

Uncertainty-Aware H_2 Control of a Wave Energy Converter Prototype

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Abstract—This paper presents a data-driven robust control framework for wave energy converters. Unlike traditional model-based approaches that neglect uncertainties, the proposed method uses a data-driven model with polytopic uncertainty identified from experimental data of a Wavestar-type prototype. The control problem is cast as a convex semidefinite program with linear matrix inequalities (LMIs), enabling the synthesis of a robust H_2 controller that maximises average energy capture. Numerical results demonstrate that the proposed controller either outperforms or matches passive, reactive, and H_∞ strategies in energy absorption and control efficiency.

I. INTRODUCTION

Maximising energy capture in wave energy converters (WECs) relies critically on effective control strategies, with controlled devices capable of absorbing up to four times more power than uncontrolled ones [1]. Most WEC controllers are based on linear potential flow theory (LPFT) [2], which assumes small motions and neglects nonlinear hydrodynamic effects, often leading to modelling inconsistencies such as negative power output [3].

These limitations reflect model uncertainty, a central challenge in WEC control alongside non-causality, physical constraints, and real-time implementability [4]. Although nonlinear hydrodynamic models offer improved accuracy, their computational cost restricts their use in control synthesis and real-time applications. As a result, linear models remain the practical choice, albeit at the expense of reintroduced uncertainty through model reduction and parameterisation.

Model uncertainty also constrains controller design. Classical impedance-matching control [2], while theoretically optimal, is non-causal and often assumes narrowband excitation, limiting its applicability in real ocean conditions. Consequently, practical implementations rely on approximate strategies such as passive or reactive controllers [5]. Recent experimental and data-driven modelling approaches aim to better capture hydrodynamic uncertainty [6], yet discrepancies between numerical and identified models persist, as evidenced by frequency-domain techniques such as empirical transfer function estimates (ETFE) [7].

Against this backdrop, this work proposes a novel causal, robust, uncertainty-aware control framework for WECs based on an H_2 performance criterion. A data-driven model with polytopic uncertainty enables a convex formulation via linear matrix inequalities (LMIs). By capturing the average energy response, the H_2 norm provides a physically meaningful objective for maximising energy extraction from stochastic ocean waves and, to the best of the authors' knowledge, represents the first application of an H_2 -based robust control framework to WEC control.

II. MODEL SPECIFICATION

This section presents the identification and parametrisation of a data-driven, uncertainty-aware, control-oriented model for a Wavestar-type WEC. The physical system is illustrated in Fig. 1 and the overall modelling workflow are illustrated in Fig. 2.

The experimental data are taken from the open-access SWELL dataset [8], obtained from a 1:20-scale Wavestar-type prototype at

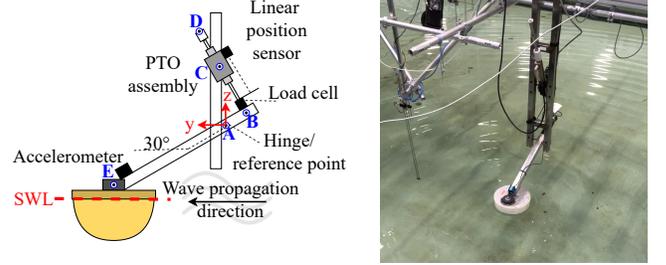


Fig. 1: Schematic and real prototype of the 1:20-scale Wavestar-type WEC (Aalborg University, May 2023).

Aalborg University (Fig. 1). A frequency-domain black-box identification approach is used to relate the wave-excitation torque about point A to the angular velocity response, where $\tau_A(t)$ is reconstructed from load-cell measurements and power take-off (PTO) kinematics (SWELL-Test 2) and $z(t) = \dot{\theta}_A(t)$ is obtained from Kalman-filter-based signal fusion (SWELL-Test 3).

To ensure broadband excitation of the dominant device dynamics, three white-noise sea states from the SWELL dataset (WNSS_1–WNSS_3) are considered. These inputs exhibit an approximately flat spectral density over the frequency range $[0.5, 10]$ rad/s. Let $i \in \{1, 2, 3\}$ index the sea-state realisations. The corresponding frequency responses are estimated using empirical transfer function estimation (ETFE), $\hat{G}_i(j\omega_k) = \frac{Z_i(\omega_k)}{F_{e,i}(\omega_k)}$, where $F_{e,i}(\omega_k)$ and $Z_i(\omega_k)$ denote the discrete Fourier transforms of the excitation torque and angular velocity, respectively. Spectral leakage and noise are mitigated by windowing and averaging over overlapping segments [9, 10, 11, 12].

A nominal frequency response is obtained as the complex average

$$\hat{G}_0(j\omega_k) = \frac{1}{3} \sum_{i=1}^3 \hat{G}_i(j\omega_k), \quad (1)$$

which is subsequently approximated by a nominal state-space model $G_0(s)$. The fit is computed by minimising the squared error between the measured and modelled frequency responses,

$$\min_{G \in \mathcal{R}} \sum_k \left| \hat{G}(j\omega_k) - G(j\omega_k) \right|^2, \quad (2)$$

where \mathcal{R} denotes the class of continuous-time, stable, strictly proper LTI models, following standard practice in wave energy system identification [6, 13, 14, 15]. A fourth-order continuous-time realisation is then selected. Additionally, variability across the three sea states is captured through the magnitude envelope

$$\overline{G}(\omega_k) = \max_i \left| \hat{G}_i(j\omega_k) \right|, \quad \underline{G}(\omega_k) = \min_i \left| \hat{G}_i(j\omega_k) \right|. \quad (3)$$

The frequency response functions were estimated using the H_1 estimator (tfestimate) with a periodic Hann window. For each experimental run, the segment length was set to $L =$

yields

$$\|H_{ez}\|_2^2 = \text{Tr}(C_{z,cl}(\alpha)W_c(\alpha)C_{z,cl}(\alpha)^\top), \quad (8)$$

where $W_c(\alpha)$ is the controllability Gramian satisfying $A_{cl}(\alpha)W_c(\alpha) + W_c(\alpha)A_{cl}(\alpha)^\top = -B_e(\alpha)B_e(\alpha)^\top$.

Let $P(\alpha) = P(\alpha)^\top \geq W_c(\alpha)$ satisfy the Lyapunov inequality

$$\begin{aligned} & A_{cl}(\alpha)P(\alpha) + P(\alpha)A_{cl}(\alpha)^\top + B_e(\alpha)B_e(\alpha)^\top \leq 0 \\ \Rightarrow & \begin{bmatrix} \mathbf{I} & A_{cl}(\alpha) & B_e(\alpha) \end{bmatrix} \begin{bmatrix} 0 & P(\alpha) & 0 \\ P(\alpha) & 0 & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ A_{cl}(\alpha)^\top \\ B_e(\alpha)^\top \end{bmatrix} \leq 0. \end{aligned} \quad (9)$$

Then,

$$\|H_{ez}\|_2^2 \leq \text{Tr}(C_{z,cl}(\alpha)P(\alpha)C_{z,cl}(\alpha)^\top). \quad (10)$$

An equivalent condition to (10) is given by

$$\begin{aligned} & \|H_{ez}\|_2^2 \leq \text{Tr}(X), \quad \text{s.t.} \\ & C_{z,cl}(\alpha)P(\alpha)C_{z,cl}(\alpha)^\top - X \leq 0, \\ \Rightarrow & \begin{bmatrix} \mathbf{I} & C_{z,cl}(\alpha) \end{bmatrix} \begin{bmatrix} -X & 0 \\ 0 & P(\alpha) \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ C_{z,cl}(\alpha)^\top \end{bmatrix} \leq 0, \end{aligned} \quad (11)$$

where $X \in \mathbb{R}^{n_z \times n_z}$. Applying Finsler's lemma [17] to (9) and (11) yields, respectively,

$$\begin{aligned} & \begin{bmatrix} 0 & P(\alpha) & 0 \\ P(\alpha) & 0 & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} + \begin{bmatrix} X_1 & X_4 \\ X_2 & X_5 \\ X_3 & X_6 \end{bmatrix} \begin{bmatrix} A_{cl}(\alpha)^\top & B_e(\alpha)^\top \\ -\mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{bmatrix}^\top \\ & + \begin{bmatrix} A_{cl}(\alpha) & B_e(\alpha) \\ -\mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{bmatrix} \begin{bmatrix} X_1^\top & X_4^\top \\ X_2^\top & X_5^\top \\ X_3^\top & X_6^\top \end{bmatrix} \leq 0, \end{aligned} \quad (12)$$

and

$$\begin{aligned} & \begin{bmatrix} -X & 0 \\ 0 & P(\alpha) \end{bmatrix} + \begin{bmatrix} -V \\ -J \end{bmatrix} \begin{bmatrix} -C_{z,cl}(\alpha)^\top & \mathbf{I} \end{bmatrix} \\ & + \begin{bmatrix} -C_{z,cl}(\alpha) \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} -V^\top \\ -J^\top \end{bmatrix} \leq 0 \Rightarrow \\ & \begin{bmatrix} -X + VC_{z,cl}(\alpha)^\top + C_{z,cl}(\alpha)V^\top & -V + C_{z,cl}(\alpha)J^\top \\ -V^\top + JC_{z,cl}(\alpha)^\top & P(\alpha) - J - J^\top \end{bmatrix} \leq 0. \end{aligned} \quad (13)$$

The LMIs (7b)–(7c) are recovered by applying the substitutions $V = 0$, $J = G^\top$, $X_1 = G^\top$, $X_2 = \beta G^\top$, $X_6 = H(\alpha)^\top$, $X_3 = X_4 = X_5 = 0$, and $Z = KG$. \square

The power absorbed by the WEC is $P_{mec}(t) = \tau_A(t)z(t)$, and the control objective is to maximise the extracted energy $E = \int_0^T P_{mec}(t) dt = \int_0^T \tau_A(t)z(t) dt$, where T denotes the time horizon. In this work, the LMIs (7b)–(7c) impose an upper bound on the H_2 norm from $\tau_A(t)$ to $z(t)$. Energy maximisation is achieved by relaxing δ , thereby increasing the variance of the angular-velocity response to stochastic wave excitation within the operating frequency band.

IV. NUMERICAL RESULTS

This section presents a numerical assessment of the simulation results obtained by applying the robust H_2 controller described in the previous section to the system characterised in Section II. The controller gain was computed by formulating and solving the LMI conditions using the YALMIP parser ([18]), the ROLMIP toolbox ([19]), and the SeDuMi solver ([20]). In this work, the optimisation was limited to the parameters β and δ , which were explored using a logarithmic grid. The search intervals were set to

$[10^{-5}, 10^0]$ for β and $[10^0, 10^{10}]$ for δ , based on practical experience indicating that feasible LMI solutions are most likely to be obtained within these ranges.

All the cases presented in this section refer to the nominal model, $G_0(s)$, introduced in Section II. For validation, three irregular sea states from the SWELL dataset, defined by JONSWAP spectra, are considered (Table I). These conditions differ from those used for identification and include both narrow- and broadband cases, allowing assessment of the controller's performance under realistic, stochastic wave excitation. Benchmark passive and reactive control results provided in SWELL are used for comparison.

To demonstrate the effectiveness of the proposed controller, Fig. 4 compares the excitation torque and the resulting angular position of the WEC in uncontrolled and H_2 -controlled cases. For the same excitation, the controlled system exhibits greater motion amplitude, consistent with the goal of WEC control, to amplify motion and thereby increase energy absorption from the waves.

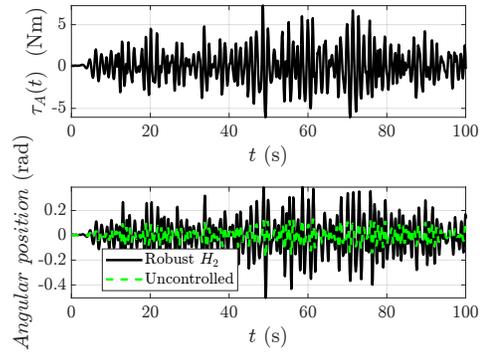


Fig. 4: Time snippet of the excitation torque (top), and time-domain response of the WEC under both uncontrolled and controlled conditions (bottom), the latter governed by the proposed robust H_2 controller for ISS1.

Additionally, the performance of the proposed H_2 controller is benchmarked against the passive and reactive controllers [5], as well as a classical robust approach, the H_∞ controller, whose design conditions are available in ([21]). Table II summarises the tuning parameters of all controllers.

Figure 5 compares the controller performance in terms of WEC output velocity and control force. It can be seen that, while requiring a smaller control effort, the H_2 controller achieves superior performance in maximising WEC motion compared with the other approaches.

As none of the existing non-optimisation-based WEC controllers can effectively handle hard constraints [22], a common approach in the case of reactive and passive controllers is to tune them by approximating the values derived from the IM principle (Table II), thereby avoiding physical constraint violations. Accordingly, Fig. 6 presents the results of Test 4 from the SWELL dataset, corresponding to the passive and reactive cases without such violations, along with the H_∞ and H_2 controllers. Table III summarises the total energy produced in this experiment. From the results, it can be concluded that the H_2 controller behaves similarly to the reactive controller, which typically injects power back into the system during parts of the cycle [23]. Across all sea states, the H_2 controller generated more energy than the H_∞ , reactive, and passive cases. As expected, the H_2 controller outperformed the H_∞ controller, as it minimises the average energy of the system response to stochastic wave excitations,

TABLE I: Irregular sea-state conditions from the SWELL dataset used for controller validation.

Layout	Sea state	Test file	Height H_s [m]	Period T_p [s]	Peakedness γ	Length [s]
L0	ISS-1	11_ISS1_1	0.063	1.412	3.3	300
L0	ISS-2	13_ISS2_1	0.104	1.836	3.3	300
L0	ISS-3	15_ISS3_1	0.0208	0.988	1	300

Paths follow Tests/L0/(11{15})_ISS(1{3})_1.mat.

TABLE II: Controller tuning parameters.

Parameter	H_2 Controller	H_∞ Controller
β	1.0×10^{-1}	1.0×10^{-2}
δ	1.0×10^8	1.0×10^{10}
Polynomial degree $P(\alpha)$	3	0
Polynomial degree $H(\alpha)$	15	0

ISSs	Passive Controller		Reactive Controller	
	K_P		K_p	K_i
ISS1	3.6103		0.8983	-15.5601
ISS2	5.7974		0.7729	-19.6628
ISS3	1.3681		1.1310	-4.8946

Note: The polynomial degrees refer to the homogeneous dependence of the decision variables on the scheduling parameter α , as defined by the the ROLMIP toolbox ([19])

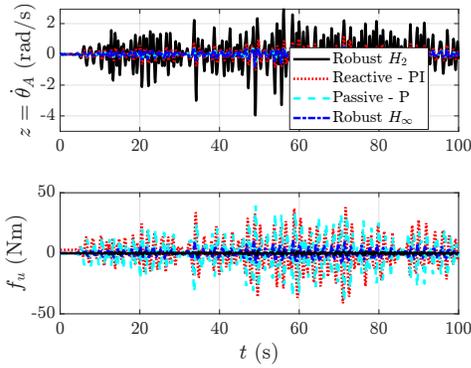


Fig. 5: Comparison of the proposed robust H_2 controller with the passive, reactive, and H_∞ strategies for ISS2. The top plot shows a time snippet of the WEC angular velocity, and the bottom plot the corresponding control torque. The H_2 controller yields greater motion amplitude with lower control effort.

whereas the H_∞ formulation is designed to guarantee performance only under worst-case conditions.

TABLE III: Energy generated [J].

Controller	ISS1	ISS2	ISS3
H_∞	30.7462	55.4733	6.7178
Passive	69.0878	145.0654	13.5941
Reactive	159.5929	217.1432	13.2706
H_2	257.6155	413.6734	56.3616

V. CONCLUSIONS

The results demonstrate the effectiveness of the proposed robust H_2 controller for wave energy conversion. Compared with passive, reactive, and robust H_∞ strategies, the H_2 controller achieved improved energy capture while maintaining balanced control effort and compliance with physical limits. The hypothesis that the H_2 formulation performs well in wave energy systems, by accounting for the average contribution of excitation frequencies, was confirmed. In contrast to H_∞ control, which targets worst-case scenarios, the H_2 approach provides an effective trade-off between energy absorption and robustness under stochastic sea states. These results were obtained by tuning only two parameters, β and δ , highlighting

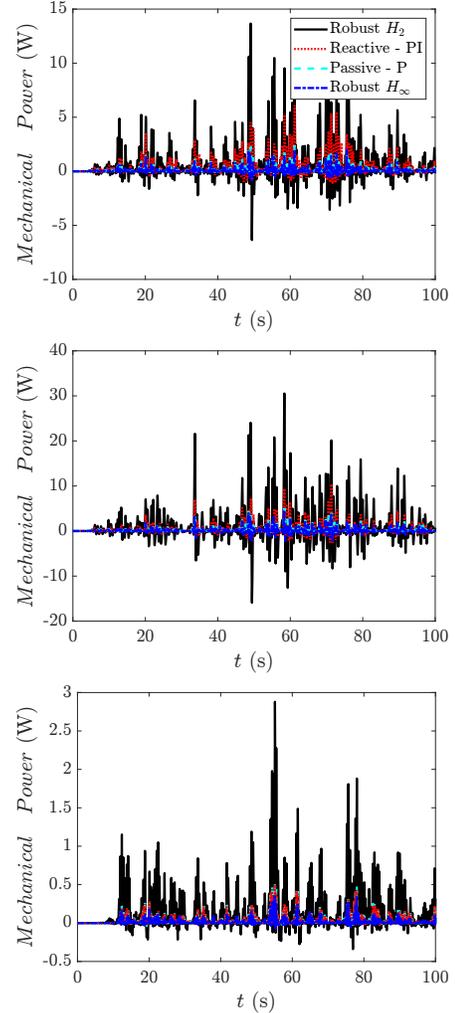


Fig. 6: Instantaneous mechanical power for different control strategies and sea states. The passive and reactive cases correspond to those available in the SWELL dataset ([8]), where no physical violations occur.

the method's simplicity and practical applicability. The study also underlines the potential of data-driven control in mitigating modelling uncertainties and limitations inherent to model-based frameworks. Future work will extend the framework to incorporate explicit state constraints and PTO dynamics, and benchmark the approach against other advanced WEC control strategies.

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