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ORIGINAL RESEARCH

Forecasting human development with an improved Theta method based on forecast combination

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Abstract
Forecasting human development is important for tracking sustainable growth and societal progress. However, this task presents statistical challenges. The primary difficulty is the limited nature of the available data, which is a typical problem encountered in forecasting many social time series. In this paper, we propose a novel approach for forecasting short time series based on the Theta method. The classical Theta method decomposes the time series into trend and short-run components. We propose an improved version of the Theta method, called θ-comb, based on the combination of alternative forecasts for the short-run component. We apply the proposed method to forecast worldwide human development, measured with the Human Development Index, from 1990 to 2022. The results show that the θ-comb method significantly improves the out-of-sample accuracy in comparison to existing approaches.

Keywords Predictions · Short time series · Theta method · Forecast combination · Human Development Index

1 Introduction

Forecasting human development has significant implications for economic and policy-making perspectives. Accurate forecasting of human development can inform targeted policy interventions, help monitoring progress towards global development goals, and ensure that resources are effectively allocated. The Human Development Index (HDI) is a comprehensive measure of a country's human development, incorporating three fundamental dimensions, namely; longevity as measured by the life expectancy index, knowledge as measured by the education index and standard of living as measured by the Gross National Income index.

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The HDI allows policymakers to move beyond the Gross Domestic Product (GDP) and obtain a more robust understanding of human welfare, as the HDI allows the three socioeconomic dimensions to be isolated and examined independently. By using the HDI, policymakers can identify specific areas in need of intervention and tailor economic and social policies accordingly, thereby increasing the effectiveness of development strategies (Shaydullina, 2020; Korankye et al., 2020). For instance, understanding the distribution of educational opportunities can inform targeted investments in educational infrastructure, which, in turn, can enhance human capital and productivity. As noted by Okoh et al. (2020), human development is fundamental for expanding people's choices and capabilities, which underscores the importance of equitable access to education and health services.

From a policy perspective, the dimension of knowledge used in the HDI is well discussed by economists, who have defined "human capital" as the investment that humans can make in their own capabilities through, education, training and skills (similar to a stock of productive capital). Investments in education and training yield returns in the labour market, as individuals with higher levels of education tend to secure a higher market wage. This relationship highlights the need for policies that not only increase the quantity of education, but also improve its quality through complementary investments in educational infrastructure (IT facilities, laboratories, etc.) and resources that serve to upskill the labour force to a higher value-added productivity level. Such educational policies can lead to enhanced productivity and, consequently, a higher standard of living, which directly influences longevity through improved health awareness and access to healthcare services. Furthermore, macroeconomic policies that focus on both width growth factors (such as the amount of labour and capital employed in production) can significantly impact the standard of living. Policies to improve the utilisation of labour and capital, as well as policies to improve total factor productivity, are crucial for sustainable development. These efforts can create a positive feedback loop in which improved living standards contribute to better health outcomes, thereby increasing life expectancy. The interplay between these dimensions of the HDI illustrates the complexity of human development and the need for integrated policy approaches that address economic, social and environmental factors (Palchoudhuri et al., 2015; Thomas & Allen, 2021).

Therefore, forecasting human development through the lens of the HDI provides valuable insights for policymakers. By understanding the links between longevity, knowledge and living standards, and by addressing disparities within these dimensions, policymakers can design more effective and equitable development strategies. This holistic approach not only supports economic growth and the associated positive spillover effects from a more developed socioeconomic society, but also improves the overall quality of life of citizens, in line with the broader goals of sustainable development (Okoh et al., 2020; Deng et al., 2018).

However, forecasting HDI poses statistical challenges. One of the main difficulties lies in the nature of the data available. The HDI values are typically reported annually and often span only a few decades. Therefore, the HDI is a typical example of a short social time series. We notice that short time series can also be found in other domains, such as management (e.g. see Thomakos et al, 2023) and tourism (e.g. see Yu and Schwartz, 2006). The limited sample size makes traditional time series forecasting methods less effective, as these models typically rely on longer data series to produce accurate and robust predictions. Additionally, the HDI's multifaceted composition introduces complexities related to the interaction of its subcomponents over time.

Given these challenges, a particularly promising approach for dealing with short and complex time series such as the HDI is the Theta method (Nikolopoulos & Thomakos, 2019). The Theta method decomposes a time series into different components to improve the accuracy of predictions. In particular, the original Theta method, proposed by Assimakopoulos and

Nikolopoulos (2000), operates by decomposing the original series into two modified versions. The first extrapolates the linear trend of the series, effectively capturing the long-term growth or decline within the series. The second component, on the other hand, is modelled using Simple Exponential Smoothing (SES) to capture short-term variations. The final forecast is obtained by averaging the two forecasts.

Despite the usefulness of the SES model in dealing with its effectiveness can be limited when applied to short-time series data, as is the case for the HDI and its components. Indeed, the inherent variability and complexity of social indicators may not be adequately captured by a simple smoothing approach. Therefore, in this paper, we propose an enhanced Theta method that improves on the traditional approach by incorporating a combination of different forecasting techniques for the second Theta component. By introducing forecast combination into the Theta method, we aim to synthesize the strengths of multiple models in forecasting the short-run component, creating a more robust and flexible framework.

We forecast worldwide human development, measured with the HDI, considering data from 1990 to 2022. It should be noted that not all 195 countries show the full history of available time series. Therefore, we consider for each dimension only the countries with full data from 1990 to 2022. The resulting application involves 142 short time series. We find that the proposed approach considerably improves the out-of-sample accuracy compared to existing approaches, namely the standard Theta method and the more recent optimal Theta developed by Fiorucci et al. (2016). In particular, we find that the Theta-related methods all provide better forecast accuracy compared to simple existing statistical methods that are applicable for short time series, that is, ARIMA, SES and Random Walk. Among the Theta-related methods, the proposed approach based on forecast combination provides the best results. We also find that the proposed method provides better results than the forecast combination of the considered base statistical methods. Therefore, the joint use of Theta decomposition and forecast combination allows for improvements in forecasting accuracy of human development.

The rest of the paper is structured as follows. Section 2 describes how human development is measured by the United Nations, describing the Human Development Index calculation and its components. Moreover, Sect. 2 presents the data considered for the application. Section 3 presents the Theta method in details and the proposed Theta approach based on forecast combination. Section 4 shows the main forecasting results to the Human Development Index. In the end, Sect. 5 concludes with final remarks and future research directions.

2 Measuring human development

2.1 The HDI index

The Human Development Index (HDI) is a composite indicator developed by the United Nations Development Programme (UNDP) to assess and compare the social and economic development of countries. It consists of three key dimensions, that are health, education, and standard of living.

The Life expectancy at birth is the indicator representing the health dimension and is computed as the average number of years a newborn is expected to live, assuming constant mortality rates at each age. For the education dimension, two variables are considered, i.e. the "Mean Years of Schooling" and "Expected Years of Schooling". The "Mean Years of Schooling" is the average number of years of education received by people aged 25 and older,

Table 1 Goalposts for the HDI dimensions

Dimension	Indicator	Min.; Max.
Health	Life expectancy at birth (years)	20; 85
Education	Expected years of schooling (years)	0; 18
	Mean Years of Schooling (years)	0; 15
Standard of living	GNI per capita (2017 PPP$)	100; 75000

based on educational attainment levels, while the expected years of schooling is computed as the total number of years of schooling a child entering school is expected to receive, assuming age-specific enrollment ratios remain constant throughout the child's life. In the end, the Gross National Income (GNI) per Capita is the indicator selected for summarizing the standard of living dimension. It reflects the average income of a country's citizens and is adjusted for purchasing power parity (PPP) to account for differences in the cost of living and inflation rates.

The raw values of each indicator are normalized using the minimum-maximum scaling. Minimum and maximum values are used to transform the indicators expressed in different units into indices between 0 and 1. These goalposts parameters act as "the natural zeros" and "aspirational targets" respectively, from which component indicators are normalized.

Therefore, the dimension indices are calculated using the formula

$$Z_{pt} = \frac{I_{pt} - \min(I_{pt})}{\max(I_{pt}) - \min(I_{pt})}, \tag{1}$$

where I_{pt} is the actual value of the indicator p at time t and minimum and maximum for I_{pt} are defined according to Table 1.

For the education dimension, the arithmetic mean of the indices for expected years of schooling and "Mean Years of Schooling" is taken. Next, the normalized values for the three indicators, within each dimension are averaged to obtain the dimensional index, and are aggregated through the geometric mean. In formula, we have that HDI at a given time point t is computed as follows

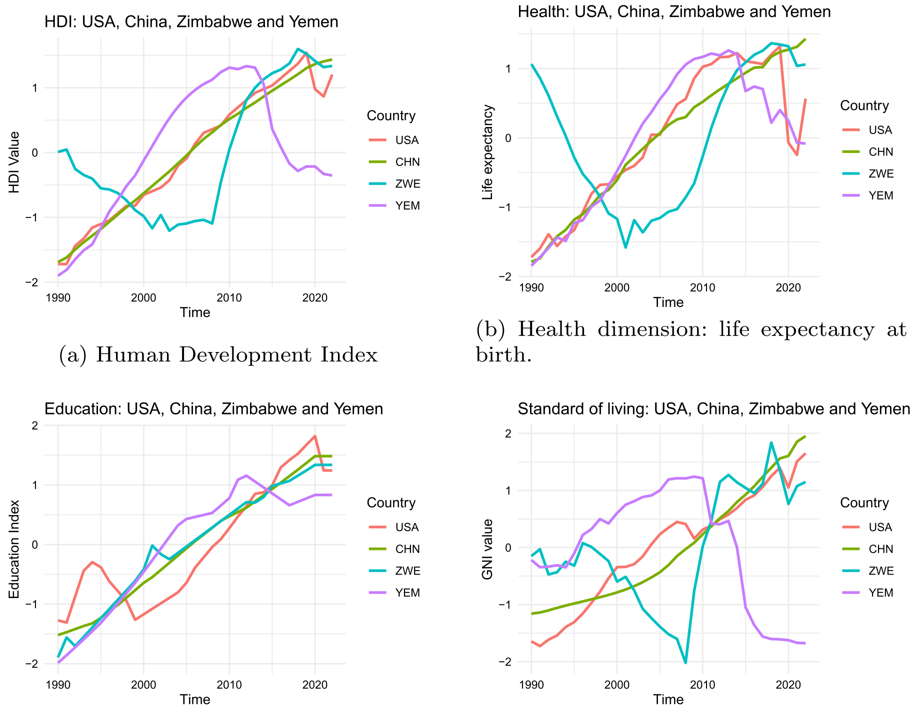
$$X_t = \left(\prod_{i=1}^{P} Z_{pt}\right)^{1/P}, \tag{2}$$

where $P = 3$ and Z_{pt} represents the p-th dimensional indicator at time t. We remark that the normalized indicators Z_{pt} refer to a generic i-th Country, but we omitted the i-th subscript for readers convenience. The calculation of the index is the same for all the $i = 1, \ldots, N$ Countries.

2.2 Data

We consider data on human development from the United Nations Development Program (UNDP) website.[1] Our aim is to forecast the Human Development Index and its components shown in Table 1, i.e. health, education and standard of living dimensions. To forecast the

[1] Data associated with this paper is available at the following link: https://hdr.undp.org/data-center/documentation-and-downloads.



(a) Human Development Index

(b) Health dimension: life expectancy at birth.

(c) Education dimension: average between expected and "Mean Years of Schooling".

(d) Standard of living dimension: GNI per-capita.

Fig. 1 Human development time series for USA, China, Zimbabwe, and Yemen. The values in the figure are standardized to compare countries with different values

health dimension we consider the life expectancy at birth variable, while for education we forecast the average of expected years of schooling and "Mean Years of Schooling" variable. Finally, we forecast the GNI per capita to predict the future standard of living. The number of Countries with full available history of data, that is, with time series are available from 1990 to 2023, is $N = 142$. The considered time series have length $T = 33$, and thus are short, presenting significant challenges for their predictions.

Figure 1 shows an example of the four-dimensional time series, i.e., the HDI and its constituents, for four countries: the USA, Iran, Zimbabwe, and Yemen. These countries were chosen to represent a range of Human Development Index (HDI) levels: very high (USA), high (China), medium (Zimbabwe), and low (Yemen). This selection highlights the varied development paths and socio-economic conditions across different regions. The USA, with its consistently high HDI, demonstrates a steady upward trend driven by strong economic, educational, and health indicators. China, which is also classified as a highly developed country, has demonstrated comparable progress to that of the USA. While the USA has consistently exhibited higher values than China in terms of health, the reverse is true in the context of education. Finally, both countries have exhibited convergent trends in terms of standard of living, particularly in recent years. Zimbabwe represents a medium HDI country according to the last HDI values, and exhibits significant variability over time reflecting periods of economic crisis and recovery influencing its development trajectory. In particular, Zimbabwe HDI time series shows a decreasing trend for the first decade (1990–2000) and then an increasing trend from 2010. Similar patterns can be found in all human development dimensions. Finally, Yemen, an example of low HDI, exhibits a contrasting trend compared to the aforementioned countries, demonstrating an increasing stable trend from 1990 to 2010, followed by a pronounced decline, further exacerbated by ongoing conflict and humanitarian challenges, which impact its development. As demonstrated in Fig. 1, both long-term trends and short-term fluctuations are highlighted. The former is attributed to various shocks, including political changes, economic recessions and natural disasters. This variability underscores the importance of employing a method capable of accurately capturing both the broader trends and short-term deviations to improve predictive analysis and inform policy decisions effectively.

3 Methodology

3.1 Theta method

Forecasting worldwide human development requires forecasting a relatively large amount of short time series. Therefore, simple but robust forecasting methods for univariate time series are very important in this context. Given the short-length feature of these types of time series, simple methods—without many parameters to estimate—are the main candidates to be used for forecasting. The Theta method (Assimakopoulos & Nikolopoulos, 2000) is a relatively simple model, which performed very well in the M3-Competition (Makridakis & Hibon, 2000). Therefore, it is commonly considered a benchmark in more recent forecasting competitions (e.g. see Athanasopoulos et al., 2011; Makridakis et al., 2020).

The Theta method decomposes the seasonally adjusted data into two “theta lines”, the first removing the curvature of the data to estimate the long-term trend component and the second approximating its short-term behaviour. Therefore, the Theta method can be considered as a forecasting with decomposition approach.

The Theta method can be breifly outlined as follows in the context of human development forecasting. Let us defining an univariate time series for the i-th country ($i = 1, \dots, N$) as $\{X_{i1}, \dots, X_{it}, \dots, X_{iT}\}$. Assimakopoulos and Nikolopoulos (2000) define the following transformed time series $\{Y^{\theta}_{i1}, \dots, Y^{\theta}_{it,}, \dots, Y^{\theta}_{iT,}\}$ such that for $t = 3, \dots, T$ it holds

$$\nabla^2 Y^{\theta}_{it} = \theta \nabla^2 X_{it}, \tag{3}$$

with ∇ indicating the difference operator, so that $\nabla X_{it} = X_{it} - X_{it-1}$. Hyndman and Billah (2003) noted that (3) is a second difference equation with solution

$$Y^{\theta}_{it} = (1 - \theta)\,(a + bt) + \theta X_{it}, \tag{4}$$

where a and b are constants and $t = 1, \dots, T$. Therefore, the series $Y_{it,\theta}$—called the θ-line—is an additive function of the series X_{it} and a linear trend.

It is clear from (4) that different θ parameters can be selected, leading to alternative decompositions. In general, the θ parameter is chosen to adjust the shape of the data, as it determines the influence of the trend and the local curvatures in the adjusted series. More precisely, we can define the following scheme (Petropoulos & Nikolopoulos, 2017).

- For $\theta = 1$ we obtain the original time series X_{it};
- For $0 < \theta < 1$ the long-term pattern is amplified, thus implying a smoothing of the data that emphasizes the overall trend;

- For $\theta = 0$, the series is a linear trend, that is, $Y_{it} = a + bt$;
- For $\theta > 1$ the curvature is amplified, and the decomposition emphasizes short-term fluctuation;
- For $\theta < 0$ the resulting θ curve reflects the opposite pattern of the linear trend.

In the original Theta method adopted in the M3 competition, Assimakopoulos and Nikolopoulos (2000) selected two θ lines, that is, $\theta = 0$ and $\theta = 2$, leading to $Y_{it}^{(\theta=0)} = a + bt$ and $Y_{it}^{(\theta=2)} = 2X_{it} - (a + bt)$. We remark that the parameters a and b are the minimum square coefficients of a simple linear regression of $\{X_{i1}, \ldots, X_{it}, \ldots, X_{iT}\}$ over $\{1, \ldots, t, \ldots, T\}$, thus having known closed-form solutions. Therefore an estimate of the first theta line is $\hat{Y}_{it}^{(\theta=0)} = \hat{a} + \hat{b}t$, while the second is $\hat{Y}_{it}^{(\theta=2)} = 2X_{it} - \hat{a} - \hat{b}t$ given that X_{it} is known. The forecast at time $t + 1$ is obtained as

$$\hat{X}_{it+1} = \omega \hat{Y}_{it+1}^{(\theta=0)} + (1 - \omega)\hat{Y}_{it+1}^{(\theta=2)}. \tag{5}$$

Hence, a linear ω-combination of the forecasts obtained for the two theta lines, where the first is simply obtained by extrapolation of the linear trend, that is $\hat{Y}_{it+1}^{(\theta=0)} = \hat{a} + \hat{b}(t + 1)$, while the second is obtained with Simple Exponential Smoothing (SES).

Originally, Assimakopoulos and Nikolopoulos (2000) assumed equal combination weights, i.e. $\omega = 0.5$, while more recently Fiorucci et al. (2016) developed an approach for optimal weights ω selection. However, Thomakos and Nikolopoulos (2014) shown that the assumption $\omega = 0.5$ in the case of two theta lines decomposition with $\theta = 0$ and $\theta = 2$ corresponds to the optimal solution when the Data Generating Process (DGP) for X_{it} is a unit root autoregressive model with uncorrelated innovations. Thomakos and Nikolopoulos (2014) explain that a possible explanation of the very good performances of the Theta method could be that this DGP assumption was compatible for many time series considered in the M3 competition.

According to Petropoulos (2019), the standard theta method can be extended by considering several deviations from the standard set-up, such as the inclusion of multiple theta lines (Petropoulos & Nikolopoulos, 2013), the use of unequal combination weights (Fiorucci et al., 2016; Spiliotis et al., 2020), or the adoption of alternative extrapolation methods. In what follows, we aim at extending the standard Theta method in the last direction, by proposing the use of a combination of alternative extrapolation approaches.

3.2 Improved Theta method with combination

Despite the utility of the SES model for handling short-term fluctuations, its effectiveness can be limited when applied to short time series data, as is the case of HDI. The inherent variability and complexity of social indicators may not be adequately captured by a simple smoothing approach. For this reason, we propose an enhanced Theta method that improves on the traditional approach by incorporating a combination of different forecasting techniques for the second Theta component.

In particular, we consider the combination of forecasts for the time series $Y_{it+1}^{(\theta=2)}$ using a set of K alternative forecasting methods, where $k = 1, \ldots, K$. Let $\hat{Y}_{it+1}^{(\theta=2)}$ denote the forecast generated by the k-th method. To construct the combined forecast, we define λ_k as the combination weights associated with each forecasting method. The combined forecast for $Y_{it+1,\theta=2}$ is expressed as

$$\hat{Y}_{it+1}^{(\theta=2)} = \sum_{k=1}^{K} \lambda_k \hat{Y}_{it+1,k}^{(\theta=2)}, \tag{6}$$

where λ_k are combination weights that sum to one, that is, $\sum_{k=1}^{K} \lambda_k = 1$.

The benefit of combining forecasts lies in leveraging the strengths of different models, which can reduce forecast variability and improve overall predictive accuracy (e.g. see see Timmermann, 2006). This approach reduces the risk of relying solely on a single model, particularly when individual models may have limitations due to the short length of the time series or specific data characteristics.

As a result, the θ-comb approach forecast can be written as

$$\hat{X}_{it+1} = \omega \hat{Y}_{it+1}^{(\theta=0)} + (1-\omega)\left(\sum_{k=1}^{K} \lambda_k \hat{Y}_{it+1,k}^{(\theta=2)}\right). \tag{7}$$

In accordance to Assimakopoulos and Nikolopoulos (2000) and to equally balance the contributions of the short-run and long-run components we keep $\omega = 0.5$. We remark that this choice is motivated by the limitations of short time series, where data scarcity makes it challenging to estimate optimal weights reliably. While optimization-based approaches could in principle be applied, these solutions tend to perform poorly and are prone to overfitting and instability when dealing with limited data. A simple, equal-weight combination provides a more robust and interpretable solution, avoiding the issues related to data-driven optimization in this context.

The main issue for implementing the (7) with $\omega = 0.5$ is the determination of combination weights λ_k. The simplest and often most effective method for combining forecasts is the sample average, where $\lambda_k = 1/K \ \forall k = 1, \ldots, K$. The combined forecast at time $t+1$ can be expressed as

$$\hat{Y}_{it+1}^{(\theta=2)} = \frac{1}{K} \sum_{k=1}^{K} \hat{Y}_{it+1,k}^{(\theta=2)}. \tag{8}$$

However, literature proposed a great variety of more complex approaches, selecting the λ_k weights optimally (for a review, see Wang et al., 2023). Bates and Granger (1969) proposed a method, similar to portfolio selection, for finding optimal weights by minimizing the variance of the combined forecast error. They discussed only combinations of pairs of forecasts, while Newbold and Granger (1974) extended the method to combinations of more than two forecasts. In general, the "minimum variance" approach in the case of K different alternative methods involves the minimization of the following objective function

$$\min_{\boldsymbol{\lambda}} \boldsymbol{\lambda}' \boldsymbol{\Sigma}_e^{-1} \boldsymbol{\lambda}, \tag{9}$$

where $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_K]'$ is the K-dimensional vector of unknown optimal weights and $\boldsymbol{\Sigma}_e$ the K-dimensional forecast error covariance matrix. The solution to (9) give the optimal weights

$$\lambda_k = \frac{1/\sigma_k^2}{\sum_{k=1}^{K} 1/\sigma_k^2}, \quad k = 1, \ldots, K \tag{10}$$

if correlations across forecast errors are ignored, while are equal to

$$\boldsymbol{\lambda} = \frac{\boldsymbol{\Sigma}_e^{-1}\mathbf{1}_K}{\mathbf{1}_K'\boldsymbol{\Sigma}_e^{-1}\mathbf{1}_K}. \tag{11}$$

if correlations across K methods are considered, and where $\mathbf{1}_K$ is a K-dimensional vector of ones, while $\boldsymbol{\Sigma}_e^{-1}$ is the inverse of the K-dimensional forecast error covariance matrix, which is unknown and need to be estimated—for instance using the sample estimator—and σ_k^2 is the k-th diagonal element of the matrix $\boldsymbol{\Sigma}_e$.

The minimum-variance approaches belong to the class of the performance-based weights, given that methods with lower forecast error variance are weighted more than others. An alternative performance approach is the Inverse Weighting (InvW,Aiolfi and Timmermann, 2006), where the weights λ_k depend on the ranking of the methods in terms of some predefined accuracy measure, usually the Root Mean Square Error (RMSE). Therefore, larger weights are given to methods ranked first.

Finally, another relatively simple combination approach are those based on regression methods (e.g. see Granger and Ramanathan., 1984). In this case, the weights are chosen by the Ordinary Least Square (OLS)

$$Y_{it}^{(\theta=2)} = \lambda_0 + \sum_{k=1}^{K} \lambda_k \hat{Y}_{it,k}^{(\theta=2)} + \varepsilon_t. \tag{12}$$

This approach ensures unbiasedness of the resulting combined forecast even if some of the K forecasts $\hat{Y}_{it,k}^{(\theta=2)}$ are biased given the presence of the constant λ_0. However, the Constrained Least Square (CLS)

$$\begin{gathered} Y_{it}^{(\theta=2)} = \sum_{k=1}^{K} \lambda_k \hat{Y}_{it,k}^{(\theta=2)} + \varepsilon_t, \\ \text{s.t.} \sum_{k=1}^{K} \lambda_k = 1, \lambda_k \geq 0, \end{gathered} \tag{13}$$

which does not ensure unbiasedeness but previous research (e.g. see see Aksu and Gunter, 1992) show that (13) provides better results in out-of-sample compared to (12).

This paper constitutes an investigation into the results obtained through the implementation of our methodology in conjunction with a variety of combination approaches. However, we remark that, in the context of short time series, the estimation of the unknown quantities required to compute the optimal weights may be more challenging compared to usual settings. Moreover, cross-validation approaches are not applicable in this setting (Hyndman & Athanasopoulos, 2021) and, therefore, the combination weights will be selected considering in-sample information. Despite these challenges, we aim to assess whether these approaches based on optimal combination offer improvements over the standard Theta method proposed by Assimakopoulos and Nikolopoulos (2000). The K forecasting methods to combine are discussed in the next Sect. 4.1.

4 Forecasting human development: results

4.1 Forecasting experiment design

To evaluate the appropriateness of the method in forecasting the time series described in Sect. 2, we adopt the following rolling-window approach. We consider the first $M = 15$ years as estimation window and obtain a forecast at time $t = M + 1$, i.e. $\hat{X}_{iM+1}$ with a generic forecasting method using information from $t = 1$ to $t = M$. Then, to forecast the time series at time $t = M + 2$, i.e. $\hat{X}_{iM+2}$ using information from $t = 2$ to $t = M + 1$. Therefore, in the rolling approach we recursively remove the oldest observation and include the newest. As a result we obtain a series of $T - M = 18$ years of out-of-sample testing where we evaluate the accuracy of alternative forecasting methods. As forecast accuracy measures, we consider both out-of-sample Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

Given the relatively short nature of the time series in our application, the number of models we can use is quite limited. In what follows, we consider the SES, the AutoRegressive Integrated Moving Average (ARIMA) and the naive models.

Given a generic time series $\{Y_{i1}, \ldots, Y_{it}, \ldots, Y_{iT}\}$, the SES model forecasts at $t + 1$ is given by applying an exponentially weighted average of past observations, that is,

$$\hat{Y}_{it+1} = \alpha Y_{it} + (1 - \alpha)\hat{Y}_{it}, \tag{14}$$

where $0 < \alpha \leq 1$ is the smoothing parameter, which can be estimated by minimizing the squared sum of errors. The ARIMA(p, q) model adopts p lagged values and q moving average components for modelling and it can be written as follows

$$(1 - \phi_1 B - \cdots - \phi_p B^p)(1 - B)^d Y_{it} = c + (1 + \gamma_1 B + \cdots + \gamma_q B^q)\varepsilon_{it}, \tag{15}$$

where B is the backshift operator such that $BY_{it} = Y_{it-1}$, $\phi_1, \ldots, \phi_p$ are the p autoregressive parameters, $\gamma_1, \ldots, \gamma_q$ are the q moving average parameters, d is the degree of differencing, c is the constant term and ϵ_t represents white noise process. The model parameters are estimated using maximum likelihood, while the optimal model order is determined following the stepwise procedure explained in Hyndman and Khandakar (2008), which aims at finding the model with minimum AICc (Hurvich & Tsai, 1989), that is the Akaike Information Criterion with small-sample correction to account for the short length of the considered time series. Finally, the naive method assumes that the forecast is equal to the most recent observation, that is,

$$\hat{Y}_{it+1} = Y_{it}. \tag{16}$$

Moreover, to evaluate if the combination procedure improves compared to the existing Theta methods, we consider two additional benchmarks. First, we provide predictions with the Assimakopoulos and Nikolopoulos (2000) approach based on SES and equal combination weights $\omega = 0.5$. Second, we also consider the Optimal Theta method (OTM), developed by Fiorucci et al. (2016), where ω is chosen optimally. The OTM shows more accurate forecasts on the M3-competition data. However, the results shown in Fiorucci et al. (2016) are in terms of average accuracy measures. Therefore, it is difficult to evaluate if the improvements in the out-of-sample accuracy are achieved for the short time series. In what follows, we evaluate the ability of the OTM model to forecast short social time series as those about human development. Finally, we remark that to implement the θ-comb approach, we combine the forecasts of the three aforementioned simple univariate time series methods to forecast the second component of the Theta decomposition, i.e. SES, ARIMA and naive.

Table 2 Average out-of-sample forecast accuracy over the $N = 142$ countries

Forecasting method	RMSE	MAE
RW	68.7490	55.6416
ARIMA	61.1430	42.6846
SES	69.1237	56.2927
Theta	57.9163	43.1612
OTM	60.1228	44.1128
θ-comb (SA)	*55.0501*	*39.7363*

The best results is highlighted in italic

Table 3 Percentage of N countries showing improvements using the θ-comb approach as defined in (8), compared to the forecasting method in the row

Forecasting method	RMSE	MAE
RW	92.2535	95.0704
ARIMA	83.0986	69.7183
SES	*93.6620*	*97.1831*
Theta	73.9437	79.5775
OTM	75.3521	76.0563

Table 4 Resulting p-values associated with the Equal Predictive Accuracy test (Diebold & Mariano, 2002)

Forecasting method	RMSE	MAE
RW	*1.393e−27*	*3.422e−32*
ARIMA	*6.889e−13*	*2.354e−05*
SES	*1.642e−30*	*4.056e−37*
Theta	*2.042e−09*	*3.174e−14*
OTM	*9.405e−10*	*3.287e−11*

A natural last benchmark is the use of forecast combination to the time original time series. We apply the same combination procedure described in Sect. 3.2, but to the original forecasts. Therefore, the objective of this paper is twofold: firstly, to evaluate whether the combination is beneficial in itself; and secondly, to determine whether it enhances accuracy if included in the theta forecasting framework.

4.2 Main results

We first evaluate the out-of-sample performance of the naive combination procedure based on equal weights (Simple Average, SA). Table 2 shows the average out-of-sample forecast accuracy for the $N = 142$ countries. Both RMSE and MAE are computed, for each country, considering the 18 years in the out-of-sample dimension. The first interesting result of Table 2 is that the original Theta method of Assimakopoulos and Nikolopoulos (2000) performs better than the single benchmark methods, that is, RW, ARIMA and SES, therefore confirming that the Theta method improves the forecast accuracy for short time series compared to standard methods. Moreover, we find that the Theta method interestingly performs better than the optimally weighted approach of Fiorucci et al. (2016) (OTM in Table 2). Therefore, our results indicate that the use of equal weights between the two selected theta lines $\theta = 0$ and $\theta = 2$ provides better out-of-sample forecasts. The most interesting result shown in Table 2,

Table 5 Average out-of-sample forecast accuracy over the $N = 142$ countries

Forecasting method	RMSE	MAE
θ-comb (SA)	*55.0501*	39.7363
Theta-comb (BG)	55.1778	39.7467
Theta-comb (NG)	59.9590	43.3576
Theta-comb (InvW)	55.1346	*39.5866*
Theta-comb (CLS)	57.5882	41.4190

The best model is highlighted in italic

Table 6 Percentage of N countries showing improvements using "optimal" θ-comb approaches, compared to the forecasting method in the row

Forecasting method	Measures RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
	θ-comb (BG)		θ-comb (NG)		θ-comb (InvW)		θ-comb (CLS)	
RW	*92.9577*	*94.3662*	78.1690	85.2113	92.2535	92.9577	83.8028	88.7324
ARIMA	82.3944	75.3521	56.3380	51.4085	*82.3944*	*75.3521*	76.7606	70.4225
SES	92.2535	*95.0704*	80.2817	85.2113	*93.6620*	94.3662	85.2113	90.1408
Theta	69.7183	76.0563	35.2113	47.1831	*70.4225*	*78.8732*	45.7746	54.9296
OTM	*74.6479*	*77.4648*	54.2254	57.0423	73.9437	75.3521	61.2676	64.7887

The best model is highlighted in italic

however, is that the improved Theta method based on forecast combination—the so-called θ-comb—provides the best results according to both the considered metrics, RMSE and MAE. More precisely, the θ-comb with Simple Average (SA) combination has an RMSE (MAE) equal to 55.05 (39.73), which is about 5% (8%) lower than the standard Theta method.

The results therefore indicate that the proposed approach improves both the Theta method and its extension for optimal weights. This result can be explained by the well-known benefit of forecast combination (e.g. see Timmermann, 2006). Instead of using the SES model for the second theta line capturing short-run fluctuations of the time series around the trend, the use of alternative forecasting approaches allows better forecasting of the short-run component. To further investigate if substantial improvements in the forecasting accuracy are achieved by the use of θ-comb approach, we analyze the percentage of N countries showing improvements using the θ-comb approach, as defined in (8), compared to the benchmark forecasting methods. Table 3 shows the result for both squared and absolute forecasting errors. A value larger than 50% indicates that the θ-comb improves the out-of-sample forecasting for more than half of the N countries considered in the sample.

The θ-combination approach proves to be a robust and generalizable forecasting strategy, capable of delivering significant improvements over a wide variety of baseline methods, underscoring the effectiveness of the θ-comb in enhancing forecasting accuracy across diverse metrics and contexts. Table 4 shows the p-values associated with the Equal Predictive Accuracy test (Diebold & Mariano, 2002). Under the null hypothesis we have the the two methods provide statistically equal forecasts out of the sample. The forecasting methods in the row are compared with the θ-comb (SA). We reject the null hypothesis for all the considered cases, thus concluding that the θ-comb (SA) provides statistically more accurate forecasts in out-of-sample.

Table 7 Resulting p-values associated with the Equal Predictive Accuracy test (Diebold & Mariano, 2002): θ-comb (SA) versus alternative combination schemes

Forecasting method	RMSE θ-comb (SA)	MAE
Theta-comb (BG)	4.655e−01	9.583e−01
Theta-comb (NG)	*1.348e−07*	*1.888e−06*
Theta-comb (InvW)	6.790e−01	5.232e−01
Theta-comb (CLS)	*1.150e−07*	*3.590e−04*

Table 8 Average out-of-sample forecast accuracy over the $N = 142$ countries

Forecasting method	RMSE	MAE
SA	59.7599	45.4308
BG	*55.8067*	*39.7924*
NG	60.5362	42.7949
InvW	56.7790	41.2765
CLS	59.4279	42.1651

The classical forecast combination approaches are compared with the benchmark methods. The best model is highlighted in italic

4.3 Do sophisticated combination approaches improve the naive?

The results in Tables 2 and 3 highlight the effectiveness of the θ-combination (θ-comb) approach under a Simple Average (SA) combination strategy. Table 5 extends this analysis by exploring whether more sophisticated combination methods can further enhance forecasting accuracy. Specifically, the table compares different θ-comb strategies, including Bates and Granger (1969) (BG), Newbold and Granger (1974) (NG), Inverse Weighting (InvW, Aiolfi and Timmermann., 2006), and Constrained Least Squares (CLS, Granger and Ramanathan, 1984), against traditional forecasting methods and both the standard Theta method and the optimal weights Theta method (OTM, Fiorucci et al., 2016). In terms of both RMSE and MAE, the results clearly indicate that θ-comb strategies consistently outperform traditional forecasting methods across both metrics. The best results are achieved by the θ-comb (InvW) method, with RMSE and MAE values of 55.13 and 39.59, respectively. These values are highlighted in italic, confirming that the Inverse Weighting approach leads to the most accurate forecasts among all considered methods. The improvement over the standard Theta method is substantial, reducing RMSE by approximately 4.8% and MAE by 8.3%. The CLS strategy, while not as effective as InvW or BG, still outperforms traditional methods like RW, ARIMA, and SES and the existing Theta methods. Table 5 highlights the versatility and robustness of the θ-comb approach in improving forecasting accuracy. Although the standard Theta method provides a strong baseline, the use of advanced combination techniques such as InvW enhances its predictive performance significantly. This finding underscores the importance of incorporating data-driven weighting schemes in combination models to better account for varying forecasting patterns across countries. By leveraging these sophisticated techniques, the proposed θ-comb method demonstrates its capacity to achieve state-of-the-art results in large-scale forecasting problems.

However, the SA method, whose performance is shown in Table 2, yields very competitive results. Indeed, the θ-comb (SA) provides better performances than sophisticated

Table 9 Average out-of-sample forecast accuracy over the $N = 142$ countries

Forecasting method	RMSE	MAE
SA	59.7599	45.4308
BG	55.8067	39.7924
NG	60.5362	42.7949
InvW	56.7790	41.2765
CLS	59.4279	42.1651
θ-comb (SA)	*55.0501*	39.7363
θ-comb (BG)	55.1778	39.7467
θ-comb (NG)	59.9590	43.3576
θ-comb (InvW)	55.1346	*39.5866*
θ-comb (CLS)	57.5882	41.4190

The classical forecast combination approaches are compared with the proposed θ-comb methods. The best method is highlighted in italic

approaches in terms of RMSE, while the InvW combination provides more accurate out-of-sample predictions in terms of MAE. Therefore, the difference between the two methods is not substantial.

The good performance of the SA combination approach is not surprising. Indeed, despite the existence of sophisticated combination methods, our results confirm the large empirical evidence (e.g. see Blanc and Setzer, 2016; Hsiao and Wan, 2014; Smith and Wallis, 2009) showing that the simple average with equal weights outperforms more complicated weighting schemes. The majority of explanations concerning the reasons why simple averaging might prevail over complex combinations in practice have focused on the errors that occur when estimating the combination weights. This is particularly evident in the context of short time series data.

Table 6 presents the percentage of countries N for which different "optimal" θ-combination (θ-comb) approaches outperform traditional forecasting methods, evaluated using RMSE and MAE as error metrics. Each row represents a baseline forecasting method, and the columns correspond to the performance of different θ-comb approaches. The best-performing θ-comb approach for each baseline method is highlighted in italic. The outcomes emphasize the adaptability and efficacy of the θ-comb framework, with specific methodologies exhibiting pronounced enhancements relative to the benchmark.

Moreover, the results also demonstrate that sophisticated methods like InvW and BG consistently deliver the highest improvement rates, particularly for benchmarks like RW, SES, and ARIMA. Even in cases where improvement rates are closer to 70%, such as against the Theta method and OTM, the performance of the θ-comb approaches remains notable, as these improvements apply to the majority of countries. Overall, the table highlights the potential of the θ-comb framework to generalize effectively and achieve state-of-the-art results across a variety of forecasting contexts. Table 7 shows the p-values associated with the Equal Predictive Accuracy test, comparing the simple average combination in the Theta-comb method with sophisticated approaches. Under the null hypothesis, the simple average combination provides statistically equal accuracy compared to sophisticated combination approaches. The results highlight that the θ-comb method based on simple average combination is statistically better than NG and CLS approaches, while the results are comparable considering BG and InvW combination methods.

Table 10 Percentage of N countries showing improvements using "optimal" θ-comb approaches, compared to the forecasting method in the row

Forecasting method	Measures RMSE θ-comb (SA)	MAE	RMSE θ-comb (BG)	MAE	RMSE θ-comb (NG)	MAE	RMSE θ-comb (InvW)	MAE	RMSE θ-comb (CLS)	MAE
SA	*87.3239*	88.7324	83.8028	81.6901	54.9296	64.0845	82.3944	78.8732	63.3803	70.4225
BG	66.9014	60.5634	*75.3521*	64.0845	26.0563	30.2817	67.6056	*64.7887*	36.6197	42.2535
NG	84.5070	73.9437	*86.6197*	*76.0563*	57.0423	51.4085	84.5070	76.0563	71.8310	66.9014
InvW	81.6901	74.6479	73.2394	69.7183	33.0986	42.9577	*73.2394*	*72.5352*	42.2535	53.5211
CLS	82.3944	71.1268	*82.3944*	*73.9437*	51.4085	47.1831	81.6901	73.9437	69.7183	67.6056

The best model is highlighted in italic

Table 11 Resulting p-values associated with the Equal Predictive Accuracy (Diebold & Mariano, 2002): θ-comb (SA) vs standard forecast combination

Forecasting method	RMSE θ-comb (SA)	MAE
SA	*3.165e−02*	8.762e−01
BG	*2.559e−12*	*4.240e−06*
NG	*2.132e−08*	*7.138e−07*
InvW	*2.485e−11*	*1.165e−04*
CLS	*5.879e−17*	*2.147e−20*

Table 12 Average out-of-sample forecast accuracy over the $N = 142$ countries for the HDI components

Forecasting method	RMSE LE	EDU	GNI	MAE LE	EDU	GNI
RW	55.1099	16.0151	74.4854	43.5096	12.1342	60.7923
ARIMA	54.9521	14.1424	72.4252	35.1601	9.7391	52.8677
SES	55.5731	16.0546	75.2697	44.0413	12.1810	61.6712
Theta	50.5321	13.6763	70.5229	35.5432	10.0772	55.3820
OTM	53.5212	14.6243	71.6099	37.0637	10.7944	55.6097
θ-comb (SA)	*49.1629*	*13.0388*	*67.8889*	*32.9654*	*9.4022*	*52.6513*

The best method is highlighted in italic

4.4 Does the θ-comb method improve forecast combination?

In what follows we evaluate the effectiveness of the θ-comb method compared to standard combination strategies that do not employ the θ decomposition. Table 8 presents the average out-of-sample of the considered benchmark approaches with respect to the combination procedures. We consider the same combination of approaches adopted for the implementation of the θ-comb method.

The results highlight that, among the classical forecast combination methods, the BG approach performs better than benchmarks, achieving the lowest RMSE and MAE, as highlighted in italic. However, all the considered combination approaches improve compared to the benchmarks, that is, RW, ARIMA, SES, standard Theta method and the OTM approach. We then evaluate if forecast combination has more value if included within the Theta decomposition scheme. In this regard, Table 9 compares the performance of classical forecast combination methods with the θ-comb methods.

The results show that θ-comb methods outperform classical combinations according to all accuracy measures. These results underscore the added value of incorporating Theta decomposition into the combination process. Furthermore, the performance gap between θ-comb methods and classical combinations demonstrates the advantage of using Theta decomposition to better capture the underlying structure of the data. In particular, θ-comb (InvW) reduces the MAE to 39.58, further improving on the best classical combination method (BG). Table 10 shows the percentage of countries showing improvements in forecast accuracy using θ-comb methods compared to classical methods. The results again demonstrate substantial gains across the board. In particular, we notice that the θ-comb (SA) approach improves RMSE and MAE in 87.32% and 88.73% of countries, respectively, compared to the classical SA method. Similarly, θ-comb (InvW) outperforms its classical counterpart in

Table 13 Resulting p-values associated with the Equal Predictive Accuracy (Diebold & Mariano, 2002) for the HDI components

Forecasting method	θ-comb (SA)					
	LE		EDU		GNI	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
RW	*5.403e−11*	*1.202e−26*	*8.949e−27*	*9.398e−24*	*1.003e−05*	*1.016e−06*
ARIMA	*4.319e−12*	*5.993e−04*	*1.739e−08*	*6.104e−02*	*4.694e−03*	8.943e−01
SES	*1.916e−12*	*2.368e−28*	*3.048e−28*	*7.021e−25*	*1.161e−06*	*8.542e−09*
Theta	*4.663e−02*	*6.558e−06*	*2.670e−09*	*2.072e−10*	*1.207e−02*	*8.687e−04*
OTM	*4.792e−05*	*2.143e−05*	*1.283e−16*	*5.891e−15*	9.682e−02	7.927e−02

73.24% (RMSE) and 72.54% (MAE) of countries. These findings highlight the robustness and adaptability of θ-comb methods in diverse datasets.

Finally, we evaluate if the differences in the out-of-sample accuracy are statistically significant. Table 11 shows the p-values of the Equal Predictive Accuracy test. The results show that the combination approach is more valuable if included in the theta decomposition. In fact, for all the consisdered standard co

4.5 Forecasting the HDI components

Finally, we apply the proposed θ-comb method under simple average combination to forecast the components of the Human Development Index (HDI), namely Life Expectancy (LE), Education (EDU), and Standard of living (GNI), across the considered set of countries. Following the same scheme of the main results, we first present the average out-of-sample forecast accuracy for each HDI component using multiple forecasting methods, including the Random Walk (RW), ARIMA, Simple Exponential Smoothing (SES), the classical Theta method, the Optimal Theta Method (OTM), and the newly introduced θ-comb (SA) method. The results are summarized in Table 12.

The forecast accuracy is assessed in terms of both the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE). As shown in Table 12, the θ-comb (SA) method outperforms all other methods across both error measures for all three HDI components. For instance, in the case of Life Expectancy (LE), θ-comb (SA) achieves the lowest RMSE (49.16) and MAE (32.97), outperforming the next best method, the Theta method, significantly. In a similar manner, for the Education (EDU) and Income (GNI) components, the combination of theta-comb (SA) has been demonstrated to provide netter results in both RME and MAE, thereby substantiating its superior forecasting capability across all constituent elements of the HDI.

In order to assess the statistical significance of these results, we conduct the Equal Predictive Accuracy (EPA) test, which compares the performance of the θ-comb (SA) method against each of the other forecasting methods. The p-values associated with the EPA test (Diebold & Mariano, 2002) are reported in Table 13.

For Life Expectancy and Education, the θ-comb (SA) method shows extremely small p-values when compared to RW, ARIMA, SES, and Theta, indicating that the improvements in accuracy are statistically significant. In the case of GNI, the θ-comb (SA) method shows statistically significant improvements compared to all methods except for OTM, where the test suggests that the two models are statistically equivalent on average.

5 Conclusions

Forecasting human development is crucial for policymakers, researchers, and international organizations committed to achieving sustainable growth and societal progress. Human development encompasses multiple dimensions, including health, education, and income, which collectively contribute to the well-being and quality of life of individuals within a society. Accurate forecasting of these indicators can inform targeted policy interventions, help monitor progress towards global development goals, and ensure that resources are effectively allocated. This paper explored forecasting human development through the Human Development Index (HDI) using an enhanced Theta method, addressing challenges posed by the HDI's multifaceted composition and the constraints of short time series data. Our findings underscore the critical importance of designing integrated policy strategies that simultaneously target the HDI's core dimensions-longevity, knowledge, and standard of living.

Forecasting HDI, however, is fraught with statistical complexities due to limited data availability and the intricate interplay of its components. Traditional forecasting methods such as ARIMA, Simple Exponential Smoothing (SES), and Random Walk struggle to achieve high accuracy in this context. To address these limitations, the study advanced the Theta method by integrating forecast combination strategies, leveraging the strengths of multiple models. This approach improves on the traditional Theta method by offering a more nuanced representation of long-term trends and short-term fluctuations, enhancing prediction reliability. Using HDI data spanning from 1990 to 2022 across 142 countries with complete datasets, our enhanced Theta method demonstrated superior out-of-sample forecast accuracy compared to both the standard Theta method and recent iterations like the optimal Theta model. It also outperformed traditional statistical methods tailored to short time series, validating its robustness in handling the variability and complexity of social indicators.

We proposed a type of the decomposition-combination approach which creates a more effective framework for projecting human development trends. These improved forecasts provide policymakers with a valuable tool for anticipating changes in HDI dimensions and devising equitable development strategies. Ultimately, this methodology supports the broader goal of supporting sustainable development by aligning economic growth with enhanced quality of life and resilience in the face of social and environmental challenges.

Future research can further develop the proposed framework, by comparing more sophisticated combination approaches considered in the paper. For example, while our work focuses on improving the short-run forecasts through a combination of alternative methods, a natural extension would be to explore different trend representations for the long-run component. This could involve combining multiple trend models to capture diverse growth patterns or allowing for polynomial trends with varying components to better adapt to structural changes in the data. Such developments could further improve the flexibility and accuracy of the Theta method in different forecasting contexts.

Moreover, we highlight that, while our study focuses on human development indicators, the proposed improved Theta method can be applied to other fields where the use of short time series is prevalent. For example, in macroeconomics many indicators are reported annually and often cover limited time spans, making short-time series forecasting essential. This fact is particularly salient in the context of developing economies. Similarly, in business and marketing, newly launched products generate only a few initial data points, requiring accurate forecasting methods for early-stage demand estimation. Another interesting application can be found in the demographic and social sciences, where indicators such as migration patterns or educational attainment rates may be available only for short historical periods. These examples illustrate the broader applicability of our approach and suggest future research directions.

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Data availability Data used in the paper are public (source UNDP, United Nations Development Program).

Declarations

Conflict of interest There is no Conflict of interest to declare.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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