Impedance Matching Network Based on Bianisotropic Metasurfaces for Antennas Operating at High Microwave, Millimeter-Wave or Terahertz Frequencies

Mohammad Alibakhshikenari\textsuperscript{1,*}, Bal S. Virdee\textsuperscript{2}, Chan H. See\textsuperscript{3,4}, Raed A. Abd-Alhameed\textsuperscript{5}, Francisco Falcone\textsuperscript{6}, and Ernesto Limiti\textsuperscript{1}

\textsuperscript{1} Electronic Engineering Department, University of Rome “Tor Vergata”, Via del Politecnico 1, 00133, Rome, ITALY
\textsuperscript{2} London Metropolitan University, Center for Communications Technology & Mathematics, School of Computing & Digital Media, London N7 8DB, UK
\textsuperscript{3} School of Engineering & the Built Environment, Edinburgh Napier University, 10 Colinton Rd., Edinburgh, EH10 5DT, UK
\textsuperscript{4} School of Engineering, University of Bolton, Deane Road, Bolton, BL3 5AB, UK
\textsuperscript{5} Faculty of Engineering and Informatics, University of Bradford, Bradford, BD7 1DP, UK
\textsuperscript{6} Electrical and Electronic Engineering Department, Public University of Navarre, 31006 Pamplona, SPAIN
*alibakhshikenari@ing.uniroma2.it

\textbf{Abstract} - In this paper a bianisotropic metasurface unit-cell is theoretically modelled and boundary conditions determined that show it’s possible to utilize bianisotropic metasurfaces for wideband impedance matching such as for antennas operating that operate at high microwave, millimeter-wave or terahertz frequencies. Analytical equations are derived for the image impedances and the corresponding insertion-loss of the bianisotropic metasurface unit-cell. Also derived are expressions that determine the effective electric and magnetic responses and the magnetoelectric coupling for achieving wideband impedance transformation when realized with transmission-line stubs.

\textbf{Keywords} - Impedance matching network, metasurfaces, effective electric and magnetic responses, effective magnetoelectric coupling, two different dielectric media, wideband impedance transformation, T-matching network.

\section{I. INTRODUCTION}

Matching networks are important to maximize transfer of power between two media, as illustrated in Fig. 1. At microwave and higher frequencies impedance transformers include stub tuners and $\lambda/4$-impedance inverters \cite{1-2} have a limited bandwidth and they suffer from tradeoff between size and bandwidth. It has been shown that bianisotropic metasurfaces, which are sub-wavelength thin structures, can refract signals with no reflections and loss. This proves that such materials can be used as effective impedance transformers \cite{1}. Bianisotropy occurs when the material exhibits magnetoelectric coupling between the electric and magnetic responses. Bianisotropic metasurfaces have been shown provide a high radiation efficiency \cite{3}; however further study is required to determine the relationship between bandwidth and impedance matching between two dielectric media when exposed to EM waves, which is the aim of this paper.

We have realized the proposed bianisotropic metasurface to investigate impedance matching capabilities using a T-matching network which exposed to electromagnetic plane-waves impinging the structure normally. Theoretical modelling is used to predict the magnetoelectric coupling that is required to achieve wideband matching. Bianisotropic metasurface is then applied on a dielectric to dielectric interface to show impedance transformation over a wideband when the sub-wavelength structure is exposed to normal incident EM plane-waves.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Impedance matching of two dielectric media with a sub-wavelength thin bianisotropic metasurface interface.}
\end{figure}

\section{II. WIDEBAND IMPEDANCE TRANSFORMATION BASED ON BIANISOTROPIC METASURFACES}

Initially it is necessary to determine conditions for impedance transformation over a wideband between two dielectric media are determined. Expressions are obtained for the input/output image-impedances of a bianisotropic metasurface unit-cell. The image-impedance at port#1 is $Z_{\text{image}_1}$ which is seen when port#2 is terminated to its corresponding image impedance $Z_{\text{image}_2}$ and vice-versa.

\subsection{A. Image Impedances and Insertion-Loss}

Input and output ports of the bianisotropic metasurface unit-cell are terminated with image impedances $Z_{\text{image}_1}$ and $Z_{\text{image}_2}$, respectively. Hence, when the port#1 is terminated to $Z_{\text{image}_1}$, $\frac{v_1}{i_1} = -Z_{\text{image}_1}$ is substituted in the general $Z$-matrix expression, and the input impedance at port#2 is:

$$Z_{\text{in}} = \frac{v_2}{i_2} = -Z_{\text{image}_1} = Z_{22} - \frac{z_{12}z_{22}}{z_{11} + z_{22}}$$

Similarly, by terminating port#2 to $Z_{\text{image}_2}$ and substituting $\frac{v_2}{i_2} = -Z_{\text{image}_2}$ in the general $Z$-matrix expression, the output impedance of port#1 is:
Substituting $Z_{in} = Z_{\text{image}2}$ and $Z_{out} = Z_{\text{image}1}$ in Eqn. (1) and Eqn. (2), impedance transformation obtained is given by:

$$\frac{Z_{\text{image}2}}{Z_{\text{image}1}} = \frac{Z_{22}}{Z_{11}} (3)$$

$$Z_{\text{image}1}Z_{\text{image}2} = Z_{22}Z_{11} - Z_{21}Z_{12} (4)$$

with the individual image impedances given by:

$$Z_{\text{image}2} = \frac{Z_{22}}{\sqrt{Z_{11}}} (Z_{22}Z_{11} - Z_{21}Z_{12}) (5)$$

$$Z_{\text{image}1} = \frac{Z_{21}}{\sqrt{Z_{22}}} (Z_{11}Z_{22} - Z_{12}Z_{21}) (6)$$

We can derive the nodal equations of the proposed T-matching network in Fig.2. The $Z$-matrix is obtained by solving for the nodal voltages when the currents in either ports $\#1$ or $\#2$ is set to zero. Hence, the $Z$-matrix elements are:

$$Z_{out} = \frac{v_4}{v_2} = Z_{\text{image}2} - Z_{\text{image}1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_{\text{image}2}} \quad (2)$$

$$Z_{21} = Z_{12} = \frac{Z_{11}}{2} - \frac{Z_{22}}{2} \left(1 - \frac{N_1^2}{N_2^2}\right) (8)$$

$$Z_{11} = \frac{Z_{11}}{2} - \frac{Z_{22}}{2} \left(1 - \frac{N_1^2}{N_2^2}\right)^2 (9)$$

where $Z_1$ and $Z_2$ are the shunt and series impedances of the proposed T-matching network; $N_1$ & $N_2$ are the number of turns of the ideal transformers used; and $Z_{12} = Z_{21}$. By substituting Eqn. (7) - (9) in Eqn. (5) and (6) for the $Z$-matrix of the proposed T-matching network, the input & output image impedances are given by:

$$Z_{\text{image}2} = \sqrt{Z_{22}Z_{11}} \frac{Z_{11} + Z_{21} \left(1 - \frac{N_1^2}{N_2^2}\right)}{Z_{22} + Z_{\text{image}2}} (10)$$

$$Z_{\text{image}1} = \sqrt{Z_{22}Z_{11}} \frac{Z_{21} + Z_{11} \left(1 - \frac{N_2^2}{N_1^2}\right)}{Z_{22} + Z_{\text{image}2}} (11)$$

When the input and output ports are terminated to the image impedances $S_{12}$ is given by [5]:

$$S_{12} = \frac{\sqrt{8[Z_{\text{image}2}I_1]I_2[Z_{\text{image}1}I_1]}Z_{11}Z_{21} \left(1 - \frac{N_1^2}{N_2^2}\right)}{2Z_{11}Z_{22}Z_{22} - Z_{11}Z_{21} \left(1 - \frac{N_1^2}{N_2^2}\right)} (12)$$

The inverse expressions of $Z_1$, $Z_2$, & $\frac{N_1}{N_2}$ can be determined from Eqn. (7) - (9) to be:

$$Z_2 = \frac{1}{2} (Z_{11} + Z_{22} - 2Z_{21}) (13)$$

$$Z_1 = 2 \frac{Z_{11}Z_{22} - Z_{21}^2}{Z_{11}Z_{22} - Z_{22}Z_{21}} (14)$$

$$\frac{N_1}{N_2} = \frac{Z_{11}Z_{22} - Z_{21}^2}{Z_{11}Z_{22} - Z_{22}Z_{21}} (15)$$

The bianisotropic metasurface unit-cell’s $E$–$H$ terminal relations of are given by [8]:

$$\frac{1}{2} \begin{bmatrix} a \times [\hat{n} \times (\hat{E}_1 + \hat{E}_2)] \end{bmatrix} = -Z_{e\text{surface}} \hat{n} \times (\hat{H}_1 - \hat{H}_2) - K_{m\text{coupling}} \hat{n} \times [\hat{n} \times (\hat{E}_1 + \hat{E}_2)]$$

$$\frac{1}{2} \begin{bmatrix} a \times [\hat{n} \times (\hat{H}_1 + \hat{H}_2)] \end{bmatrix} = Y_{m\text{surface}} \hat{n} \times (\hat{E}_1 - \hat{E}_2) + K_{m\text{coupling}} \hat{n} \times [\hat{n} \times (\hat{H}_1 - \hat{H}_2)]$$

where $Z_{e\text{surface}}$ and $Y_{m\text{surface}}$ are the effective electric surface impedance and effective magnetic surface admittance of the bianisotropic metasurface, respectively, and $K_{m\text{coupling}}$ is the magnetoelectric coupling coefficient between the effective electric and magnetic responses. By assuming the fields are linearly polarized, the boundary conditions for bianisotropic transition in the $Z$-matrix form are given by:

$$Z_{\text{bianisotropic}} = \begin{bmatrix}
Z_{e\text{surface}} - \frac{1}{4} K_{m\text{coupling}}^2 & Z_{e\text{surface}} + \left[1 + 2K_{m\text{coupling}}^2\right] \frac{1}{4} Y_{m\text{surface}} \\
Z_{e\text{surface}} + \left[1 + 2K_{m\text{coupling}}^2\right] \frac{1}{4} Y_{m\text{surface}} & Z_{e\text{surface}} - \frac{1}{4} K_{m\text{coupling}}^2
\end{bmatrix}$$

Fig.2. Proposed bianisotropic metasurface unit-cell, (a) impedance sheet, (b) corresponding impedance sheet that generates identical S-parameter response as a metasurface with electric, magnetic and magnetoelectric surface parameters, (c & d) metasurface unit-cell with impedance sheet modelled as a T-network.
By substituting Eqn. (18) in Eqn. (13) - (15), the following mapping expressions enables the proposed T-matching network in Fig.2 to be modelled as a bianisotropic metasurface:

\[
Z_2 = \frac{1}{2Y_{\text{surface}}} \quad (19)
\]
\[
Z_1 = 2Z_{e_{\text{surface}}} \quad (20)
\]
\[
\frac{N_1}{N_2} = 2K_{m_{\text{coupling}}} \quad (21)
\]

By applying the mapping expressions in Eqn. (19) - (21) to Eqn. (10) - (12) we obtain the following image impedances and transfer function of bianisotropic metasurface:

\[
Z_{\text{images}} = \left(\frac{1}{Z_{e_{\text{surface}}}}\right)\left(\frac{4Z_{e_{\text{surface}}}Y_{\text{surface}} + (1+2K_{m_{\text{coupling}}})}{4Z_{e_{\text{surface}}}Y_{\text{surface}} - (1+2K_{m_{\text{coupling}}})}\right)^\frac{1}{2} \quad (22)
\]
\[
Z_{\text{images}} = \left(\frac{1}{Z_{e_{\text{surface}}}}\right)\left(\frac{4Z_{e_{\text{surface}}}Y_{\text{surface}} + (1-2K_{m_{\text{coupling}}})}{4Z_{e_{\text{surface}}}Y_{\text{surface}} - (1-2K_{m_{\text{coupling}}})}\right)^\frac{1}{2} \quad (23)
\]
\[
S_{12} = \frac{(9[2_{\text{images}}])^\frac{1}{2}}{4Z_{e_{\text{surface}}}Y_{\text{surface}}} \times \frac{(1+4K_{m_{\text{coupling}}}^2)}{[4Z_{e_{\text{surface}}}Y_{\text{surface}} + (1+2K_{m_{\text{coupling}}})]^2 - 4K_{m_{\text{coupling}}}^2[Y_{\text{surface}}]} \quad (24)
\]

B. Bianisotropic Metasurface Parameters

In general, we can assume that the bianisotropic metasurface is lossless and realizes perfect impedance matching between two dielectric media with intrinsic impedances \(\eta_1\) and \(\eta_2\), i.e. \(S_{11} = S_{22} = 0\) and \(S_{12} = S_{21} = e^{-j\theta}\). The corresponding Z-matrix is [6]:

\[
Z = \begin{bmatrix}
-j\eta_1\sec(\theta) & -j\eta_1\eta_2\sin(\theta) \\
-j\eta_2\sin(\theta) & -j\eta_2\sec(\theta)
\end{bmatrix} \quad (25)
\]

When Eqn.(25) is substituted in Eqn.(13) - (15) we obtain:

\[
Z_1 = \frac{-j\tan(\theta)}{\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \sec(\theta)} \quad (26)
\]
\[
Z_2 = -\frac{j\tan(\theta)}{\eta_1 - \eta_2 + 2(\eta_1\eta_2) \sec(\theta)} \quad (27)
\]
\[
\frac{N_1}{N_2} = \frac{\eta_1 + \eta_2}{\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \sec(\theta)} \quad (28)
\]

When applied to mapping conditions in Eqn. (19) - (21) we obtain:

\[
Z_{e_{\text{surface}}} = \eta_1\eta_2 \frac{j\cot(\theta)}{\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \sec(\theta)} \quad (29)
\]
\[
Y_{m_{\text{surface}}} = \eta_1\eta_2 \frac{\cot(\theta)}{\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \sec(\theta)} \quad (30)
\]
\[
K_{m_{\text{coupling}}} = \frac{\eta_1 + \eta_2}{2\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \sec(\theta)} \quad (31)
\]

The expressions in Eqn. (26) - (31) can be expressed as

\[
Z = \frac{1}{2Y_{e_{\text{surface}}}} \quad (32)
\]
\[
K_{m_{\text{coupling}}} = \frac{1}{2\eta_1 - \eta_2 + 2\sqrt{\eta_1\eta_2} \cosec(\theta)} \quad (34)
\]

The above expressions reveal that it is possible to achieve wideband impedance matching. The bianisotropic metasurface doesn’t require to have both effective electric and magnetic responses with coupling between them, but the frequency dispersion of these two responses needs to be identical with opposite duality, i.e. if one is capacitive, the other is required to be inductive. This assures any reflections from one is canceled by the reflections from the other. Also, the ratio of \(Z_{e_{\text{surface}}}\) and \(Y_{m_{\text{surface}}}\) needs to be independent of frequency, which agrees with expressions in Eqn. (22) - (24). For impedance matching between dielectric media with different intrinsic impedances \(\eta_1 \neq \eta_2\) the coupling coefficient \(K_{m_{\text{coupling}}}\) needs to be zero only when the two dielectric media are the same.

CONCLUSION

Theoretical analysis presented provides an understanding of what is required from bianisotropic metasurface unit-cells to realize wideband matching. The theoretical expressions demonstrate how certain boundary conditions when applied to the bianisotropic metasurface model can be used to realize wideband matching between two different dielectric media.

ACKNOWLEDGMENT

This work is partially supported by innovation programme under grant agreement H2020-MSCA-ITN-2016 SECRET-722424 and the financial support from the UK Engineering and Physical Sciences Research Council (EPSRC) under grant EP/E0/22936/1.

REFERENCES