Metasurface Structures with Bianisotropic Characteristics for Phased Array Antennas

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Abstract—It is theoretically shown that by cascading isotropic impedance sheets it is possible to obtain scattering performance as an idealized isotropic metasurface with electric, magnetic and magneto-electric surface parameters. ABCD matrices, scattering matrices, impedance matrices, and hybrid matrices are used to analyze the propagation of electromagnetic waves through layered media. It is also shown that metasurfaces can exhibit magnetoelectric coupling which is important for wide-angle impedance matching (WAIM) in phased-array antennas to reduce variation of reflection-coefficient with scan angle and polarization.

Index Terms—Metasurfaces, impedance matching network, antennas, surface parameters, scattering parameters.

I. INTRODUCTION

Metamaterials are well established three-dimensional structures that are engineered to exhibit unique electromagnetic properties not normally found in nature [1-4]. Metamaterials are engineered by simply arranging electrically small scattering structures in 3D space to obtain negative-index material (NIM). Metasurface are metamaterials realized on a 2D surface. The effective permittivity and effective permeability of these materials are simultaneously negative in a specified frequency band. Advantage of metasurfaces are they use less physical space than the 3D metamaterial structures. Potential applications of metasurface include: (i) reconfigurable electromagnetic (EM) surfaces, (ii) miniature resonators, (iii) waveguide structures, (iv) electromagnetic absorbers, (v) biomedical devices, (vi) switches, (vii) tunable frequency-agile materials, (viii) EM shielding, (ix) low-reflection materials, and (x) antennas. Metamaterials have been used to design various components in the microwave to optical region of the EM spectrum to enhance system performance.

Reflection-coefficient at the surface of phased-array antennas tends to vary with the scanning angle and the polarization of the radiation. The variation in reflection becomes significant in large planar arrays used for wide-angle scanning. Conventional matching techniques in the individual element feedlines cannot compensate for such variation. The reflection can adversely affect the phased-array antenna’s radiation efficiency, radiation pattern, polarization, and stability of RF amplifiers at the transmitter or receiver. Wide-angle impedance matching (WAIM) is one technique to address such reflections at the aperture-air interface phased-array antennas [5]. In order to achieve a wide angular range with minimal return-loss it is necessary to optimize the WAIM layer’s dielectric until an acceptable return-loss is achieved. In practice it is challenging to match the antenna at all angles using the limited set of available dielectric materials. This issue can be overcome if we can engineer anisotropic materials whose $\varepsilon$ or $\mu$ can be precisely controlled. In this paper, theoretical expressions are derived that show metasurfaces can exhibit magnetoelectric coupling. These metasurfaces with bianisotropic properties can be developed for application as WAIM layers for application in phased-array antennas.

II. REALIZING META SURFACE SHEETS AS IMPEDANCE MATCHING NETWORKS BETWEEN TWO DIFFERENT MEDIA

Metasurfaces are engineered from subwavelength unit-cell structures created on a surface, while metamaterials are subwavelength structures created in a volumetric space [7-10]. EM interaction in metamaterial occurs in the bulk of the material however in the case of metasurfaces the EM interaction occurs at its 2D surface [7, 8]. Typical examples of metasurface structures are shown in Fig.1.

Metasurfaces can be described using effective material parameters ($\varepsilon$ and $\mu$) as well as in terms of impedances/admittances [11-13].
A) Modelling Metasurfaces Based on Surface Parameters

Fig.2 shows a 2D metasurface which is composed of an array of closely spaced, polarizable particles. Such an array of subwavelength particles can be represented with a surface polarizability matrix where the averaged tangential fields are related to the induced dipole moments per unit area.

Fig.2. Metasurface of closely spaced two-dimensional polarizable particles.

\[
\left( \vec{p}^s \right)_{\text{mes}} = \left( \vec{a}^s_{\text{e}} \vec{a}^s_{\text{m}} \vec{a}^s_{\text{m}} \right) \left( \vec{E}_t \right)_\text{avg} \quad (1)
\]

Idealized model of a metasurface when time-varying dipole moment is represented as a surface current is expressed by

\[
\left( \vec{J} \right)_{\text{mes}} = j \omega \left( \vec{p}^s \right)_{\text{mes}} = j \omega \left( \vec{a}^s_{\text{e}} \vec{a}^s_{\text{m}} \right) \left( \vec{E}_t \right)_\text{avg} = \left( \vec{p} \right) \left( \vec{E}_t \right)_\text{avg} \quad (2)
\]

The electric, magnetic and magneto-electric properties of a metasurface can be described using surface parameters \( \vec{p}, \vec{X}, \vec{Y} \), and \( \vec{Z} \), thus

\[
\begin{align*}
\left( J_x \right)_{\text{mes}} &= j \omega \left( \vec{p}^s \right)_{\text{e}} \left( \vec{p}^s \right)_{\text{m}} = \left( \vec{Y}_{xx} \vec{Y}_{xy} \vec{Y}_{yx} \vec{Y}_{yy} \right) \left( \vec{E}_x \right)_\text{avg} \\
\left( J_y \right)_{\text{mes}} &= j \omega \left( \vec{p}^s \right)_{\text{e}} \left( \vec{p}^s \right)_{\text{m}} = \left( \vec{Z}_{xx} \vec{Z}_{xy} \vec{Z}_{yx} \vec{Z}_{yy} \right) \left( \vec{E}_y \right)_\text{avg} \\
\left( J_z \right)_{\text{mes}} &= j \omega \left( \vec{p}^s \right)_{\text{e}} \left( \vec{p}^s \right)_{\text{m}} = \left( \vec{Z}_{xx} \vec{Z}_{xy} \vec{Z}_{yx} \vec{Z}_{yy} \right) \left( \vec{E}_z \right)_\text{avg} \quad (3)
\end{align*}
\]

Metasurfaces that are reciprocal and lossless, defined in below, have 10 degrees of freedom.

\[
\begin{align*}
\vec{p} &= \vec{p}^T \quad (4) \\
\vec{Y} &= -\vec{Y}^T \quad (5) \\
\vec{Z} &= \vec{Z}^T \quad (6)
\end{align*}
\]

Electric properties of a metasurfaces can be described using scattering parameters \( \tilde{S}_{11}, \tilde{S}_{12}, \tilde{S}_{21} \) and \( \tilde{S}_{22} \), thus

\[
\begin{align*}
\left( \vec{E}_1 \right) = \begin{pmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{21} & \tilde{S}_{22} \end{pmatrix} \left( \vec{E}_2 \right) \\
\end{align*}
\quad (7)
\]

Time-varying dipole moment can be represented as a surface current, resulting in the following idealized model of a metasurface

\[
\begin{align*}
\left( \vec{E}_{1x} \vec{E}_{1y} \vec{E}_{1z} \right) &= \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{pmatrix} \\
\left( \vec{E}_{2x} \vec{E}_{2y} \vec{E}_{2z} \right) &= \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} E_{2x} \\ E_{2y} \\ E_{2z} \end{pmatrix} \quad (8)
\end{align*}
\]

B) Boundary Conditions for a Surface Supporting \( \vec{J} \) and \( \vec{M} \)

Boundary condition for a surface that supports normal and tangential current densities, which is shown in Fig.3, is expressed by

\[
-\hat{z} \times \left( \vec{E}_2 - \vec{E}_1 \right) = \vec{M}_s + \frac{\omega \mu_0}{\epsilon_0} \vec{J}_n \quad (9)
\]

\[
\hat{z} \times \left( \vec{H}_2 - \vec{H}_1 \right) = \vec{J}_s + \frac{\omega \mu_0}{\epsilon_0} \vec{M}_n \quad (10)
\]

Fig.3. Boundary condition for a surface supporting the normal and tangential current densities.

Boundary condition for a surface that supports tangential current density only, shown in Fig.4, can be defined as follows

\[
-\hat{z} \times \left( \vec{E}_2 - \vec{E}_1 \right) = \vec{M}_s \quad (9)
\]

\[
\hat{z} \times \left( \vec{H}_2 - \vec{H}_1 \right) = \vec{J}_s \quad (10)
\]

Fig.4. Boundary condition for a surface supporting the tangential current density only.

The tangential current densities as an interaction between the average, tangential electric and magnetic-fields and bianisotropic sheet parameters can be expressed by

\[
\begin{align*}
\left( \vec{J}_s \right)_{\text{mes}} &= \left( \vec{p} \right) \left( \vec{E}_t \right)_\text{avg} \\
\left( \vec{J}_n \right)_{\text{mes}} &= \left( \vec{p} \right) \left( \vec{H}_t \right)_\text{avg} \\
\end{align*}
\quad (11)
\]

The boundary conditions on the surface are defined as

\[
\begin{align*}
\hat{z} \times \left( \vec{H}_2 - \vec{H}_1 \right) &= \vec{J}_s \quad (12) \\
-\hat{z} \times \left( \vec{E}_2 - \vec{E}_1 \right) &= \vec{M}_s \quad (13)
\end{align*}
\]

By inserting Eqn.(11) into Eqn.(12), and Eqn.(11) into Eqn.(13), we respectively obtain

\[\]
\[ z \times (H_2 - H_1) = Y E_{t-avg} + \hat{x} H_{t-avg} \quad (14) \]
\[ -z \times (E_2 - E_1) = Y E_{t-avg} + \hat{Z} H_{t-avg} \quad (15) \]

Interactions of tangential fields with the metasurface are captured by this boundary condition. The scattering parameters of a metasurface can be represented by surface parameters. In general, a metasurface is a four-ports device, as shown in Fig. 5, i.e. there are two polarizations on each side of the metasurface resulting in each scattering parameter being a 2x2 matrix.

\[ S_{nn} = \begin{pmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{pmatrix} \quad (16) \]

![Diagram](image)

Fig.5. General view of a metasurface as a four-ports device with two polarizations on its each side.

The interaction between scattering parameters and surface parameters allows a metasurface to be designed to perform a specific scattering function.

Assuming the metasurface is isotropic and there is no polarization conversion, the surface parameters are then represented by

\[ Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (17) \]
\[ Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (18) \]
\[ \hat{Y} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (19) \]
\[ \hat{X} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (20) \]

then

\[
\begin{bmatrix}
J_x^s \\
J_y^s \\
M_x^s \\
M_y^s
\end{bmatrix} =
\begin{bmatrix}
Y & 0 & 0 & \chi \\
0 & Y & -\chi & Z \\
0 & Y & Z & 0 \\
-\gamma & 0 & 0 & \gamma
\end{bmatrix}
\begin{bmatrix}
E_{x-avg} \\
E_{y-avg} \\
H_{x-avg} \\
H_{y-avg}
\end{bmatrix}
\quad (21)
\]

This matrix equation can be written as

\[
\begin{align*}
J_x^s &= Y E_{t-avg} - \chi \hat{x} \times \hat{H}_{t-avg} \\
M_x^s &= -Y \hat{x} \times E_{t-avg} + \hat{Z} H_{t-avg}
\end{align*}
\quad (22) \]
\[
\begin{align*}
J_y^s &= Y E_{t-avg} - \chi \hat{x} \times \hat{H}_{t-avg} \\
M_y^s &= -Y \hat{x} \times E_{t-avg} + \hat{Z} H_{t-avg}
\end{align*}
\quad (23)
\]

By analyzing the metasurface as an electric admittance \((Y)\) and magnetic impedance \((Z)\) sheet exposed to by a normal incident plane wave, as shown in Fig.6, we have

\[
\begin{bmatrix}
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
\hat{y} \times (H_2 - H_1) \\
\hat{z} \times (E_2 - E_1)
\end{bmatrix}
\quad (24)
\]

![Diagram](image)

Fig.6. Metasurface as an electric admittance \((Y)\) and magnetic impedance \((Z)\) sheet.

Incident, reflected, and transmitted waves are defined by

\[
\begin{align*}
\text{Incident wave:} \quad & \hat{k}^+ = k \hat{x}, \quad E^+ = E^+ \hat{y}, \quad H^+ = -\frac{E^+}{\eta} \hat{x} \\
\text{Reflected wave:} \quad & \hat{k}^- = -k \hat{x}, \quad E^- = \Gamma E^+ \hat{y}, \quad H^- = \frac{E^+}{\eta} \hat{x} \\
\text{Transmitted wave:} \quad & \hat{k}^t = k \hat{x}, \quad E^t = \tau E^+ \hat{y}, \quad H^t = -\frac{\tau E^+}{\eta} \hat{x}
\end{align*}
\quad (25) \]

The magnetic-field boundary condition remains unaffected as the electric admittance sheet case; however, the electric-field boundary condition is changed.

\[
\begin{align*}
\hat{J}^s &= \hat{y} \times (\hat{H}_2 - \hat{H}_1) = YE_{t-avg} \\
\hat{M}^s &= \hat{z} \times (\hat{E}_1 - \hat{E}_2) = ZH_{t-avg}
\end{align*}
\quad (28) \]
\[(29) \]

Eqs. (28) and (29) indicate that the tangential magnetic-field is identical to the electric surface current density, and the tangential electric-field is identical to the negative magnetic surface current density. These boundary conditions can be applied to the tangential electric and magnetic-fields to determine the reflection and transmission-coefficients. The magnetic-field boundary condition from Eqn. (28) gives

\[
\hat{J}^s = \frac{1}{\eta} \left( \frac{1}{\eta} \hat{x} - \frac{E^+}{\eta} \hat{x} + \frac{E^-}{\eta} \hat{x} \right) = \frac{1}{2} (E^+ + E^- + \tau E^+) \hat{y} \rightarrow
\]
\[
\frac{1}{\eta} (E^+ - \Gamma E^+ - \tau E^+) = \frac{1}{2} (E^+ + \Gamma E^+ + \tau E^+) \rightarrow
\]
\[
(1 - \Gamma - \tau) = \frac{\eta}{2} (1 + \Gamma + \tau)
\quad (30)
\]

In addition, from Eqn. (29), the electric-field boundary condition gives

\[
\hat{M}^s = \frac{1}{2} \left( \frac{1}{2} \hat{x} (H^+ + H^- - H^t) \hat{x} \rightarrow
\quad (E^+ + \Gamma E^+ - \tau E^+) = \frac{1}{2} \left( E^+ - \Gamma E^+ + \tau E^+ \right) \rightarrow
\]
\( (1 + \Gamma - \tau) = \frac{z}{z_0} (1 - \Gamma + \tau) \)  

(31)

By solving Eqns. (30) and (31), the reflection and transmission-coefficients are given by

\[
\Gamma = \frac{1}{2s} \left( \frac{z - z_0}{1 + \frac{z}{z_0}} \right)^2 \quad (32)
\]

\[
\tau = \left( \frac{1 - \frac{z}{z_0}}{1 + \frac{z}{z_0}} \right)^2 \quad (33)
\]

By considering the magneto-electric coupling between two electric and magnetic boundaries, Eqns. (28) and (29) can be extended to

\[
\mathbf{J} = \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \gamma E_{\text{avg}} - \chi \mathbf{n} \times \mathbf{H}_{\text{avg}} \quad (34)
\]

\[
\mathbf{M} = \mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = -\gamma \mathbf{n} \times \mathbf{E}_{\text{avg}} + \mathbf{Z}\mathbf{H}_{\text{avg}} \quad (35)
\]

For a single polarization, the surface parameters are expressed by

\[
\begin{pmatrix}
Y \\
X \\
Z
\end{pmatrix} =
\begin{pmatrix}
B \\
R \\
JX
\end{pmatrix}
\]

(36)

where \( B, R \) and \( X \) are real numbers. The three distinct surface parameters can be chosen to achieve three desired scattering properties. In the case of the isotropic, reciprocal and lossless, the metasurface is expressed by

\[
\begin{pmatrix}
J_x^y \\
J_y^x \\
M_x^y \\
M_y^x
\end{pmatrix} =
\begin{pmatrix}
B & 0 & 0 & R \\
0 & jB & -R & 0 \\
R & jX & 0 & H_x^{\text{avg}} \\
0 & jX & 0 & H_y^{\text{avg}}
\end{pmatrix}
\]

(37)

In compact form it can be expressed thus

\[
\mathbf{J} = jB E_{\text{avg}} - R\mathbf{n} \times \mathbf{H}_{\text{avg}} \quad (38)
\]

\[
\mathbf{M} = -R\mathbf{n} \times \mathbf{E}_{\text{avg}} + \mathbf{B}\mathbf{H}_{\text{avg}} \quad (39)
\]

The three distinct surface parameters can be chosen to achieve three desired scattering properties.

L-networks and T-networks, as shown in Fig.7, can be used for impedance transformation.

Matching a complex impedance \( Z_L \) to a complex impedance \( Z_{\text{in}} \) requires manipulation of reactance \( X_1 \) and \( X_2 \). The third reactance \( X_3 \) to form a T-network enables the phase of the circuit to be controlled.

An impedance matrix can also be used to represent the metasurface as a two-ports network. If the two-ports impedances are different, the metasurface can be viewed as an impedance matching layer, represented in Fig.8.

The impedance matrix equation of the two-port network is given by

\[
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} =
\begin{pmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix}
\]

(40)

Input impedance and the load impedance of the network are defined by

\[
Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = \frac{v_1}{i_1} \quad (41)
\]

\[
Z_L = R_L + jX_L = -\frac{v_2}{i_2} \quad (42)
\]

The lossless condition requires entries of the impedance matrix to be imaginary. Thus,

\[
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = j \begin{pmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{pmatrix}
\begin{pmatrix}
\frac{v_1}{z_{\text{in}}} \\
\frac{v_2}{z_L}
\end{pmatrix}
\]

(43)

The lossless condition can also be used to define \( V_2 \) in terms of \( V_1 \), since power must be conserved across a lossless surface

\[
\text{Re}(V_1 I_1^*) = \text{Re}(V_2 I_2^*) \Rightarrow \text{Re} \left( \frac{V_1}{Z_{\text{in}}} \right) = \text{Re} \left( \frac{V_2}{Z_L} \right) \Rightarrow |V_2| = |V_1| \frac{R_L X_{21} + X_{22} R_{\text{in}}}{R_L X_{21} + X_{22} R_{\text{in}}} = |V_1| R \quad (44)
\]

Considering the transmission phase between two ports

\[
\phi_{21} = \delta V_2 - \delta V_1 = \phi_2 - \phi_1 \quad (45)
\]

The input impedance, load impedance, and impedance matrix equations can be written as

\[
Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = |Z_{\text{in}}| e^{i\phi_{21}} \quad (46)
\]

\[
Z_L = R_L + jX_L = |Z_L| e^{i\phi_{22}} \quad (47)
\]

\[
\begin{pmatrix}
V_1 \\
V_2 e^{i\phi_2}
\end{pmatrix} = j \begin{pmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{pmatrix}
\begin{pmatrix}
\frac{v_1}{z_{\text{in}}} \\
\frac{v_2}{z_L} e^{i\phi_2}
\end{pmatrix} \quad (48)
\]
Separating into real and imaginary parts, and solving these equations for the elements of the impedance matrix results in

\[
[Z] = j \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} \rightarrow \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} = \begin{pmatrix} \frac{|Z_{11}| R \cos(\phi_{11} - \phi_{21})}{|Z_{11}|} & \frac{|Z_{12}| R \sin(\phi_{12} - \phi_{21})}{|Z_{12}|} \\ \frac{|Z_{21}| R \cos(\phi_{21} + \phi_{22})}{|Z_{21}|} & \frac{|Z_{22}| R \sin(\phi_{22} + \phi_{21})}{|Z_{22}|} \end{pmatrix}
\]

By cascading three isotropic impedance sheets it is possible to obtain scattering performance as an idealized isotropic metasurface with electric, magnetic and magneto-electric surface parameters. This is because three sheet impedance values \(Z_{s1}, Z_{s2}, \text{ and } Z_{s3}\) provide three degrees of freedom to mimic three surface properties: \(Z, Y, \chi = Y\). The metasurface unit-cell with cascaded impedance sheets can be modelled as a \(T\)-network, as shown in Fig.7 (b), or a transmission-line loaded with shunt impedances separated by the short distances as shown in Fig.9.

\[\text{Fig.9. Metasurface unit-cell with three cascaded impedance sheets, (a) metasurface with electric impedance sheets, and (b) transmission-line loaded with shunt impedances separated by the short distances.}\]

Shunt impedances can be calculated by matching the response of the three cascaded sheets to the desired response of the surface parameters. The shunt impedances are separated by transmission-lines. The cascade of electric sheets has the following ABCD matrix

\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{casc}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{MS}} \rightarrow \begin{pmatrix} Z_{s1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\beta d) & jZ_0 \sin(\beta d) \\ jZ_0 \sin(\beta d) & \cos(\beta d) \end{pmatrix} \begin{pmatrix} Z_{s1} & 0 \\ 0 & 1 \end{pmatrix}
\]

where \(Z_0\) is the characteristic impedance of the transmission-lines between the shunt impedances. The first, third, and fifth matrices on the right side represent shunt impedances. The second and fourth matrices on the same side represent transmission-lines. The ABCD matrix of the metasurface in terms of impedance matrix is given by

\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{MS}} = \begin{pmatrix} Z_{s1} & Z_{s2} \\ Z_{s3} & Z_{s4} \end{pmatrix}
\]

as the metasurface impedance matrix, the two ABCD matrices presented by Eqsns. (50) and (51) can now be equated, and the sheet impedances solved for

\[
\begin{align*}
\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{casc}} &= \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{MS}} \\
Z_{s1} &= \frac{Z_0}{\sqrt{R_L}} \sin(\beta d) - \frac{Z_0^2}{2} \sin(2\beta d) \\
Z_{s2} &= \frac{Z_0^2}{2} \sin(2\beta d) + \frac{Z_0}{\sqrt{R_L}} \sin(\beta d) \\
Z_{s3} &= \frac{Z_0}{\sqrt{R_L}} \sin(\beta d) - \frac{Z_0^2}{2} \sin(2\beta d)
\end{align*}
\]

For zero reflection, the input impedance of the impedance matrix must be equal to the wave impedance of the surrounding wavefront. If \(Z_{\text{in}} = Z_L\), then \(Z_{s1} = Z_{s3}\), and the metasurface only exhibits electric and magnetic responses \(Y = Y\). If \(Z_{\text{in}} \neq Z_L\), then \(Z_{s1} \neq Z_{s3}\), and the metasurface exhibits a bianisotropic response \(\chi, Y = 0\). If the input and output impedances are real \((Z_{\text{in}} = R_{\text{in}})\) and \((Z_{L} = R_{L})\), and the desired phase delay be \(\phi_{21}\), then these equations simplify significantly and the necessary impedance matrix reduces to

\[
[Z] = j \frac{1}{\sin(\phi_{21})} \begin{pmatrix} R_{\text{in}} \cos(\phi_{21}) & \sqrt{R_{\text{in}} R_L} \\ \sqrt{R_{\text{in}} R_L} & R_{\text{L}} \cos(\phi_{21}) \end{pmatrix}
\]

The sheet impedance values become

\[
\begin{align*}
Z_{s1} &= \frac{jZ_0 \sin(\beta d) - \frac{Z_0^2}{2} \sin(2\beta d)}{\sqrt{R_{\text{in}} \cos(\phi_{21}) - \frac{Z_0^2}{2} \sin(2\beta d)}} \\
Z_{s2} &= \frac{-jZ_0^2 \sin(\beta d)}{Z_0 \sin(2\beta d) + \sqrt{R_{\text{in}} R_L} \sin(\phi_{21})} \\
Z_{s3} &= \frac{Z_0 \sin(2\beta d) - \frac{Z_0^2}{2} \sin(2\beta d)}{\frac{Z_0}{\sqrt{R_{\text{in}} \cos(\phi_{21}) - \frac{Z_0^2}{2} \sin(2\beta d)}}}
\end{align*}
\]

Denominator of \(Z_{s1}\) includes the term \(\sqrt{R_L/R_{\text{in}}}\), and denominator of \(Z_{s3}\) includes \(\sqrt{R_{\text{in}}/R_L}\). This is because the cascade of impedance sheets becomes asymmetric when \(Z_{\text{in}} \neq Z_L\).
The impedance matrix can be realized as a $T$-network, or as three shunt impedances separated by a short distance.

III. CONCLUSION

Theoretical expressions are derived that relate the reflection and transmission-coefficients of a general bianisotropic metasurface to its constituent surface parameters. ABCD matrices, scattering matrices, impedance matrices, and hybrid matrices are used to analyze the propagation of electromagnetic waves through layered media. Wave matrices are found for magneto-electric sheet boundaries. These results are then used to develop an analytical synthesis approach for metasurfaces consisting of a cascade of sheets separated by dielectric spacers.

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