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Distributed Leader-Following Formation of Discrete-Time Multi-Agent LPV Systems

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ABSTRACT

This paper addresses the leader-following formation consensus problem for multi-agent systems (MASs) with agents represented by discrete-time linear parameter-varying (LPV) models. The scenario where each agent can be modeled with distinct time-varying scheduling parameters is investigated with respect to compensation signals. Using Lyapunov stability arguments, sufficient conditions are derived for designing a distributed gain-scheduled observer-based consensus protocol that ensures formation tracking. Furthermore, we explore the case where the effects of the desired formation and different parameters are considered internal perturbations. Under this assumption, we propose sufficient design conditions to ensure that the combined estimation and tracking error dynamics are ℓ_∞-norm bounded. The effectiveness of the proposed leader-following framework is illustrated through numerical examples.

1 | Introduction

Multi-agent systems (MASs) embrace several practical applications, including spacecraft formation, cooperation of robotic systems, sensor networks, among others [1]. An interesting feature of MASs is their cooperative actions, which allow them to solve complex problems by modifying the agents' behavior to achieve a common goal. The most investigated cooperative approaches are classified into consensus, formation, and flocking problems [2].

In the leader-following formation problem, one agent acts as the leader, and the remainder, designated as followers, must track the trajectories of the leader at a desired distance or offset, maintaining a predefined geometric shape [3]. This problem has been investigated for different classes of MASs in different scenarios, including second-order nonlinear multi-agent systems under fixed, directed, and switching communication topology [4, 5], high-order nonlinear MASs with an uncertain leader [6], discrete-time heterogeneous linear MASs [7], nonlinear MASs with event-based communication and quantized leader signal [8], and consensus of fractional-order fuzzy multiagent systems under DoS attacks [9].

The importance of communication topologies and the exchange of information among agents in designing cooperative protocols for MASs is widely recognized. However, due to physical and economic constraints, complete communication among agents and the measurement of all system states might not be possible in practical scenarios. To address this issue, the design of distributed observer-based protocols that limit the exchange of estimated information to only agents within the same neighborhood can be a viable solution [10, 11]. Hence, several approaches focused on the design of distributed observer-based protocols have been proposed for leader-following strategies considering continuous [12–18] and discrete-time dynamics [19–23].

Further discussing continuous-time approaches, [12] investigates a time-varying group formation for general linear time-invariant MASs, considering group leaders with nonzero bounded inputs. The leader's inputs are treated as disturbances, and a nonlinear function is introduced into the consensus protocol to mitigate their effects. In [13], an observer-based quadratic boundedness protocol is proposed to achieve leader-following consensus in the presence of bounded disturbances. The observer and consensus gain are designed simultaneously based on sufficient conditions to ensure that the states of the closed-loop MASs remain enclosed within an invariant ellipsoid. The boundedness of formation errors is also ensured in [14] by designing observer-based event-triggered protocols for MASs under switching directed communication topologies. To achieve time-varying formations for heterogeneous linear MASs subject to disturbances, uncertainties, and various topology scenarios, [15] proposes a robust distributed observer-based protocol with adaptive observers. Moreover, considering the methods for agents with discrete-time dynamics, a two-layer consensus approach is proposed in [19] for nonlinear strict-feedback MASs. The methods of [20] and [21] focus on the output regulation problem for MASs with jointly connected switching networks and the output tracking formation for heterogeneous MASs subject to time-varying faults, respectively. With the design of reduced-order observers, the work of [23] investigates the formation problem for linear MASs with switching communication topology.

Although distributed observer-based protocols have been developed for various scenarios within the context of the leader-follower formation problem, the case where agents are described by linear parameter-varying (LPV) models remains underexplored. The main approaches that employ the LPV framework for MASs can be found in [1, 24–36]. The leaderless consensus problem of homogeneous continuous-time LPV MASs has been investigated in [1, 26–28]. Among these results, the approach of [27] proposes a gain-scheduled observer-based consensus protocol, in which both the controller and the observer gains depend on the time-varying scheduling parameters. The design is performed through sufficient LMI conditions obtained considering Polya's Theorem. A similar gain-scheduled observer-based approach is also considered in [28]. However, different from [27], the presence of additive and multiplicative faults in the sensors and actuators is considered. Virtual actuators and sensors are introduced to compose a fault-tolerant consensus protocol to deal with this faulty scenario. Moreover, the presence of actuator faults modeled as polytopic uncertainties is also considered in [26]. Furthermore, a gain-scheduled consensus protocol is proposed in [1], considering the presence of time-varying delays in the communication among the agents.

The use of homogeneous continuous-time LPV models for agents is also explored in [24, 25]. As in [27, 28], observer-based protocols are introduced, but with attention directed to the leader-following consensus problem. Furthermore, the presence of actuator faults is also considered in [24]. The leaderless consensus of heterogeneous continuous-time LPV MASs and the leader-following consensus problem for MASs with heterogeneous parameter-dependent linear fractional transformation dynamics are investigated in [29] and [30], respectively. Further investigations in MASs with continuous-time heterogeneous LPV modeling are performed in [31–33]. These approaches focus on the output regulation problem considering distributed observers [31], distributed adaptive observers [32], and distributed event-triggered adaptive observers [33]. In contrast to the previously discussed scenario of continuous-time LPV MASs, approaches for discrete-time LPV MASs have received limited attention. In [34], a leaderless consensus approach based on finite frequency fault estimators and adaptive event-triggered mechanisms is proposed, and in [35, 36], the event-triggered ℓ_2-optimal output formation is investigated.

Among the approaches discussed above, only [24, 25, 30] focus on designing leader-following consensus protocols for continuous-time LPV MASs. To the best of the authors' knowledge, the development of distributed gain-scheduled observer-based protocols for the leader-following formation problem in discrete-time LPV MASs remains an open issue. Thus, to address this gap in the literature, the first motivation of this paper is:

- To propose a distributed observer-based consensus protocol to ensure the leader-following formation consensus of discrete-time LPV MASs with scheduling parameters mismatch.

The scheduling parameters mismatch among the agents appears in an LPV modeling where each agent has independent time-varying scheduling parameters. The first work to evaluate the influence of different scheduling parameters was [27], where it is shown that if a consensus protocol designed considering equal parameters is implemented in a continuous-time LPV MAS with different parameters, a non-synchronization scenario appears. In this case, the parameter mismatch can be seen as internal perturbations that disrupt the dynamics of the consensus tracking error. Moreover, it is known that the properties of the desired formation can also affect consensus errors [37]. Thus, to deal with these issues, the second motivation of this paper is:

- To propose a distributed observer-based consensus protocol to guarantee the practical leader-following formation consensus of discrete-time LPV MASs with scheduling parameters mismatch, and ensure that the trajectories of the consensus-tracking error converge exponentially to an attractive bounding region, for the cases in which the internal perturbations are not compensated.

Based on the above discussion, the main contributions of the proposed approach can be summarized as:

- Inspired on compensation signals usually considered only to expand the set of feasible formations [14, 18, 37, 38], we propose a novel compensated distributed observer-based consensus protocol capable of simultaneously expanding the set of feasible formations, deal with unmeasurable states, and compensate the internal perturbations raised on the formation tracking errors by the scheduling parameters mismatch among the discrete-time LPV modeling of the agents.

- Novel LMI-based synthesis conditions are proposed for designing the gains of both local observers and the consensus protocol such that the exponential stability of the tracking-error system is guaranteed.
- For the scenarios where the proposed compensation signals cannot be designed, by modeling the effects of the desired formation and heterogeneous scheduling parameters as internal perturbations, we propose a novel LMI-based condition to guarantee that the tracking-error dynamics is ℓ_∞ bounded.

The remainder of this paper is organized as follows. Section 2 describes the proposed distributed gain-scheduled observer-based consensus protocol and formulates the main problems addressed in this paper. Section 3 provides the proposed LMI-based design conditions. Section 4 presents two numerical simulations to evaluate the effectiveness of the proposed approach. Finally, Section 5 concludes this work.

Notation and List of Symbols: The identity matrix of order n is denoted by I_n and the null matrix of order $n \times m$ by $0_{n\times m}$. If the dimensions of both identity and null matrices are straightforwardly deduced, they are omitted. In a given symmetric matrix, the term deduced by symmetry is denoted by "$\star$". Moreover, the following Table 1 provides a summary of other mathematical symbols and their corresponding descriptions.

TABLE 1 | List of symbols.

Symbol	Description
$\mathbb{R}^n$	Is the n-dimensional Euclidean space
$\mathbb{R}^{m\times n}$	Is the set of $m \times n$ real matrices
$\mathbb{N}$	Is the set of non-negative integers
ℓ^∞	Is a Banach space
$X^\dagger$	Denotes the Moore-Penrose pseudo-inverse of X
$\mathrm{diag}(A, B)$	Denotes a block diagonal matrix whose elements are A, B
$A \otimes B$	Denotes the standard Kronecker product between matrices A and B
$\mathrm{He}(X)$	Corresponds to $\mathrm{He}(X) = X + X^\top$
$\mathbb{N}_{\leq m}$	Denotes the set $\{1, \ldots, m\}$ for some $m \in \mathbb{N}$
$\mathcal{G}(\mathcal{V}, \mathcal{E})$	Is the graph representing the communication among agents
$\mathcal{V}$	Set of vertices of the graph
$\mathcal{E}$	Set of edges of the graph
$\mathcal{N}_i$	Neighborhood set of an agent i
$\overline{\mathcal{L}}$	Laplacian matrix of the overall communication graph
$\mathcal{D}$	Degree matrix of the graph
$\mathcal{A}$	Adjacency matrix of the graph
Λ	Diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$ with the eigenvalues λ_i of the Laplacian matrix
η	Diagonal matrix $\eta = \mathrm{diag}(\eta_1, \ldots, \eta_N)$ with the pinning parameters η_i
f_i	Desired formation for the i-th agent
$\rho_i(k)$, $\rho_s(k)$	Vector of time-varying scheduling parameters of the agents and leader
$x_i(k)$	State vector of the i-th agent
$\hat{x}_i(k)$	Estimation of the state vector of the i-th agent
$y_i(k)$	Output vector of the i-th agent
$z_i(k)$	Estimation error of the i-th agent
$\delta_i(k)$	Consensus tracking error of the i-th agent
$u_i(k)$	Designed consensus law of the i-th agent
$v_i(k)$	Designed compensation signal of the i-th agent
$r_i(k)$	Designed compensation signal of the i-th agent
$s(k)$	State vector of the leader
$e(k)$	Augmented error vector
$w(k)$	Internal perturbation vector
γ	Positive scalar that corresponds to the ℓ_∞ performance level
$\sigma \in (0, 1)$	Scalar parameter of the decay rate
$\|\|x\|\|_{\ell_\infty}$	For a sequence of vectors $\{x(k)\}_{k\in\mathbb{Z}^+}$ denote $\|\|x\|\|_{\ell_\infty} = \sup_{k\geq 0} \|\|x(k)\|\| < \infty$

2 | Problem Formulation

This section first presents the preliminaries of the graph theory considered to model the communication among the agents. Then, the considered leader-following formation and the assumptions required for the proposed observer-based consensus law are discussed. Moreover, based on the obtained modeling of the estimation and consensus-tracking errors, the addressed control problems are formally stated.

2.1 | Graph Theory and Communication Setup

The communication among the N agents is represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices, and $\mathcal{E} = \{e_{ij} = (i, j) \in \mathcal{V} \times \mathcal{V}\}$ is the set of edges. The neighborhood set of an agent i is represented by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$.

The elements of the adjacency matrix $\mathcal{A} = [a_{ij}]$, $i, j \in \mathcal{V}$, are defined as

$$a_{ij} = \begin{cases} 0, & \text{if } i = j \text{ or } e_{ij} \notin \mathcal{E}, \\ 1, & \text{if } e_{ij} \in \mathcal{E} \end{cases}$$

and the elements of the diagonal degree matrix $\mathcal{D} = \mathrm{diag}(d_1, \ldots, d_N)$ are computed as $d_i = \sum_{j=1}^{N} a_{ij}$, $i \in \mathcal{V}$. The Laplacian matrix associated with the graph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Moreover, the communication among the leader and the following agents is represented by $\eta = \mathrm{diag}(\eta_1, \ldots, \eta_N)$, where the pinning parameters η_i indicate whether the i-th follower has access to the leader dynamics ($\eta_i = 1$) or not ($\eta_i = 0$). Therefore, the overall communication can be represented by $\overline{\mathcal{L}} = \mathcal{L} + \eta$.

The matrix $\overline{\mathcal{L}}$ can be written in terms of its spectral decomposition, such that $\overline{\mathcal{L}} = T\Lambda T^{-1}$, where the orthogonal matrix $T \in \mathbb{R}^{N\times N}$ constitutes the eigenvectors of $\overline{\mathcal{L}}$, and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_N) \in \mathbb{R}^{N\times N}$ is a diagonal matrix with the eigenvalues of $\overline{\mathcal{L}}$ ordered as $\lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$.

2.2 | Formation Consensus of Discrete-Time LPV MASs

Consider an MAS comprising a leader and N followers represented by discrete-time LPV systems. The dynamics of the followers are given by

$$x_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k),$$
$$y_i(k) = C(\rho_i(k))x_i(k) \tag{1}$$

where $x_i(k) \in \mathbb{R}^{n_x}$ is the state vector, $u_i(k) \in \mathbb{R}^{n_u}$ is the consensus law (control input), $y_i(k) \in \mathbb{R}^{n_y}$ is the output, and $\rho_i(k) \in \mathbb{R}^p$ is the vector of time-varying scheduling parameters, which are functions of measured exogenous signals. Meanwhile, the leader system is described as

$$s(k+1) = A(\rho_s(k))s(k) \tag{2}$$

where $s(k) \in \mathbb{R}^{n_x}$ is the state vector and $\rho_s(k) \in \mathbb{R}^p$ is the vector of time-varying scheduling parameters of the leader.

The parameter-dependent matrices of the followers $A(\rho_i(k)) \in \mathbb{R}^{n_x \times n_x}$, $B(\rho_i(k)) \in \mathbb{R}^{n_x \times n_u}$, and $C(\rho_i(k)) \in \mathbb{R}^{n_y \times n_x}$, the parameter-dependent matrix of the leader $A(\rho_s(k))$, and all the parameter-dependent matrices to be designed in this paper belong to a polytopic domain and can be written as functions of the scheduling parameters as follows:

$$G(\rho(k)) = \sum_{h=1}^{N_v} \alpha_h(\rho(k))G_h$$

where N_v is the number of vertices of the polytopic domain and $\alpha_h(\rho(k))$ satisfy the convex sum property:

$$\sum_{h=1}^{N_v} \alpha_h(\rho(k)) = 1, \quad \text{and} \quad \alpha_h(\rho(k)) \geq 0, \quad \forall i \in \mathbb{N}_{\leq N}$$

Notice that the parameter-dependent matrices of the agents are homogeneous with respect to the vertices of the polytopic domain. However, they have independent time-varying scheduling parameters, which induce heterogeneity in global dynamics. Moreover, the leader-dynamics $A(\rho_s(k))$ also share the same vertices of the followers' representation.

The main goal of this paper is to design distributed observer-based consensus laws $u_i(k)$, $\forall i \in \mathbb{N}_{\leq N}$, such that the followers described in Equation (1) track the trajectory generated by the leader-dynamics (2) according to a desired formation, as established in Definition 1.

Definition 1 ([18, 23]). The MAS (1) and (2) achieves leader-following formation consensus if

$$\lim_{k\to\infty} ||x_i(k) - s(k) - f_i|| = 0, \quad \forall i \in \mathbb{N}_{\leq N} \tag{3}$$

for any initial condition $x_i(0)$, where f_i denotes the desired constant formation vector of agent i with respect to the leader trajectory.

2.3 | Observer-Based Consensus Law

To ensure that the MAS achieves the leader-following formation consensus (3), the proposed gain-scheduled observer-based distributed consensus law is:

$$u_i(k) = K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i}[\overline{x}_i(k) - \overline{x}_j(k)] + \eta_i\big(\overline{x}_i(k) - s(k)\big)\right) + v_i(k) + r_i(k) \tag{4}$$

$$v_i(k) = M(\rho_i(k))f_i \tag{5}$$

$$r_i(k) = \eta_i R(\rho_i(k))s(k) \tag{6}$$

where $K(\rho_i(k)) \in \mathbb{R}^{n_u \times n_x}$, $M(\rho_i(k)) \in \mathbb{R}^{n_u \times n_x}$, and $R(\rho_i(k)) \in \mathbb{R}^{n_u \times n_x}$, are the gain-scheduled functions to be designed, $\overline{x}_i(k) = \hat{x}_i(k) - f_i$, and $\hat{x}_i \in \mathbb{R}^{n_x}$ is the estimated state of the local agent.

For each agent, the local estimated state is given by

$$\hat{x}_i(k+1) = A(\rho_i(k))\hat{x}_i(k) + B(\rho_i(k))u_i(k) + L(\rho_i(k))(C(\rho_i(k))\hat{x}_i(k) - y_i(k)) \tag{7}$$

where $L(\rho_i(k)) \in \mathbb{R}^{n_x \times n_y}$ is the gain-scheduled observer gain to be designed.

Remark 1. The proposed observer-based gain-scheduled distributed consensus law (4) can be partitioned into three components. The first term, $K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i}\left[\overline{x}_i(k) - \overline{x}_j(k)\right] + \eta_i\big(\overline{x}_i(k) - s(k)\big)\right)$, concerns the distributed relative information with respect to the neighboring agents and the leader. Moreover, the components $v_i(k)$ and $r_i(k)$ are compensation signals introduced to deal with the desired formation f_i and the mismatch among the scheduling parameters of agents and the leader, respectively. Notice that if $f_i = 0, \forall i \in \mathbb{N}_{\leq N}$, this term is reduced to a leader-following consensus protocol. Compensation signals similar to $v_i(k)$ can be found in the literature of formation tracking for continuous-time linear MASs [14, 18, 37, 38]. However, a key distinction of the proposed approach is the design of $r_i(k)$, which is proposed in this work to deal with the mismatch among the scheduling parameters in the formation tracking of discrete-time LPV MASs.

2.4 | Estimation and Formation-Tracking Error Dynamics

The proposed design conditions will be developed considering a global error composed of local estimation and consensus-tracking errors. From the discrete-time LPV model of the agents (1), and the defined local observer structure (7), it is possible to write the dynamics of the local estimation errors $z_i(k) = x_i(k) - \hat{x}_i(k)$ as

$$z_i(k+1) = A(\rho_i(k))\big(x_i(k) - \hat{x}_i(k)\big) - L(\rho_i(k))\left(C(\rho_i(k))\hat{x}_i(k) - y_i(k)\right),$$
$$z_i(k+1) = \big(A(\rho_i(k)) + L(\rho_i(k))C(\rho_i(k))\big)z_i(k)$$

Let $\alpha_{hl}(\rho_k) = \text{diag}(\alpha_h(\rho_1(k))\alpha_l(\rho_1(k)), \dots, \alpha_h(\rho_N(k))\alpha_l(\rho_N(k)))$, and $z(k) = (z_1(k), \dots, z_N(k))$. We then have that

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}(\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes (A_h + L_l C_h))$$

Meanwhile, by setting $\delta_i(k) = x_i(k) - s(k) - f_i$ as the local leader-following consensus tracking error, it is possible to compute

$$\delta_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k) - A(\rho_s(k))s(k) - f_i$$

By the definitions of local estimation and consensus tracking error, we have $\hat{x}_i(k) = x_i(k) - z_i(k)$, and $x_i(k) = \delta_i(k) + s(k) + f_i$, resulting in $\hat{x}_i(k) = \delta_i(k) + s(k) + f_i - z_i(k)$. Replacing $\hat{x}_i(k)$ in the consensus law (4), we obtain

$$u_i(k) = K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i} \left[(\delta_i(k) - z_i(k)) - (\delta_j(k) - z_j(k)) \right] + \eta_i(\delta_i(k) - z_i(k)) \right) + v_i(k) + r_i(k), \quad \forall i \in \mathbb{N}_{\leq N}$$

allowing us to write the dynamics of the local consensus tracking error as

$$\begin{aligned}\delta_i(k+1) &= A(\rho_i(k))\delta_i(k) + (A(\rho_i(k)) - I + B(\rho_i(k))M(\rho_i(k)))f_i \\ &+ (A(\rho_i(k)) - A(\rho_s(k)) + \eta_i B(\rho_i(k))R(\rho_i(k)))s(k) \\ &+ \eta_i B(\rho_i(k))K(\rho_i(k))(\delta_i(k) - z_i(k)) \\ &+ B(\rho_i(k))K(\rho_i(k)) \sum_{j\in\mathcal{N}_i} \left[(\delta_i(k) - z_i(k)) - (\delta_j(k) - z_j(k)) \right]\end{aligned}$$

Let $\alpha_{hlq}(\overline{\rho}_k) = \text{diag}(\alpha_h(\rho_1(k))\alpha_l(\rho_1(k))\alpha_q(\rho_s(k)), \ldots, \alpha_h(\rho_N(k))\alpha_l(\rho_N)(k)\alpha_q(\rho_s(k)))$, $f = (f_1, \ldots, f_N)$, $\overline{s}(k) = (\mathbf{1} \otimes s(k))$, and $\delta(k) = (\delta_1(k), \ldots, \delta_N(k))$. Then, similarly to the estimation error, it is possible to write

$$\begin{aligned}\delta(k+1) = &\sum_{h=1}^{N_v}\sum_{l=1}^{N_v} (\alpha_{hl}(\rho_k) \otimes I_{n_x}) \Big\{ (I_N \otimes A_h + \overline{\mathcal{L}} \otimes B_h K_l)\delta(k) \\ &- (\overline{\mathcal{L}} \otimes B_h K_l)z(k) + (I_N \otimes (A_h + B_h M_l - I_{n_x}))f \Big\} \\ &+ \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v} (\alpha_{hlq}(\overline{\rho}_k) \otimes I_{n_x})(I_N \otimes (A_h - A_q) - \eta \otimes B_h R_l)\overline{s}(k)\end{aligned}$$

Finally, defining the augmented error system $e(k) = (z(k), \delta(k))$, we have

$$e(k+1) = \bar{A}(\rho_k)e(k) + \overline{B}_w w(k) \tag{8}$$

where $w(k) = w_f(k) + w_s(k)$, and

$$\overline{A}(\rho_k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \begin{bmatrix} \alpha_{hl}(\rho_k) \otimes (A_h + L_l C_h) & 0 \\ -\alpha_{hl}(\rho_k)\overline{\mathcal{L}} \otimes B_h K_l & \alpha_{hl}(\rho_k) \otimes A_h + \alpha_{hl}(\rho_k)\overline{\mathcal{L}} \otimes B_h K_l \end{bmatrix},$$

$$w_f(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} (\alpha_{hl}(\rho_k) \otimes (A_h + B_h M_l - I_{n_x}))f, \ \overline{B}_w = \begin{bmatrix} 0_{Nn_x} \\ I_{Nn_x} \end{bmatrix},$$

$$w_s(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v} ((\alpha_{hlq}(\overline{\rho}_k) \otimes I_{n_x})(I_N \otimes (A_h - A_q) - \eta \otimes B_h R_l))\overline{s}(k)$$

With the error system (8), it is possible to conclude that the leader-following formation consensus (3) can be achieved only if $w(k) = 0$. The term $w(k)$ can be seen as an internal perturbation that arises in the closed-loop error system due to the desired formation ($w_f(k)$), and heterogeneity among the scheduling parameters of the agents and the leader ($w_s(k)$). The compensation signals (5) and (6) introduced in the consensus law (4) are designed such that the augmented error system (8) is reduced to $e(k+1) = \bar{A}(\rho_k)e(k)$, which allows the design of the control and the observer gains, $K(\rho_i(k))$ and $L(\rho_i(k))$, by employing a separation principle argument.

Remark 2. As discussed in [18], without the compensation signal (5), the leader-following formation consensus (3) is achieved only for specific formations f_i that satisfy $(A(\rho_i(k)) - I) f_i = 0$. Therefore, the compensation signal (5) is essential to extend the range of feasible formations. Another outstanding contribution of this paper is to deal with heterogeneity among the scheduling parameters of the agents and the leader using the additional compensation signal (6). Similarly, if (6) is not introduced in the consensus law, the consensus on the leader-follower formation (3) can be achieved if all scheduling parameters are equal, that is, $\rho_1(k) = \rho_2(k) = \cdots = \rho_N(k) = \rho_s(k)$, once it results in $A(\rho_i(k)) = A(\rho_s(k), \forall\, i \in \mathbb{N}_{\leq N}$.

The design of the compensation signal (5) requires only local information. However, to cope with the heterogeneity among the scheduling parameters, the design of (6) requires access to the states and scheduling parameters of the leader, as defined in the next Assumption.

Assumption 1. The leader states $s(k)$, and the scheduling parameters $\rho_s(k)$ are available for all agents of the MASs (1), that is, $\eta_i = 1, \forall\, i \in \mathbb{N}_{\leq N}$.

Based on the previous discussion, the first problem addressed in this work can be stated as follows.

Problem 1. Given the LPV MAS described in Equation (1), assuming that Assumption 1 holds, design both the distributed observer-based consensus law (4), and the observer (7), such that the origin of the error system (8) is exponentially stable and the leader-following formation consensus (3) is achieved.

In this paper, we are also interested in investigating the case where Assumption 1 does not hold and the compensation signals cannot be designed. In this case, we have that if $M(\rho_i(k)) = 0_{n_u \times n_{n_x}}$ and $R(\rho_i(k)) = 0_{n_u \times n_{n_x}}$, the internal perturbations of the augmented error system (8) can be rewritten as

$$w_f(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} (\alpha_{hl}(\rho_k) \otimes (A_h - I_{n_x}))f \tag{9}$$

$$w_s(k) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\sum_{q=1}^{N_v} ((\alpha_{hlq}(\overline{\rho}_k) \otimes I_{n_x})(I_N \otimes (A_h - A_q)))\overline{s}(k) \tag{10}$$

For the sequence of vectors $\{w_f(k)\}_{k\in\mathbb{N}}$, and $\{w_s(k)\}_{k\in\mathbb{N}}$, define $||w_f||_{\ell_\infty} = \sup_{k\geq 0} ||w_f(k)|| < \infty$, and $||w_s||_{\ell_\infty} = \sup_{k\geq 0} ||w_s(k)|| < \infty$. Consequently, $||w||_{\ell_\infty} \leq ||w_f||_{\ell_\infty} + ||w_s||_{\ell_\infty}$. Due to the effects of these internal perturbations, the leader-following consensus (3) cannot be achieved. Therefore, the second problem addressed in this paper is to guarantee that in the absence of the compensation signals, the error system (8) is bounded, as described in the following problem.

Problem 2. Given the LPV MAS described in Equation (1), if Assumption 1 does not hold, design a distributed observer-based consensus law in the form

$$u_i(k) = K(\rho_i(k))\left(\sum_{j\in\mathcal{N}_i} \overline{x}_i(k) - \overline{x}_j(k) + \eta_i(\overline{x}_i(k) - s(k)) \right) \tag{11}$$

such that the augmented error system (8), with the internal perturbations given by (9) and (10), is bounded for any initial condition $e(0)$, and any sequence $\{w(k)\}_{k\in\mathbb{N}} \in \ell_\infty$. That is, there exists an upper bound $\varphi(e(0), ||w||_{\ell_\infty})$ such that

$$||e(k)|| \le \varphi(e(0), ||w||_{\ell_\infty}), \forall k \ge 0$$

and

$$\lim_{k\to\infty} \sup \ ||e(k)|| < \gamma ||w||_{\ell_\infty} \tag{12}$$

where $\gamma > 0$ corresponds to the ℓ_∞ performance level.

3 | Main Results

This section presents the main results of this work. First, the conditions to ensure the exponential and ℓ_∞ stability of the augmented error system are defined based on the Lyapunov theory. Then, assuming that the compensation signals are properly designed, sufficient conditions are provided for designing the compensated consensus law able to ensure the leader-following consensus (3). Finally, sufficient conditions are presented for the design of the uncompensated consensus law that provides the bounding guarantees for the ℓ_∞ gain (12).

3.1 | Exponential and ℓ_∞ Stability Analysis Conditions

The proposed approach is based on the Lyapunov theory presented in the following analysis condition.

Lemma 1. *If there exist positive scalars γ, $\sigma \in (0,1)$, and symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x\times n_x}$ and $P_2 \in \mathbb{R}^{n_x\times n_x}$, such that the Lyapunov function*

$$V(e(k)) = e(k)^\top P\, e(k), \quad P = \text{diag}((I_N \otimes P_1), (I_N \otimes P_2)) \tag{13}$$

satisfies

$$\Delta_{Vk} + \sigma\left(V(e(k)) - \gamma w(k)^\top w(k)\right) < 0 \tag{14}$$

$$\begin{bmatrix} P & \star \\ I & \gamma I \end{bmatrix} > 0 \tag{15}$$

where $\Delta_{Vk} = V(e(k+1)) - V(e(k))$, then the augmented error system (8), with the rewritten internal perturbations given by (9) and (10) is bounded by

$$||e(k)|| < \sqrt{\gamma(1-\sigma)^k V(e(0)) + \gamma^2 ||w||^2_{\ell_\infty}} \tag{16}$$

and ℓ_∞-stable with performance level γ. Moreover, if $w(k) = 0\, \forall k \ge 0$, the augmented error system (8) is exponentially stable with respect to the origin, and the leader-following formation consensus (3) is achieved.

Proof. The proof follows the same reasoning as in [39]. First, notice that condition (14) can be written as

$$V(e(k+1)) < (1-\sigma)V(e(k)) + \sigma\gamma w(k)^\top w(k))$$

which recursively implies that

$$\begin{aligned} V(e(k)) &< (1-\sigma)^k V(e(0)) + \sigma\gamma \sum_{i=0}^{k-1} (1-\sigma)^i \, \|w(k-1-i)\|^2, \quad \forall k \ge 1 \\ &< (1-\sigma)^k V(e(0)) + \gamma ||w||^2_{\ell_\infty} \end{aligned} \tag{17}$$

once $\sigma \in (0,1)$. By applying the Schur complement in Equation (15) we have

$$P > \gamma^{-1} I$$

which is equivalent to

$$\gamma e(k)^\top P e(k) > e(k)^\top e(k) \tag{18}$$

By combining (17) and (18), it is possible to conclude that

$$||e(k)||^2 < \gamma V(e(k)) < \gamma(1-\sigma)^k V(e(0)) + \gamma^2 ||w||^2_{\ell_\infty}, \quad \forall k \ge 1$$

resulting in the bound defined in Equation (16). Moreover, notice that if $w(k) = 0$, the condition (17) is reduced to $V(e(k)) < (1-\sigma)^k V(e(0))$, guaranteeing that the origin of the error system is exponentially stable. This concludes the proof. □

3.2 | Compensated-Consensus Design Conditions

In the sequel, the design conditions proposed to solve Problem 1 are presented.

Theorem 1. *On the basis that Assumption 1 holds, consider the augmented error dynamics (8) obtained with discrete-time LPV MAS (1), the gain-scheduling consensus protocol (4), and the observer (7). Given positive scalars $\sigma \in (0,1)$, $\xi \in \mathbb{R}^+$, and the eigenvalues $\lambda_{\mathbf{m}}$ of $\overline{\mathcal{L}}$, $\forall \mathbf{m} = 1, \dots, N$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x\times n_x}$, $\widetilde{P}_2 \in \mathbb{R}^{n_x\times n_x}$, matrices $X_1 \in \mathbb{R}^{n_x\times n_x}$, $X_2 \in \mathbb{R}^{n_x\times n_x}$, $\widetilde{K}_l \in \mathbb{R}^{n_u\times n_x}$, and $\widetilde{L}_l \in \mathbb{R}^{n_x\times n_y}$, such that the following inequalities hold*

$$\Psi_{hh} < 0, \qquad \text{if } h = l \tag{19}$$

$$\Psi_{hl} + \Psi_{lh} < 0, \qquad \text{if } h < l \tag{20}$$

$$\Phi_{hh\mathbf{m}} < 0, \qquad \text{if } h = l, \ \forall\, \mathbf{m} = 1, \dots, N \tag{21}$$

$$\Phi_{hl\mathbf{m}} + \Phi_{lh\mathbf{m}} < 0, \qquad \text{if } h < l, \ \forall\, \mathbf{m} = 1, \dots N \tag{22}$$

with

$$\Psi_{hl} = \begin{bmatrix} P_1 - \xi\text{He}(X_1) & \star \\ \xi\widetilde{\Theta}_{hl}^\top & (\sigma-1)P_1 \end{bmatrix} \tag{23}$$

$$\Phi_{hl\mathbf{m}} = \begin{bmatrix} \widetilde{P}_2 - \xi\text{He}(X_2) & \star \\ \xi\widetilde{\Gamma}_{hl\mathbf{m}}^\top & (\sigma-1)\widetilde{P}_2 \end{bmatrix} \tag{24}$$

$$\widetilde{\Theta}_{hl} = X_1 A_h + \widetilde{L}_l C_h \tag{25}$$

$$\widetilde{\Gamma}_{hl\mathbf{m}} = A_h X_2 + \lambda_m B_h \widetilde{K}_l \tag{26}$$

for all $h, l \in \mathbb{N}_{\leq N_v}$, and if there exist compensation gains $M(\rho_i(k))$ and $R(\rho_i(k))$, such that the following conditions

$$\left(A(\rho_i(k)) - I + B(\rho_i(k))M(\rho_i(k))\right) f_i = 0 \tag{27}$$

$$\left(A(\rho_i(k)) - A(\rho_s(k)) + B(\rho_i(k))R(\rho_i(k))\right) s(k) = 0 \tag{28}$$

hold for all $i \in \mathbb{N}_{\leq N}$. Then $\{w(k) = 0\}_{k\in\mathbb{N}}$, the error dynamics (8) is exponentially stable, and the gains of the observer (7), the matrices of the Lyapunov function (13), and the remaining gains of the consensus protocol are given, respectively, by $L_l = X_1^{-1}\widetilde{L}_l$, $P_2 = X_2^{-\top}\widetilde{P}_2X_2^{-1}$, P_1, and $K_l = \widetilde{K}_lX_2^{-1}$.

Proof. First, notice that matrices P_1 and $\widetilde{P}_2$ are symmetric and positive definite. Therefore, the Lyapunov function (13) is also positive definite. Moreover, considering that the conditions (27) and (28) hold we have that $\{w(k) = 0\}_{k\in\mathbb{N}}$, and by employing the separation principle argument, the exponential stability of the augmented error system (8) can be evaluated considering the following subsystems

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\left(\alpha_{hl}(\rho_k) \otimes (A_h + L_lC_h)\right)z(k),$$

$$\delta(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\Big((\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \overline{\mathcal{L}} \otimes B_hK_l)\Big)\delta(k)$$

Defining $V(z(k)) = z(k)^\top(I_N \otimes P_1)z(k)$, the condition $V(z(k+1)) + (\sigma - 1)\, V(z(k)) < 0$, is equivalent to

$$z(k+1)^\top(I_N \otimes P_1)z(k+1) + (\sigma-1)z(k)^\top(I_N \otimes P_1)z(k) < 0$$

Replacing (25) in Equation (23), and performing the change of variables $\widetilde{L}_l = X_1\, L_l$, results in

$$\begin{bmatrix} P_1 - \xi\mathrm{He}(X_1) & \xi(X_1A_h + X_1L_lC_h) \\ \star & (\sigma-1)P_1 \end{bmatrix} \tag{29}$$

Multiplying (29) by $[A_h^\top + C_h^\top L_l^\top \;\; I_{n_x}]$, on the left and its transpose in the right, we obtain

$$(A_h + L_lC_h)^\top P_1\,(A_h + L_lC_h) + (\sigma-1)P_1$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (19) and (20) are sufficient to guarantee $V(z(k+1)) + (\sigma - 1)\, V(z(k)) < 0$. Furthermore, by defining

$$V(\delta(k)) = \delta(k)^\top(I_N \otimes P_2)\delta(k) \tag{30}$$

and considering the spectral decomposition $\overline{\mathcal{L}} = T\Lambda T^{-1}$, we define the exchange of coordinates $\widetilde{\delta}(k) = (T^{-1} \otimes I_{n_x})\delta(k)$ that allows to write the condition $V(\delta(k+1)) + (\sigma-1)V(\delta(k)) < 0$ as

$$\widetilde{\delta}(k+1)^\top(I_N \otimes P_2)\widetilde{\delta}(k+1) + (\sigma-1)\widetilde{\delta}(k)^\top(I_N \otimes P_2)\widetilde{\delta}(k) < 0$$

with

$$\widetilde{\delta}(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v}\left((\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \Lambda \otimes B_hK_l)\right)\widetilde{\delta}(k)$$

Multiplying (24) by $\mathrm{diag}(X_2^{-\top}, X_2^{-\top})$ on the left and its transpose on the right, replacing (26), and performing the exchange of variables $\widetilde{K}_l = K_lX_2$, and $P_2 = X_2^{-\top}\widetilde{P}_2X_2^{-1}$ results in

$$\begin{bmatrix} P_2 - \xi\mathrm{He}(X_2^{-\top}) & \xi(X_2^{-\top}A_h + \lambda_m X_2^{-\top}B_hK_l) \\ \star & (\sigma-1)P_2 \end{bmatrix} \tag{31}$$

Multiplying (31) by $[A_h^\top + \lambda_m K_l^\top B_h^\top \;\; I_{n_x}]$ on the left and its transpose on the right, it results in

$$(A_h + \lambda_m B_hK_l)^\top P_2\,(A_h + \lambda_m B_hK_l) + (\sigma-1)P_2$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (21) and (22) are sufficient to guarantee that $V(\delta(k+1)) + (\sigma - 1)\, V(\delta(k)) < 0$.

Combining the previous steps, it is possible to see that the LMIs (19) and (22) are sufficient to guarantee

$$V(e(k+1)) + (\sigma - 1)\, V(e(k)) < 0$$

with a Lyapunov function $V(e(k)) = V(\delta(k)) + V(z(k))$, as defined in Equation (13). Thus, if all the conditions of Theorem 1 hold, the trajectories of the error system (8) converge exponentially to the origin. This concludes the proof. □

Assuming that the compensation signals are properly designed, the presented Theorem 1 provides sufficient conditions to guarantee the exponential stability of the origin of the error system (8). If the input matrices $B(\rho_i(k))$ are invertible, the suitable design of the compensation gains can be directly performed by considering

$$M(\rho_i(k)) = B(\rho_i(k))^{-1}\left(I_{n_x} - A(\rho_i(k))\right),$$
$$R(\rho_i(k)) = B(\rho_i(k))^{-1}\left(A(\rho_s(k)) - A(\rho_i(k))\right)$$

Moreover, if the matrices $B(\rho_i(k))$ are not invertible, the Moore-Penrose pseudo-inverse $B(\rho_i(k))^\dagger$ may be applied similarly. In the above cases, the compensation signals can be designed independently of the formation, extending the range of feasible desired formations, as discussed in Remark 2 and in [18].

3.3 | Bounded-Consensus Design Conditions

In the previous approach, if the design of the compensation gains is not possible, the leader-following consensus (3) cannot be achieved. Although suitable formations can be defined to ensure $(A(\rho_i(k) - I)f_i = 0, \forall i \in \mathbb{N}_{\leq N}$, the difference among the scheduling parameters will prevent the exact consensus. To deal with these cases, we design the consensus law (11), which guarantees that the error system (8) is bounded. To solve Problem 2, we present the design conditions in Theorem 2.

Theorem 2. *Consider the augmented error dynamics (8) obtained with discrete-time LPV MAS (1), the gain-scheduling consensus protocol (11), the observer (7), and the rewritten internal perturbations (9) and (10). Given positive scalars $\sigma \in (0,1)$, $\xi \in \mathbb{R}^+$, and the eigenvalues $\lambda_{\mathbf{m}}$ of $\overline{\mathcal{L}}$, $\forall \mathbf{m} = 1, \ldots, N$, if there exist symmetric positive definite matrices $P_1 \in \mathbb{R}^{n_x\times n_x}$, $\widetilde{P}_2 \in \mathbb{R}^{n_x\times n_x}$, matrices $X_1 \in \mathbb{R}^{n_x\times n_x}$, $X_2 \in \mathbb{R}^{n_x\times n_x}$, $\widetilde{K}_l \in \mathbb{R}^{n_u\times n_x}$, and $\widetilde{L}_l \in \mathbb{R}^{n_x\times n_y}$, and* *a positive scalar $\gamma \in \mathbb{R}^+$, such that inequalities (19) and (20) hold together with*

$$\Upsilon_{hh\mathbf{m}} < 0, \qquad \text{if } h = l,\ \forall\ \mathbf{m} = 1, \ldots, N \tag{32}$$

$$\Upsilon_{hl\mathbf{m}} + \Upsilon_{lh\mathbf{m}} < 0, \qquad \text{if } h < l,\ \forall\ \mathbf{m} = 1, \ldots, N \tag{33}$$

$$\begin{bmatrix} \widetilde{P} & \star \\ \overline{X}_2 & \gamma I \end{bmatrix} < 0 \tag{34}$$

for all $h, l \in \mathbb{N}_{\leq N_v}$, where $\widetilde{P} = \mathrm{diag}(P_1, \widetilde{P}_2)$, $\overline{X}_2 = \mathrm{diag}(I_{n_x}, X_2)$,

$$\Upsilon_{hl\mathbf{m}} = \begin{bmatrix} \tilde{P}_2 - \xi \mathrm{He}(X_2) & \star & \star \\ \xi \widetilde{\Gamma}_{hl\mathbf{m}}^\top & (\sigma - 1)\tilde{P}_2 & \star \\ \xi I & 0 & -\sigma\gamma I \end{bmatrix} \tag{35}$$

and $\widetilde{\Gamma}_{hl\mathbf{m}}$ as in Equation (26). Then, the error dynamics (8) is ℓ_∞ stable with performance index γ. The gains of the observer (7), the Lyapunov function matrices (13), and the gains of the consensus protocol (11) are given, respectively, by $L_l = X_1^{-1}\widetilde{L}_l$, $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, P_1, $K_l = \widetilde{K}_l X_2^{-1}$.

Proof. Firstly, from the error system (8) with the rewritten internal perturbations (9) and (10), it is possible to see that the internal perturbations do not affect the state estimation. Therefore, the separation principle can be once again employed, and the ℓ_∞ stability of the augmented error system (8) can be evaluated considering the following subsystems

$$z(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} \alpha_{hl}(\rho_k) \otimes (A_h + L_l C_h) z(k) \tag{36}$$

$$\delta(k+1) = \sum_{h=1}^{N_v}\sum_{l=1}^{N_v} (\alpha_{hl}(\rho_k) \otimes I_{n_x})(I_N \otimes A_h + \overline{\mathcal{L}} \otimes B_h K_l)\delta(k) + w(k) \tag{37}$$

Recall that it was already shown in the proof of Theorem 1 that if the inequalities (19) and (20) hold, the trajectories of the estimation error subsystem (36) converge exponentially to the origin. Therefore, we only need to prove that subsystem (37) is ℓ_∞ stable.

The remainder of the proof follows the same steps performed in the proof of Theorem 1. Notice that considering (37), the condition

$$V(\delta(k+1)) + (\sigma - 1)V(\delta(k)) - \sigma\gamma w(k)^\top w(k) < 0$$

can be equivalently written as

$$\tilde{\delta}(k+1)^\top (I_N \otimes P_2)\tilde{\delta}(k+1) + (\sigma - 1)\tilde{\delta}(k)^\top (I_N \otimes P_2)\ \tilde{\delta}(k) - \tilde{w}(k)^\top (I_N \otimes \sigma\gamma I_{n_x})\tilde{w}(k) < 0 \tag{38}$$

with $V(\delta(k))$ as in Equation (30), and $\widetilde{w}(k) = (T^{-1} \otimes I_{n_x})w(k)$.

Replacing (26) in Equation (35), multiplying (35) on the left by $\mathrm{diag}(X_2^{-\top}, X_2^{-\top}, I_{n_x})$ and its transpose on the right, and performing the exchange of variables $\widetilde{K}_l = K_l X_2$, and $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, results in

$$\begin{bmatrix} P_2 - \xi \mathrm{He}(X_2^{-\top}) & \xi(X_2^{-\top}A_h + \lambda_m X_2^{-\top}B_h K_l) & \xi X_2^{-\top} \\ \star & (\sigma - 1)P_2 & 0_{n_x \times n_x} \\ \star & \star & -\sigma\gamma I_{n_x} \end{bmatrix} \tag{39}$$

Multiplying, (39) by

$$\mathcal{B}^\top = \begin{bmatrix} A_h^\top + \lambda_{\mathbf{m}} K_l^\top B_h^\top & I_{n_x} & 0_{n_x \times n_x} \\ I_{n_x} & 0_{n_x \times n_x} & I_{n_x} \end{bmatrix}$$

on the left and its transpose on the right, we obtain

$$\begin{bmatrix} (A_h^\top + \lambda_{\mathbf{m}} K_l^\top B_h^\top)P_2(A_h + \lambda_{\mathbf{m}} B_h K_l) + (\sigma - 1)P_2 & \star \\ P_2(A_h + \lambda_{\mathbf{m}} B_h K_l) & P_2 - \sigma\gamma I_{n_x} \end{bmatrix} \tag{40}$$

Moreover, performing a dimension adjustment, and then multiplying (40) by $[\widetilde{\delta}(k)^\top\ \widetilde{w}(k)^\top]$ on the left, and its transpose on the right, results in

$$\begin{aligned} &\widetilde{\delta}(k)^\top (I_N \otimes (A_h^\top + \lambda_{\mathbf{m}} K_l^\top B_h^\top)P_2(A_h + \lambda_{\mathbf{m}} B_h K_l))\widetilde{\delta}(k) \\ &+ \widetilde{w}(k)^\top (I_N \otimes (P_2 - \sigma\gamma I_{n_x}))\widetilde{w}(k) \\ &+ \widetilde{\delta}(k)^\top ((\sigma - 1)(I_N \otimes P_2))\widetilde{\delta}(k) \\ &+ \mathrm{He}\Big(\widetilde{\delta}(k)^\top (I_N \otimes P_2(A_h + \lambda_{\mathbf{m}} B_h K_l))\widetilde{w}(k)\Big) \end{aligned}$$

from which it is possible to conclude that due to the convex properties of time-varying parameters, the LMIs (32) and (33) are sufficient to guarantee (38). Therefore, we have that the LMIs (19) to (33) and (32) and (33) are sufficient to guarantee (14) with a Lyapunov function as defined in Equation (13).

Finally, multiplying (34) by $\mathrm{diag}(I_{n_x}, X_2^{-\top}, I_{n_x}, I_{n_x})$ on the left, and its transpose on the right, and performing the exchange of variables $P_2 = X_2^{-\top}\widetilde{P}_2 X_2^{-1}$, it is possible to see that (34) is equivalent to (15). Thus, all the conditions of the Lemma 1 are satisfied. This concludes the proof. □

Remark 3. As discussed in [39], the upper bound of the error (16) can be reduced with the minimization of γ.

4 | Numerical Examples

In this section, we presented two distinct examples to illustrate the effectiveness of the main results of this work. The first example is a numerical system adapted from [27]. In this example, the goal is to explore the possible configurations of the proposed consensus protocols in two distinct scenarios. In the first scenario, all agents have access to the measurements of the leader, and the design is performed according to Theorem 1. Additionally, in the second scenario, we explore the discussion performed in Remark 2 and make a comparison on the evolution of the norm of the augmented error system considering the design performed with Theorem 2. Moreover, in the second example, we highlight the applicability of the proposed approach on a problem with practical physical meaning, where the goal is to guarantee the exact leader-follower formation tracking of an angular positioning LPV MAS.

4.1 | Example 1

Consider the LPV MAS, adapted from [27], composed of $N = 4$ agents with communication topology described by

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}, \quad \eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Initially we take $\eta_i = 1, \forall\, i \in \mathbb{N}_{\leq 4}$, as defined in Assumption 1. The vertices of the continuous-time system are

$$\bar{A}_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; \bar{A}_2 = R_p^{-1} \begin{bmatrix} 0 & 1+p \\ -1 & 0 \end{bmatrix} R_p,$$

$$R_p = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}, \; \beta = \arctan(p), \; p = 0.5$$

$$\overline{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \overline{B}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1+10p \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 1+10p & 0 \end{bmatrix}$$

By employing the Euler discretization with $T_s = 0.01s$, we have

$$A_1 = I + T_s\bar{A}_1 = \begin{bmatrix} 1.000 & 0.010 \\ -0.010 & 1.000 \end{bmatrix},$$

$$A_2 = I + T_s\bar{A}_2 = \begin{bmatrix} 1.002 & 0.014 \\ -0.011 & 0.998 \end{bmatrix},$$

$$B_1 = T_s\overline{B}_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \; B_2 = T_s\overline{B}_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.06 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 6 & 0 \end{bmatrix}$$

To ensure that conditions (27) and (28) hold, we define

$$M(\rho_i(k)) = B(\rho_i(k))^{-1}(I_{n_x} - A(\rho_i(k)),$$
$$R(\rho_i(k)) = B(\rho_i(k))^{-1}(A(\rho_s(k) - A(\rho_i(k))$$

as the gain of the compensation signals. Then, the design of the observer-based consensus protocol gains can be carried out by invoking Theorem 1. By considering $\xi = 1$ and $\sigma = 0.01$, we obtain the following gains

$$K_1 = \begin{bmatrix} -27.8777 & -0.2284 \\ 0.1051 & -10.4342 \end{bmatrix}, \; K_2 = \begin{bmatrix} -27.9406 & -0.4216 \\ 0.0494 & -4.5015 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.3941 \\ -0.3084 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.1582 \\ -0.1234 \end{bmatrix}$$

and Lyapunov matrices

$$P_1 = \begin{bmatrix} 1.2098 & -0.3890 \\ -0.3890 & 0.4823 \end{bmatrix}, \; P_2 = \begin{bmatrix} 1.3220 & -0.0047 \\ -0.0047 & 1.3213 \end{bmatrix}$$

To validate the designed consensus protocol, we perform a simulation with initial conditions

$$x_1(0) = \begin{bmatrix} 3 \\ 10 \end{bmatrix}, \; x_2(0) = \begin{bmatrix} -7 \\ -3 \end{bmatrix}, \; x_3(0) = \begin{bmatrix} 10 \\ 1 \end{bmatrix}, \; x_4(0) = \begin{bmatrix} -5 \\ -8 \end{bmatrix},$$

$$\hat{x}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_3(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\hat{x}_4(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; s(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

exogenous time-varying scheduling parameters,

$$\begin{aligned}
\alpha_1(\rho_1(k)) &= \frac{1+\sin(2t(k))}{2} & \alpha_2(\rho_1(k)) &= 1 - \alpha_1(\rho_1(k)),\\
\alpha_1(\rho_2(k)) &= \frac{1+\cos(t(k))}{2} & \alpha_2(\rho_2(k)) &= 1 - \alpha_1(\rho_2(k)),\\
\alpha_1(\rho_3(k)) &= \frac{1+\sin(0.05t(k))}{2} & \alpha_2(\rho_3(k)) &= 1 - \alpha_1(\rho_3(k)),\\
\alpha_1(\rho_4(k)) &= \frac{1+\cos(0.05t(k))}{2} & \alpha_2(\rho_4(k)) &= 1 - \alpha_1(\rho_4(k)),\\
\alpha_1(\rho_s(k)) &= \frac{1+\cos(5t(k))}{2} & \alpha_2(\rho_s(k)) &= 1 - \alpha_1(\rho_s(k))
\end{aligned}$$

and the desired formations,

$$f_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \; f_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}, \; f_3 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \; f_4 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Considering the simulation time of 10 s and the sampling time $T_s = 0.01s$, the closed-loop trajectories of LPV MAS (1) equipped with the proposed consensus protocol (4) are depicted in Figure 1. Furthermore, the trajectories of the estimation and consensus errors are presented in Figures 2 and 3, respectively.

The initial conditions of the leader and the following agents in the x-plane are highlighted by the black hexagram ✶ and the circles ●, respectively. Similarly, the positions of the leader and the following agents in the x-plane in $t = 10s$ are highlighted by the magenta hexagram ✶ and the circles ●, respectively. From

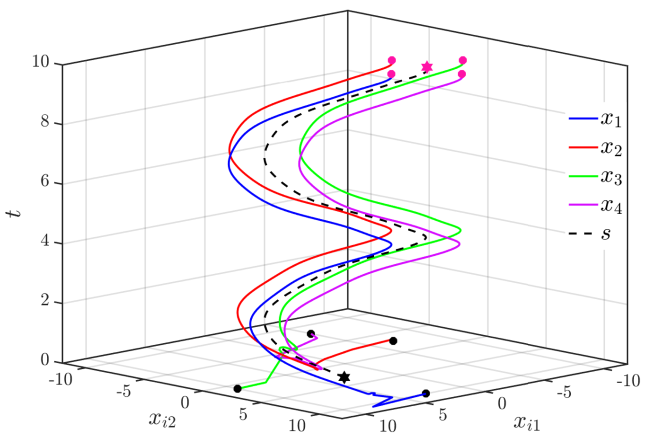

FIGURE 1 | Trajectories of the leader and following agents equipped with the proposed formation consensus protocol (4) designed with Theorem 1—Example 4.1.

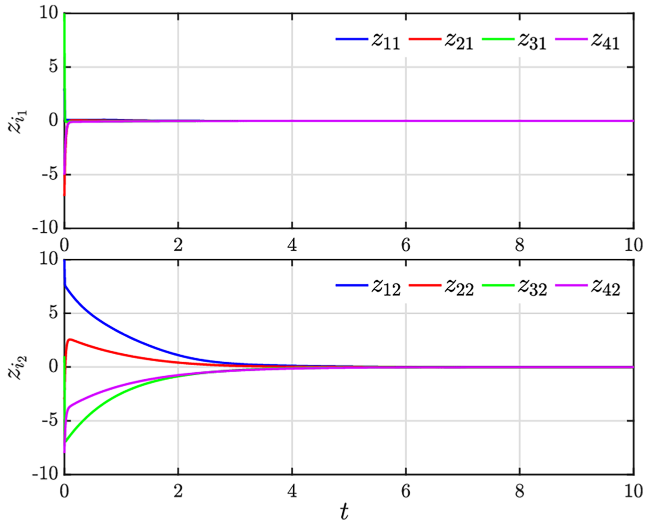

FIGURE 2 | Trajectories of the estimation error obtained with the observer designed with Theorem 1—Example 4.1.

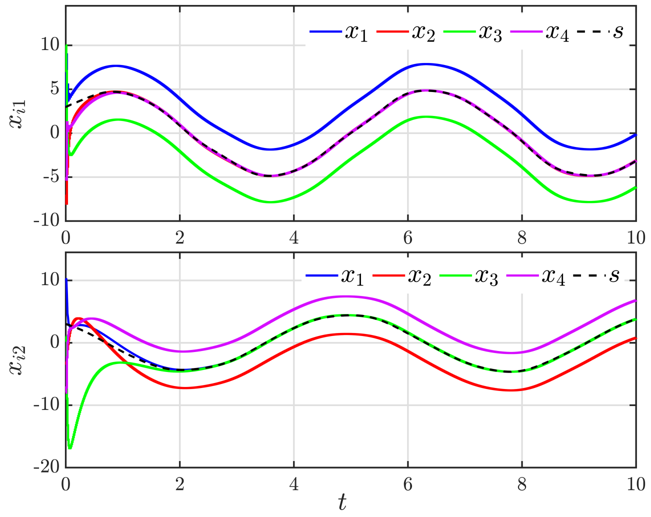

FIGURE 4 | Trajectories of the leader and following agents equipped with the proposed formation consensus protocol designed with Theorem 2—Example 4.1.

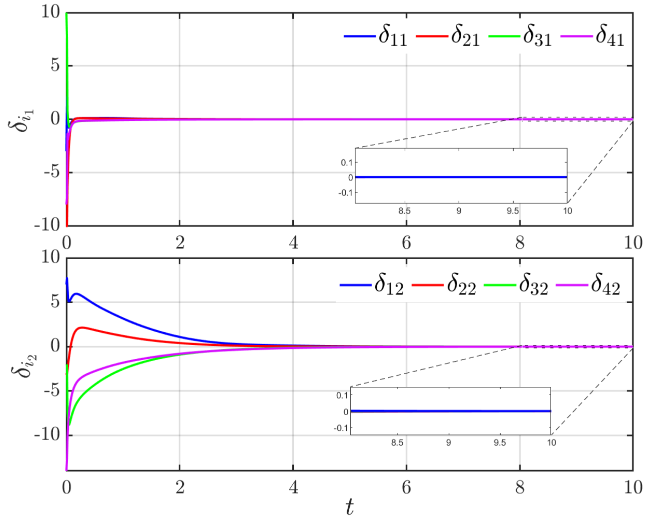

FIGURE 3 | Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 1—Example 4.1.

the results of Figure 1, it is possible to see that the following agents successfully track the leader in (- - - -) while maintaining the specified formation.

Notice that both errors converge exponentially to the origin. Therefore, it is clear that the leader-follower consensus (3) is properly achieved, illustrating the effectiveness of the proposed observer-based consensus protocol in dealing with the formation tracking problem of LPV MASs with different time-varying scheduling parameters.

As previously discussed in Remark 2, the exact leader-follower consensus obtained in the simulation results depicted in Figures 1–3 requires the proper design of the signals (5) and (6) to compensate for the internal perturbations. To illustrate the importance of the proposed compensation, especially when dealing with different scheduling parameters, consider a second scenario where information about the leader is unavailable to all agents. In this case, only agent x_1 will receive leader information, that is, $\eta_i = 0$, for $i = 2, 3, 4$.

Recall that without leader information, local controllers cannot be designed with compensation signals (6). Therefore, since the condition of the Assumption 1 does not hold for all agents, the consensus protocol must be designed by invoking Theorem 2. Then, by considering $\xi = 1.8$ and $\sigma = 0.03$ we obtain $\gamma = 34.62$, and

$$K_1 = \begin{bmatrix} -36.9253 & 2.9426 \\ -0.5473 & -13.2470 \end{bmatrix},\ K_2 = \begin{bmatrix} -27.6660 & -5.3710 \\ 2.4110 & -2.4536 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.4186 \\ -0.7467 \end{bmatrix},\ L_2 = \begin{bmatrix} -0.1644 \\ -0.2909 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 105.7084 & -27.9799 \\ -27.9799 & 15.2488 \end{bmatrix},\ P_2 = \begin{bmatrix} 0.0879 & -0.0146 \\ -0.0146 & 0.0325 \end{bmatrix}$$

Considering the same initial conditions, formations, and scheduling parameters, the trajectories of the closed-loop following agents and the consensus error of the second scenario are depicted in Figures 4 and 5. It can be seen from the results of Figure 4 that even without compensation $r_i(k)$, the following agents can track the leader dynamics. However, as shown in Figure 5, the agents do not achieve the exact desired formation, and consensus errors oscillate around the origin due to the internal perturbation $w_s(k)$ in Equation (10). This result evidences the contributions of the proposed approach, demonstrating that dealing with the formation perturbation $w_f(k)$ is not sufficient to achieve the exact formation (3) for LPV MASs in the form of (1).

Moreover, to evaluate the boundedness guarantees provided by the conditions of Theorem 2, consider a scenario where none of the compensation signals (5) and (6) are designed. The behavior of the combined internal perturbation $w(k)$, with the same initial conditions and time-varying scheduling parameters, is depicted in Figure 6. With the computation of $||w||_{\ell_\infty}$, together

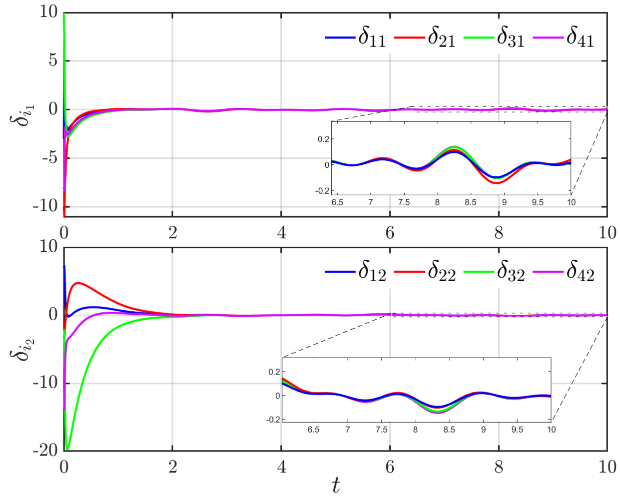

FIGURE 5 | Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 2—Example 4.1.

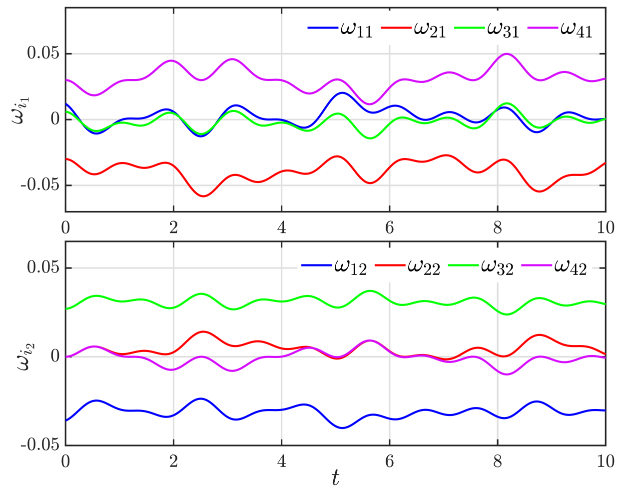

FIGURE 6 | Internal perturbations of the closed-loop consensus tracking error—Example 4.1.

with the previously presented values of σ, γ, P_1, and P_2, obtained with Theorem 2, we can compute $\varphi(e(0), ||w||_{\ell_\infty})$ as defined in Equation (16).

A comparison of the norm of the augmented error system (8), together with the obtained $\varphi(e(0), ||w||_{\ell_\infty})$, is presented in Figure 7. The exact leader-follower consensus can be achieved when considering both compensations (depicted in ▬▬). In other cases, when only one type of compensation is considered (only $r_i(k)$ depicted in ▬▬ and only $\nu_i(k)$ depicted in ▬▬), or when the compensations are neglected (depicted in ▬▬), the norm of the augmented error system is bounded by $\varphi(e(0), ||w||_{\ell_\infty})$ (depicted in - - - -). As expected, the consensus law without compensation yields the worst results, again accentuating the benefits of the proposed approach.

4.2 | Example 2

Consider an LPV MAS angular positioning inspired by the model presented in [40, 41]. The classical angular position system (APS) comprises a rotating antenna driven by an electric motor. The

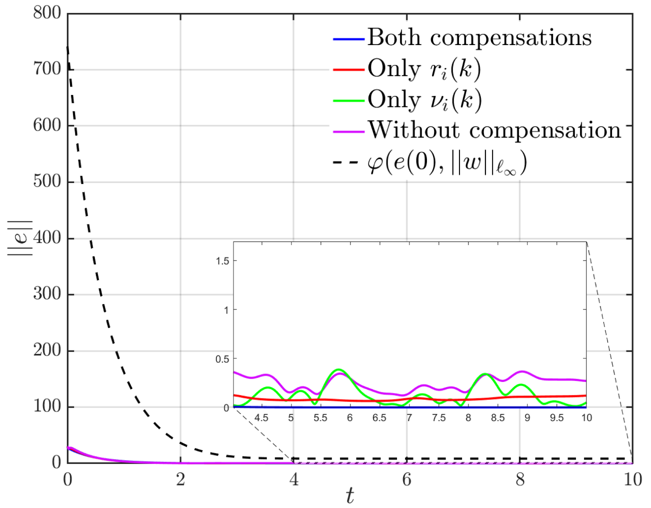

FIGURE 7 | Comparison of the norm of the augmented error system—Example 4.1.

control goal is to drive the motor to rotate the antenna so that it points in the direction of a moving target. In this paper, we assume that the target is a MAS composed of a leader-follower formation, and our goal is to design a distributed gain-scheduled observer-based consensus such that the formation of the angular positioning LPV MAS matches the formation of the targets, as illustrated in Figure 8.

The time-varying dynamics of the APS are given by

$$x_i(k+1) = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 - 0.1\rho_i(k) \end{bmatrix} x_i(k) + \begin{bmatrix} 0 \\ 0.1\kappa \end{bmatrix} u_i(k),$$
$$y_i(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_i(k)$$

where $x_i = [\theta_i^\top \ \dot{\theta}_i^\top]^\top$, θ_i [rad] is the angular position, $\dot{\theta}_i$ [rad/s] is the angular velocity, $0.1 \text{ s}^{-1} \leq \rho_i(k) \leq 10 \text{ s}^{-1}$ is proportional to the coefficient of viscous friction in the rotation parts of the antenna, and $\kappa = 0.787 \text{ rad}^{-1}\text{V}^{-1}s^{-2}$ is a given constant. Similarly to [40, 41], $\rho_i(k)$ is arbitrarily time-varying in the indicated range of variation.

We assume that the angular position of the leader-target θ_{lt} [rad] and the formation of its followers are measurable and available. Considering a simulation time of $t = 10$ s with a sampling period of $T_s = 0.01$s, the position of the leader-target is given by

$$\theta_{lt}(k) = \begin{cases} \frac{\pi}{3} & 0 \leq k \leq 300, \\ \frac{\pi}{3} + (k - 300)\frac{\pi}{6} & 300 < k \leq 400, \\ \frac{\pi}{2} & 300 < k \leq 700, \\ \frac{\pi}{2} + (k - 700)\frac{\pi}{6} & 700 < k \leq 800, \\ \frac{2\pi}{3} & 800 < k \leq 1000 \end{cases} \quad (41)$$

Moreover, we assume that the target system comprises one leader and three followers with the same distance of $\pi/12$. Therefore, the desired formation and the communication among the agents of the APS are given by

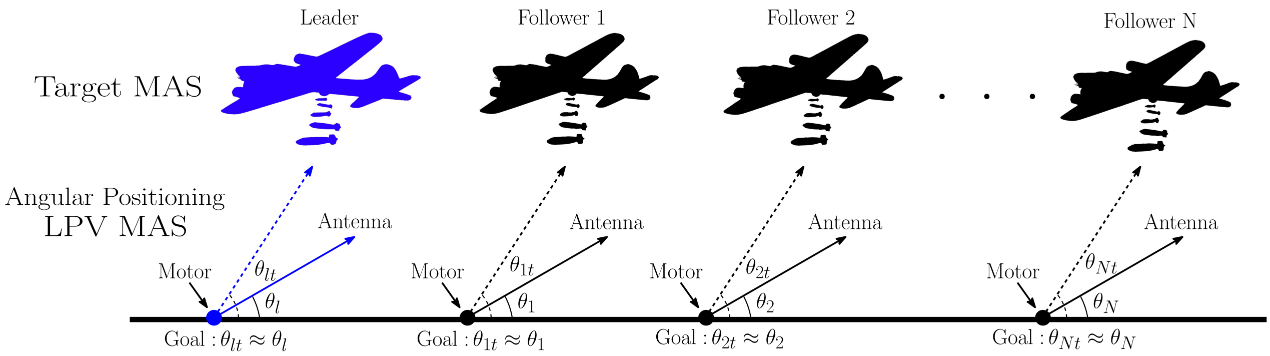

FIGURE 8 | Setup of an angular positioning MASs.

$$f_1 = \begin{bmatrix} -\frac{\pi}{12} \\ 0 \end{bmatrix}, \quad f_2 = \begin{bmatrix} -\frac{\pi}{6} \\ 0 \end{bmatrix}, \quad f_3 = \begin{bmatrix} -\frac{\pi}{4} \\ 0 \end{bmatrix},$$

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad \eta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice that a leader of the APS in the open-loop form of (2) cannot track the position of the leader-target described in Equation (41). In this case, it is necessary to consider a controlled leader in the form

$$s(k+1) = A(\rho_s(k))s(k) + B(\rho_s(k))u_s(k)$$

where the leader control input $u_s(k)$ is properly designed to track the leader-target. By the definition of the consensus error, notice that we now have

$$\delta_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k) - A(\rho_s(k))s(k) - B(\rho_s(k))u_s(k) - f_i$$

Similarly to [23], we assume that the leader control input is known by all followers, and the consensus law is modified to $u_i(k) = u_i(k) + u_s(k)$, resulting in

$$\delta_i(k+1) = A(\rho_i(k))x_i(k) + B(\rho_i(k))u_i(k) - A(\rho_s(k))s(k) - f_i$$
$$+ (B(\rho_i(k)) - B(\rho_s(k)))u_s(k)$$

Since the input matrix of the APS systems is parameter-independent, we have $B(\rho_i(k)) = B(\rho_s(k)) = B$, and the augmented error system (8) remains unchanged. However, in the more general case, the consensus law can be modified to $u_i(k) = u_i(k) + \bar{u}_s(k)$ where employing the same strategy considered in the compensation signals, $\bar{u}_s(k)$ is defined to satisfy condition $B(\rho_i(k))\bar{u}_s(k) - B(\rho_s(k))u_s(k) = 0$.

Then, the desired formations satisfy $\left(A(\rho_i(k)) - I\right) f_i = 0$. Therefore, we only design the compensation signals $r_i(k)$ considering the Moore-Penrose pseudo-inverse to obtain $R(\rho_i(k)) = B(\rho_i(k))^{\dagger}(A(\rho_s(k)) - A(\rho_i(k))$.

With $\xi = 1$, and $\sigma = 0.025$, we solve the conditions of Theorem 1 to obtain the following gain vertices:

$$K_1 = \begin{bmatrix} -3.4843 & -4.8451 \end{bmatrix}, \; K_2 = \begin{bmatrix} -3.6610 & -0.3055 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -1.0372 \\ -0.3919 \end{bmatrix}, \; L_2 = \begin{bmatrix} -1.0412 \\ 0.0359 \end{bmatrix}$$

and Lyapunov matrices

$$P_1 = \begin{bmatrix} 0.8620 & -0.2865 \\ -0.2865 & 0.7248 \end{bmatrix}, \; P_2 = \begin{bmatrix} 2.6946 & 0.6532 \\ 0.6532 & 0.7807 \end{bmatrix}$$

To validate the designed consensus protocol, we perform a simulation of the closed-loop system with trajectories starting from the initial conditions

$$x_1(0) = \begin{bmatrix} \frac{\pi}{10} \\ 0 \end{bmatrix}, \; x_2(0) = \begin{bmatrix} \frac{\pi}{12} \\ 0 \end{bmatrix}, \; x_3(0) = \begin{bmatrix} \frac{\pi}{14} \\ 0 \end{bmatrix},$$

$$\hat{x}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \hat{x}_3(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; s(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the exogenous time-varying scheduling parameters of the followers and the leader are defined as

$$\alpha_1(\rho_1(k)) = \frac{1 + \sin(3t(k))}{2} \qquad \alpha_2(\rho_1(k)) = 1 - \alpha_1(\rho_1(k)),$$
$$\alpha_1(\rho_2(k)) = \frac{1 + \cos(4t(k))}{2} \qquad \alpha_2(\rho_2(k)) = 1 - \alpha_1(\rho_2(k)),$$
$$\alpha_1(\rho_3(k)) = \frac{1 + \sin(0.01t(k))}{2} \qquad \alpha_2(\rho_3(k)) = 1 - \alpha_1(\rho_3(k)),$$
$$\alpha_1(\rho_s(k)) = \frac{1 + \cos(7t(k))}{2} \qquad \alpha_2(\rho_s(k)) = 1 - \alpha_1(\rho_s(k))$$

The closed-loop trajectories of the angular positioning LPV MAS are depicted in Figure 9. As shown in Figure 9, with the designed consensus protocol, the angular positioning system can successfully track the formation of the target. Notice that due to compensation signals $r_i(k)$, the consensus error depicted in Figure 10 converges exponentially to the origin, and the formation is maintained even during the transient period when the angular velocities are not null and $w_s(k)$ is actively disturbing the augmented error system.

Furthermore, we show by the consensus error depicted in Figure 11 that if the compensation signals $r_i(k)$ are neglected, the angular positioning system loses its desired formation during the transition times, highlighting the importance of the proposed method.

5 | Conclusions

This paper has addressed the problem of consensus in leader-follower formation for multi-agent systems represented

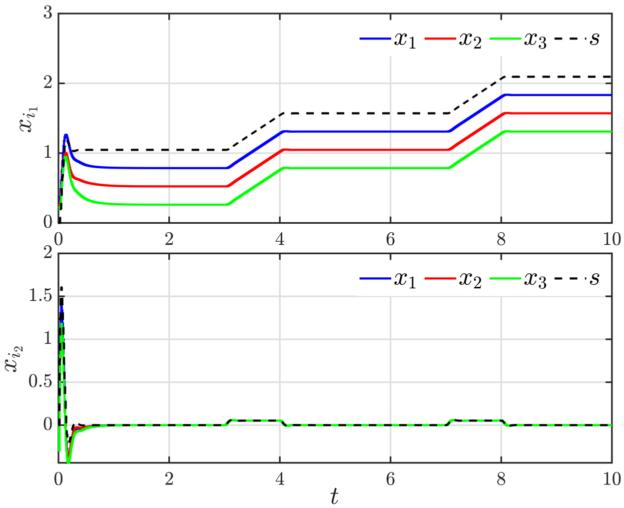

FIGURE 9 | Trajectories of the leader and following agents equipped with the proposed formation consensus protocol designed with Theorem 1 — Example 4.2.

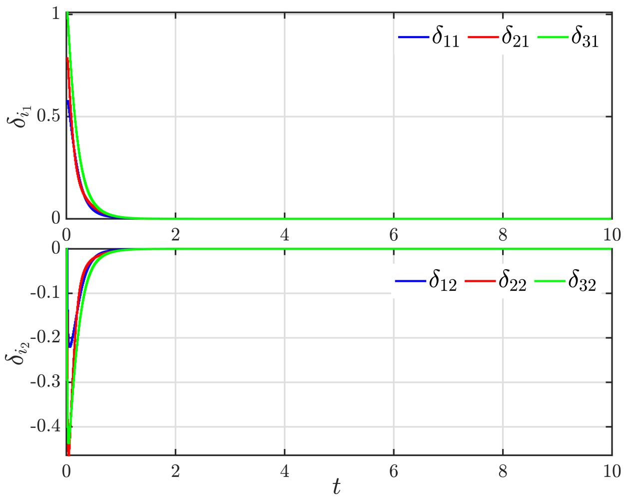

FIGURE 10 | Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 1 — Example 4.2.

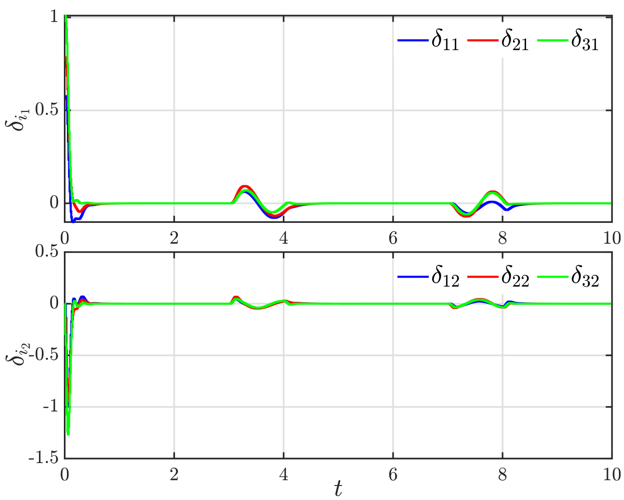

FIGURE 11 | Trajectories of the consensus error obtained with the consensus protocol designed with Theorem 2 without the compensation signals $r_i(k)$ — Example 4.2.

by discrete-time polytopic LPV models. The modeling employed investigates a more general scenario, where although all agents belong to a homogeneous polytopic domain, it allows a mismatch among the scheduling parameters of the agents and the leader. Under this scenario, the dynamics of the consensus tracking error are subject to internal perturbations that might prevent the closed-loop system from achieving the desired formation. To properly deal with this issue, we proposed a novel distributed gain-scheduled observer-based consensus protocol that, in addition to the classical relative information concerning the neighboring agents, also possesses compensation signals to cancel these perturbations. It has been shown that, with the design of these additional components, the proposed protocol enhances the set of possible formations even with mismatches in the physical modeling os the agent. Moreover, for the cases where we cannot design the compensation signals, we have presented a ℓ_∞ analysis condition that provides an upper bound for the disturbed augmented error system and guarantees a practical formation. Numerical experiments highlight the effectiveness of the proposed consensus protocol in achieving the exact leader-follower consensus even when mismatches among the time-varying scheduling parameters introduce heterogeneity into the MASs. One limitation of the proposed protocol is the requirement of ideal and undirected communication among agents. However, dealing with network issues such as communication delays, packet dropouts, and cyberattacks goes beyond the scope of this work and will be addressed in future research.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data was created or analyzed in this study.

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