ, β, and κ obtained from the optimization problem. Fig. 1 depicts the estimations of the region $\mathcal{E}(P, 1)$ (magenta line), the attractor (red line), and the set $\mathcal{D}_u$ (blue line) for each combination of (λ, μ). Furthermore, each sub-figure also depicts several time simulations that are initialized at the boundary of $\mathcal{E}(P, 1)$, all converging toward the attractor $\mathcal{E}(P, \varepsilon)$. From Fig. 1, it is possible to notice that as the noise magnitude λ increases, the size of the attractor also increases. Moreover, the approximation of the region of admissible initial conditions $\mathcal{E}(P, 1)$ remains relatively unchanged when λ increases. Furthermore, Fig. 1 also shows that the tuning parameter μ significantly influences the size of the obtained regions. For smaller values of μ, the approximation of the set of admissible initial conditions $\mathcal{E}(P, 1)$ is larger. However, the attractors' approximation is relatively poor because the trajectories are in smaller region around the origin. On the other hand, the optimization process yields a more accurate approximation of the attractor, though it significantly reduces the size of the set of admissible initial conditions. This demonstrates that the tuning parameter μ is a balancing factor between optimizing the estimations of the initial condition region and the attractor, which cannot be achieved simultaneously.

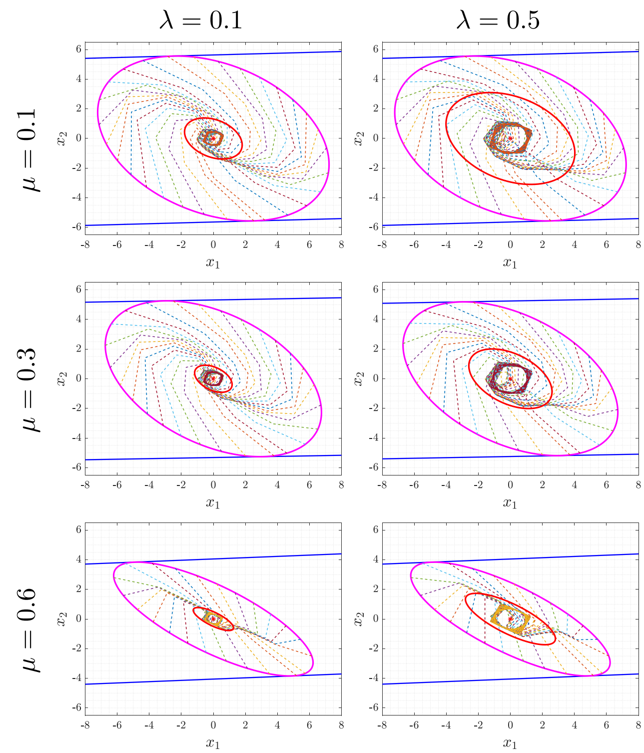

Fig. 1. Estimation of the initial condition region and the attractor obtained by solving the optimization problem (14) for different values of λ and μ.

Table 1
Parameters μ, λ, ξ, κ, and β.

λ	μ	ξ	κ	β
0.1	0.1	1000	0.0620	0.0572
0.1	0.3	200	0.0300	0.0713
0.1	0.6	2.5	0.0408	0.2255
0.5	0.1	1000	0.3102	0.0572
0.5	0.3	50	0.1501	0.0732
0.5	0.6	2.5	0.2038	0.2258

Example 2. Consider the following matrices for the discrete-time uncertain system (1):

$$A_1 = \begin{bmatrix} 1.1 & -0.3 \\ 0 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.2 & 0.5 \\ -0.5 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

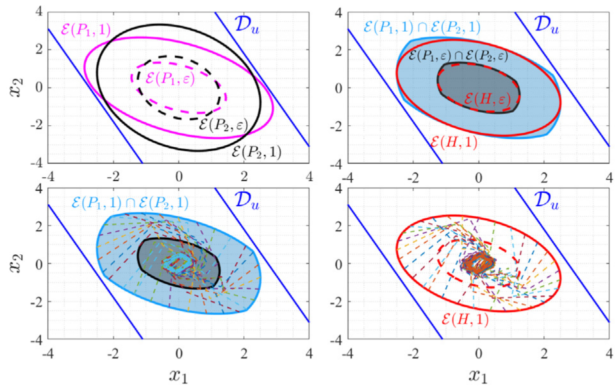

Fig. 2. Estimated regions for the set of admissible initial conditions and for the attractor.

$$B_2 = \begin{bmatrix} 1.4 \\ 0.6 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.1 & 0 \end{bmatrix},$$

where $\bar{u} = 2$. The optimization problem (14) was solved using parameters $\rho_1 = 1$ and $\rho_2 = 10^{-3}$, with $\lambda = 0.1$, $\mu = 0.2$. A grid search was conducted over the parameter ξ within the range $[0, 2]$ at intervals of 0.1. The value $\xi = 1.1$, which maximized ε, was then selected. For this setup, we obtained the following matrices:

$$P_1 = \begin{bmatrix} 0.1439 & 0.0637 \\ 0.0637 & 0.1699 \end{bmatrix}, \; P_2 = \begin{bmatrix} 0.1690 & 0.0300 \\ 0.0300 & 0.0954 \end{bmatrix},$$
$$L = -0.5637, \; T = 0.7937, \; G = \begin{bmatrix} -0.5811 & -0.2367 \end{bmatrix},$$

$\epsilon = 3.9939$, $\kappa = 0.2504$, and $\beta = 0.3137$. For comparison, by solving the optimization problem (15), we obtained the matrix $H = \begin{bmatrix} 0.1763 & 0.0538 \\ 0.0538 & 0.1729 \end{bmatrix}$.

Fig. 2 (top left) depicts the estimated regions $\mathcal{E}(P_1, 1)$ (magenta line) and $\mathcal{E}(P_2, 1)$ (black line), the attractors $\mathcal{E}(P_1, \varepsilon)$ (magenta dashed line) and $\mathcal{E}(P_2, \varepsilon)$ (black dashed line), and the boundary of $\mathcal{D}_u$ (blue line). Fig. 2 (top right) shows the results obtained for the intersection of the ellipses by using a numerical solution, and the optimization problem (15). As we can see, the solution from (15) is contained within the solution obtained numerically, that is, $\mathcal{E}(H, 1) \subset (\mathcal{E}(P_1, 1) \cap \mathcal{E}(P_1, 2)) \subset \mathcal{D}_u$. Furthermore, to carry out the simulation, the uncertain parameter was selected randomly. Fig. 2 (bottom left) presents the results of the $\mathcal{OP}_1$ with the numerical solution for the intersection of the ellipses, while Fig. 2 (bottom right) shows the solution of the $\mathcal{OP}_2$ given in (15) with several simulations, each initialized at the boundary of the set of admissible initial conditions, $(\mathcal{E}(P_1, 1) \cap \mathcal{E}(P_1, 2)) \subset \mathcal{D}_u$, $\mathcal{E}(H, 1) \subset \mathcal{D}_u$, respectively. As observed, all trajectories converge toward the attractor regions, illustrating the effectiveness of the proposed approach in handling uncertain systems.

5. Conclusion

This paper has introduced conditions for designing output-feedback controllers for uncertain discrete-time systems under input saturation and persistent disturbances. The proposed approach has ensured that trajectories starting within the set of admissible initial conditions will converge to an attractor defined by a subset of the Lyapunov function where the system states will remain confined. Optimization procedures have been proposed to enlarge the region of admissible initial conditions and to reduce the size of the attractor. Numerical examples have been provided to show the effectiveness of the proposed methods. In future work, data-driven solutions to the local SOF stabilization of systems subject to input saturation will be investigated.

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