Event-Triggered Control and Interactive LQR Tuning for Improved Control Efficiency

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***Abstract*—This work investigates the application of event-triggered control (ETC) in combination with the linear quadratic regulator (LQR) for discrete-time systems. A methodology is presented that uses an iterative approach to tune the LQR controller and reduces control effort by replacing periodic control with ETC. Lyapunov's stability theory is applied to design triggering mechanisms and analyse their effects on control design. Using the inverted pendulum on a cart as a reference system, it is demonstrated that proper tuning of the LQR controller and the implementation of an event-based control mechanism can reduce control energy and actuator effort.**

***Index Terms*—Non-linear Dynamical Systems, Event-Triggered Control, LQR, Inverted Pendulum.**

I. INTRODUCTION

Control systems constantly strive to optimise efficiency and performance. The Linear Quadratic Regulator (LQR) is known for its optimal control, balancing performance and control cost. However, selecting the Hermitian positive-definite matrices Q and R often through trial and error raises doubts about LQR's true optimality [1]. It is known that raising the values of elements in the weighting matrix Q enhances the system's dynamic response, but it comes at the cost of increased energy consumption. On the other hand, the matrix R is related to the energy consumption of the controller, but increasing the values of the matrix R too much may result in non-zero steady-state values. Several works have been developed aiming to improve the selection method of weighting matrices encompassing various techniques, such as analytical approaches [2], [3], stochastic methods [4], non-parametric optimisation [5], including genetic algorithms [6]–[8], among others.

In contrast, Event-triggered control (ETC) is acclaimed for activating control based on specific system states or outputs [9], [10], while also frequently reducing energy consumption in networked control systems (NCS). Event-driven control approach, with an emphasis on emulation or co-design, has been extensively studied and applied in various domains [11], such as power systems [12], advanced manufacturing [13], and robotics [14]. In [15], techniques for ETC in discrete-time linear systems with precisely known parameters were presented, applicable to both emulation and co-design. Furthermore, [16] introduces an extension of these techniques with a state observer. In [17] the emulation problem was considered for discrete-time linear parameter-varying systems (LPV), and [18] also presents event-triggered control techniques for LPV systems. On the other hand, [19] presents ETC techniques for non-linear systems, and in [20], [21], event-triggered control techniques for the class of non-linear control systems via network representation using quasi-LPV polytopic models were discussed.

Combining ETC with LQR could enhance system performance by cutting redundant control actions and further lowering energy consumption [19]. Since LQR already prioritises energy savings through appropriate weighting matrices, merging these techniques could lead to more efficient and sustainable control systems.

In this work, an approach is presented that simultaneously employs LQR and ETC for discrete-time systems. This novel methodology involves the design of a triggering mechanism based on the Lyapunov stability theory and the formulation of an optimisation problem in terms of Linear Matrix Inequalities (LMIs) to refine the mechanism by reducing the number of events. The impact of different weightings on the matrices Q and R on the system's performance is simultaneously investigated, and the consequences of using an ETC-based triggering device on control force concerning energy conservation and actuator effort are analysed. Finally, the effects based on the number of events observed in the trigger mechanism are also evaluated to show the efficiency of the proposed approach. While the combination of ETC and LQR is not new [22], [23], the main contribution of this paper lies in combining these two approaches, in the context of discrete-time systems, with a focus on improving control efficiency.

In this paper, the inverted pendulum on a cart serves as the reference system; however, the implications of these findings may apply to various engineering applications. It is demonstrated that with proper tuning of an LQR controller and the implementation of an event-based control mechanism, both peak and total control energy and actuator effort can be significantly reduced. This contributes to the ongoing discussion on the need for more efficient systems, promoting resource savings and addressing sustainability aspects.

II. TECHNICAL BACKGROUND

A. Linear Quadratic Regulator

LQR is a widely used control technique designed to minimise a quadratic cost function. This function typically represents the weighted quadratic deviations of the system state from a desired trajectory and the control inputs from their optimal values [24]. For a discrete-time linear system described by:

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

the quadratic cost function, J, to be minimised is:

$$J = \sum_{k=0}^{\infty} \left(x(k)^\top Q x(k) + u(k)^\top R u(k) \right), \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the control input vector, and $y \in \mathbb{R}^n$ is the system output. Both $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{n_u \times n_u}$ are weighting matrices that indicate the importance of state and control efforts in the optimisation process. Additionally, $Q = Q^\top \geq 0$ is positive semi-definite, and $R = R^\top > 0$ is positive definite.

Solving the LQR problem entails implementing optimal control, under the following static state-feedback control law, $u(k) = -Kx(k)$, and K is the gain matrix that can be calculated by solving the Riccati equation defined for its discrete form as: $A^\top SA - S + Q - A^\top SB(R + B^\top SB)^{-1}B^\top SA = 0$. With the solution $S = S^\top > 0$, the optimal gain reads $K = \left(R + B^\top SB\right)^{-1} B^\top SA$.

B. Event-Triggered Control

ETC is widely used in NCS [21], where the benefit of controlling spatially distributed systems encounters the challenge of required communication subsystems, often leading to unnecessary resource usage [19]. A more efficient strategy for conserving communication resources is to implement event-based control instead of periodic control. The main objective is to analyse system behaviour and identify key moments for generating control events. It updates the control input, $u(k) \in \mathbb{R}^{n_u}$, only when a condition is met, thus reducing overall updates and communication. This is done by formulating a function $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, such that $f(e(k), x(k)) > 0$, where $x(k)$, $e(k)$, and k represent the system states, measurement error, and discrete time instant, respectively. Each controller activation is dictated by the triggering device defined by f, implying the controller maintains the previous state until the mechanism activates again. Figure 1 illustrates the event-based mechanism, with ZOH representing the zero-order hold that retains the last control input until a new event occurs. When designing the trigger mechanism f, a key metric is comparing the system's performance with the number of activation events. The aim is to achieve optimal performance while minimising both errors and event count.

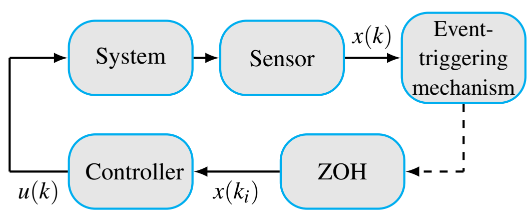

Fig. 1: Event-based control mechanism for precisely known systems.

III. MAIN RESULTS

A. Triggering Mechanism

Consider a discrete-time linear system described by (1), where the control law is the same as that defined in Section II-A and is expressed by

$$u(k) = Kx(k_i). \tag{3}$$

By using (3) in (1) one has

$$x(k+1) = Ax(k) + BKx(k_i). \tag{4}$$

Defining the measurement error in the interval $[k_i, k_{i+1})$ as $e(k) = x(k_i) - x(k)$, and replacing $x(k_i)$ in (4), the closed-loop system yields

$$x(k+1) = \mathscr{A}x(k) + \nu(k), \tag{5}$$

where, $\nu(k) = BKe(k)$, $\quad \mathscr{A} = A + BK$.

Based on the event-based control theory for linear discrete-time systems, the following Lemma provides conditions to certify the stability of the closed-loop system (5).

Lemma 1: If there exist positive definite matrices $P \in \mathbb{R}^{n \times n}$, $Q_\sigma \in \mathbb{R}^{n \times n}$, $Q_\delta \in \mathbb{R}^{n \times n}$, and matrices $X_1 \in \mathbb{R}^{n \times n}$, $X_2 \in \mathbb{R}^{n \times n}$, $X_3 \in \mathbb{R}^{n \times n}$ such that

$$\begin{bmatrix} \Psi & -X_1 + \mathscr{A}^\top X_2^\top & X_1 + \mathscr{A}^\top X_3^\top \\ X_2\mathscr{A} - X_1^\top & P - X_2 - X_2^\top & X_2 - X_3^\top \\ X_3\mathscr{A} + X_1^\top & -X_3 + X_2^\top & -Q_\delta + X_3 + X_3^\top \end{bmatrix} < 0, \tag{6}$$

hold with $\Psi = -P + Q_\sigma + X_1\mathscr{A} + \mathscr{A}^\top X_1^\top$, then the closed-loop system (5) is asymptotically stable under the event-based strategy, with the triggering function

$$f(e(k), x(k)) = \nu(k)^\top Q_\delta \nu(k) - x(k)^\top Q_\sigma x(k). \tag{7}$$

a similar trigger criterion was already introduced in the literature [15], [16].

Proof 1: Consider the following matrix

$$\beta = \begin{bmatrix} I & \mathscr{A}^\top & 0 \\ 0 & I & I \end{bmatrix}. \tag{8}$$

Multiplying (6) on the left by (8) and on the right by its transpose, results in

$$\begin{bmatrix} -P + Q_\sigma + \mathscr{A}^\top P\mathscr{A} & \mathscr{A}^\top P \\ P\mathscr{A} & P - Q_\delta \end{bmatrix} < 0. \tag{9}$$

Multiplying (9) on the left by $\begin{bmatrix} x(k)^\top & \nu(k)^\top \end{bmatrix}$ and on the right by its transpose, one has:

$$\begin{aligned} &x(k)^\top(-P + Q_\sigma + \mathscr{A}^\top P\mathscr{A})x(k) + x(k)^\top \mathscr{A}^\top P\nu(k) \\ &\qquad + \nu(k)^\top P\mathscr{A}x(k) + \nu(k)^\top P\nu(k) - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned}$$

Which can be rewritten as

$$\begin{aligned} &(\mathscr{A}x(k) + \nu(k))^\top P(\mathscr{A}x(k) + \nu(k)) \\ &\qquad - x(k)^\top Px(k) + x(k)^\top Q_\sigma x(k) - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned} \tag{10}$$

Substituting (5) in (10), results in

$$\begin{aligned} &-x(k)^\top Px(k) + x(k)^\top Q_\sigma x(k) + x(k+1)^\top Px(k+1) \\ &\qquad - \nu(k)^\top Q_\delta \nu(k) < 0. \end{aligned}$$

Considering the Lyapunov function $V(x(k)) = x(k)^\top Px(k)$, which is positive definite since $P > 0$, one can write $V(x(k+1)) - V(x(k)) < \nu(k)^\top Q_\delta \nu(k) - x(k)^\top Q_\sigma x(k) < 0$, ensuring that $V(x(k+1)) - V(x(k)) < f(e(k), x(k))$. According to Lyapunov's theory, it is guaranteed that the system converges asymptotically as long as $f(e(k), x(k)) < 0$, thus concluding the proof.

B. ETC Optimisation Mechanism

Consider the triggering function (7). To reduce the number of events, in the worst case, the following conditions must be respected

$$\nu(k)^\top Q_\delta \nu(k) \leq x(k)^\top Q_\sigma x(k), \tag{11}$$

and multiplying both sides of (11) by $(x(k)^\top Q_\sigma x(k))^{-1}$ holds $\nu(k)^\top Q_\delta \nu(k)(x(k)^\top Q_\sigma x(k))^{-1} \leq 1$. Therefore, one should minimise Q_δ and maximise Q_σ for the system to take longer to violate the constraint.

The optimisation problem can be structured as follows:

$$\begin{aligned} &\min: \quad \sigma \\ &\text{subject to:} \begin{cases} (6), \; \begin{bmatrix} X - Q_\delta & I \\ I & Q_\sigma \end{bmatrix} \geq 0, \; \operatorname{trace}(X) < \sigma, \end{cases} \end{aligned} \tag{12}$$

where $X \in \mathbb{R}^{n \times n}$, and $\sigma \in \mathbb{R}$ are the decision variables. By applying the Schur complement yields $X - Q_\delta - Q_\sigma^{-1} \geq 0$, or simply $Q_\delta + Q_\sigma^{-1} \leq X$. Taking the trace on both sides: $\operatorname{trace}(Q_\delta + Q_\sigma^{-1}) \leq \operatorname{trace}(X)$. Note that by minimising σ we are also minimising the sum of $Q_\delta + Q_\sigma^{-1}$, which will imply a smaller number of events since we are minimising Q_δ, and maximising Q_σ.

C. Tuning the LQR

The effects of different weights assigned to Q and R are evaluated based on the following criteria. Let us define the sampled values $\Omega : \{q \in \mathbb{R}, 0.0001 \leq q \leq 1000\}$ and $\Phi : \{r \in \mathbb{R}, r = 10^n \,|\, 0 \leq n \leq 2\} \cup \{r \in \mathbb{R}, r = 100i \,|\, 2 \leq i \leq 10\}$, then Q and R were iteratively varied as follows: $Q = qQ_0$ and $R = rR_0$, where $Q_0 = \mathbf{I}$ and $R_0 = 0.0001$.

To determine the controller's sensitivity in terms of the choice of matrices Q and R, a new cost function is defined:

$$J_{NEW} = J + N, \tag{13}$$

where J is computed as in (2) and N is the number of events. In this way, the choice for the matrices Q and R will consider the performance in terms of energy expenditure, and number of events. However, for real applications, when designing values for Q and R, it is important to consider two aspects: the convergence time of the response and the maximum control effort. Thus, constraint conditions are established, observing whether the peak value of the control effort falls within acceptable limits as described below:

$$\min\ J_{NEW}, \quad \text{subject to:} \begin{cases} Q \leq 1000 \times \mathbf{I}, \\ R \leq 1, \\ \text{Maximum time} = 30\ \text{s}, \end{cases} \tag{14}$$

where, $\mathbf{I} \in \mathbb{R}^{n \times n}$ is the identity matrix. The first two aspects are related to the fact that under certain conditions, the optimisation process described in the previous section does not find feasible solutions. Therefore, it was necessary to establish limits for Q and R. The third one is associated with real-time control, where a few seconds can be crucial for stabilising the system, making it necessary to limit the convergence time of the states.

IV. NUMERICAL EXPERIMENTS

The simulations were conducted using MATLAB version R2023a, the YALMIP parser [25], and the SeDuMi solver [26], executed on a system equipped with a 12-core Intel Core i9-10920XE CPU, 64 GB of RAM, and an NVIDIA T400 4GB GPU running Windows 11 Enterprise, version 22H2. The codes used for the simulations are available in [27].

A discretized-linearised model of the traditional inverted pendulum on a cart problem was considered to illustrate the proposed method. The objective is to stabilise a pendulum with mass m atop a massless rod of length L, by manipulating a cart of mass M. The cart's motion is dictated by the control force exerted in the X-axis direction, where x is the position of the cart, v is its velocity, θ is the angle of the pendulum, ω denotes the angular velocity, g is the gravitational acceleration. The states and matrices of the state-space representation of a linearised continuous system are described below:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x \\ v \\ \theta \\ \omega \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)g}{LM} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{LM} \end{bmatrix}.$$

The inverted pendulum setup was established with the following parameters: $m = 1$, $M = 0.1$, $L = 0.5$, and $g = 9.81$.

A. Q and R Weighting

Figs. 2 and 3 depict the control action in relation to the choice of R and Q, respectively, for the continuous system over 1-second simulation. Due to the difficulty of representing the Q matrix in the figure's legend, Fig. 3 details the multiplicative factor of $Q_0 = \mathbf{I}$, q, which is defined in the previous Section. The larger R and the smaller Q are, the lesser the controller's expenditure; the larger Q and the smaller R are, the faster the states converge, but at the cost of greater control action expenditure. By selecting $R = 1$ a reduction of 95.92% is noticed in the maximum value of the control input when compared with $R = 0.0001$. On the other hand, by varying Q from $Q = 1000$ to $Q = 0.0001$ a reduction of 91.03% is noticed in the maximum control input. This result elucidates that at the expense of some computational effort, iterative methods can be applied to achieve real optimal control when using LQR.

B. Event Triggered Control

As depicted in Fig. 4, the trigger device introduced in this study operates by directing the states to the target values within the specified time frame. The values that minimise the cost function indicated in Equation (13) are $Q = 10\mathbf{I}$ and $R = 1$, adopted for the system with a trigger device for Figs. 4-7. The energy of the controller can be estimated by calculating the sum of the squared terms of the control signal. For the sake of comparison, using the parameters recommended by Brunton and Kutz [28], specifically $Q = \mathbf{I}$ and $R = 1$ for the inverted pendulum simulation, the presented methodology results in a reduction of the maximum control input by 97.45% on the peak value and a decrease of 97.40% in total energy consumption.

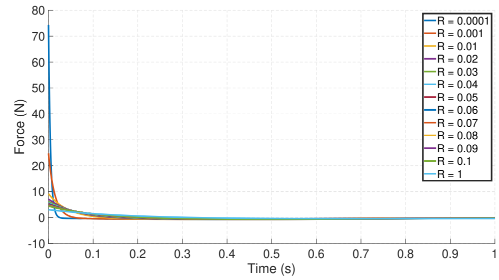

Fig. 2: Effect of varying R in the input control with fixed $Q = I$.

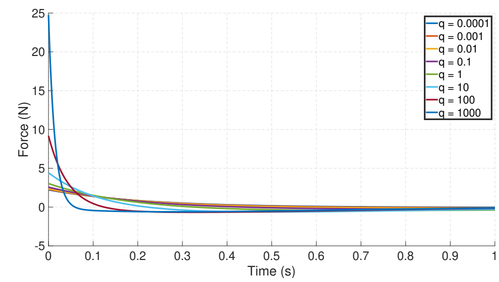

Fig. 3: Effect of varying Q in the input control with fixed $R = 1$.

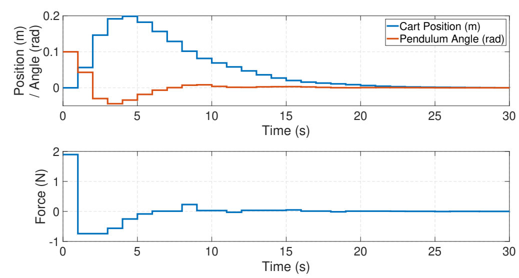

Fig. 4: Triggered control applied to inverted pendulum on a cart: states behaviours (top) and input control (bottom).

This significant energy reduction highlights that the selection of weighting matrices should not be considered a secondary factor in controller design. It plays an essential role that must be taken into account. As evidenced by our findings, an appropriate selection can lead to significant energy savings and reduced resource consumption. Such optimisation not only enhances system efficiency but also aligns with sustainable engineering practices. By minimising unnecessary energy use and optimising performance, we can achieve both environmental and economic benefits, emphasizing the interconnection of effective control design and sustainable resource management.

Further, the application of ETC illustrated that by incorporating an intermediary step of discretization, effective control performance can still be achieved. Given that controllers are predominantly designed as digital systems, selecting a discrete model over its continuous counterpart can be suitable. Ensuring congruence between the model and the digital control framework enhances performance and reduces the risk of inconsistencies stemming from model-controller misalignment. Integrating LQR with ETC can play a pivotal role in enhancing control design.

The versatility of the methodology presented allows one to consider various metrics when selecting Q and R, by simply altering the cost function presented in Equation (13). For instance, by applying the triggering device, a new cost function could be established to focus exclusively on minimising the number of events, rather than on both the events and the overall cost. This might involve imposing penalties on control expenditure and settling time. Alternatively, one could adjust Q and R to optimise the cost function for the continuous plant, among other potential strategies.

Considering the cost function presented in (13) to select the weighting matrices, we noted a minimum of 68 events with the maximum inter-event interval of 6, as shown in Fig. 5. In contrast, the periodic approach uses 120 events. Table I lists the number of events for various Q and R values, where q represents the multiplication factor of the matrix $Q = \mathbf{I}$ and the elements "-" indicate unfeasible solutions.

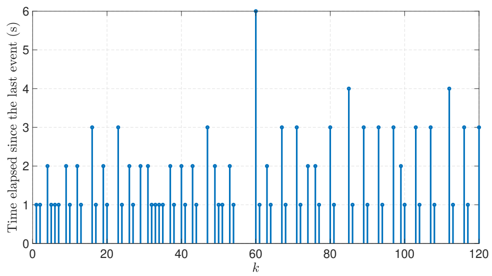

Fig. 5: Number of events for Q and R selected by (13) and inter-event samples.

TABLE I: Number of Events Related to R and $Q = qI$ Matrices.

	q							
R	0.0001	0.001	0.01	0.1	1	10	100	1000
0.0001	-	68	84	86	85	86	86	86
0.001	-	90	68	84	86	85	86	86
0.01	-	-	90	68	84	86	85	86
0.02	-	-	92	83	85	87	85	86
0.03	-	-	-	88	82	87	87	86
0.04	-	-	-	86	77	89	87	86
0.05	-	-	-	88	82	90	86	85
0.06	-	-	-	89	82	90	86	85
0.07	-	-	-	90	75	85	86	85
0.08	-	-	-	91	74	84	86	86
0.09	-	-	-	90	75	85	86	85
0.1	-	-	-	-	68	84	86	85
1	-	-	-	-	90	68	84	86

C. *Comparison Between The Models*

The continuous-time system was discretized with a sample time $T_s = 0.25$ seconds, followed by the implementation of the trigger device. Fig. 6 and 7 show the comparison between the continuous plant, the discretized plant, and the discretized plant with a trigger mechanism, in terms of state behaviour and control input, respectively.

As expected, the triggered model takes longer to converge due to the control acting only at specific moments when the states exceed the thresholds set by the triggering conditions. On the other hand, in terms of maximum effort, the triggered model performs better than the continuous one, with a reduction of 57.14% in the control input peak value and 18.14% in the total control input energy. Additionally, the cart's deviation from the equilibrium position is smaller in the triggered model compared to both the continuous and discrete models, although this improvement is not observed for the angle position.

These results point to the fact that triggered control can be used to improve energy expenditure in various systems, especially those under energy saturation, battery-operated devices, or where the control effort should be minimised to prolong the lifetime of actuators. This improvement, combined with reduced network traffic in NCS, is one of the potential benefits of this application. Another advantage is its event-centric focus, which becomes particularly vital in systems exposed to external disturbances. Rather than designing a controller that continuously adapts to all perturbations, the system can be engineered to respond exclusively to environmental changes that pose critical challenges to the system's stability.

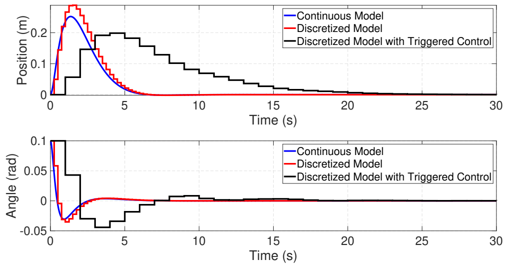

Fig. 6: Comparison between continuous model, discretized model and discretized model with trigger device for R and Q selected by Equation (13) criteria: position state behaviour (top) and angle state behaviour (bottom).

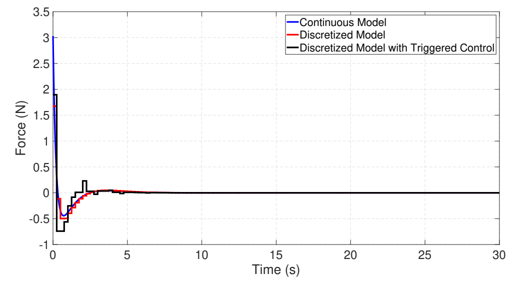

Fig. 7: Comparison between continuous model, discretized model, and discretized model with trigger device for R and Q selected by Equation (13) criteria: input control.

V. CONCLUSIONS

In this paper we presented a new methodology that uses an iterative approach for tuning the LQR controller, and a robust method to reduce control effort by replacing periodic control with an event-triggered mechanism in discrete-time systems. A new condition in the form of LMI was proposed for designing the ETC via emulation.

The new cost function is able to prioritise the optimisation of state convergence, control expenditure, and the number of controller activation events, simultaneously. While this method has proven effective, we also acknowledged that alternative objective functions can be utilised. For instance, if one's primary concern is energy expenditure and control effort, modifications can be made to emphasise these aspects. Conversely, in situations where the event count is paramount due to remote communication systems, the tuning could be adjusted to prioritize reducing the number of events. Therefore, the methodology introduced in this work is versatile and can be tailored for enhanced efficacy based on specific requirements.

Despite the iterative approach adopted in this work to analyse the effects of different Q and R matrices in LQR tuning, we confined our methodology to merely adopting Q as the identity and multiplying it by various scalar values. An alternative strategy might involve evaluating the controllability matrix of a system and weighting Q differently for each state, assessing which states exert the most influence on system behaviour. As a potential avenue for future research, one could develop a mathematical methodology to propose alternative optimisation techniques for LQR tuning.

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