An Extended NATREX Model for China

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Abstract

This paper extends, for the first time, Stein’s (1995a) NATREX model to China and other similar emerging market economies. We incorporate fundamentals that have not been studied by the existing literature on the NATREX model to capture the unique characteristics of the Chinese economy. Based on dynamic stability analysis, we derive the medium-run and long-run real equilibrium exchange rates and relative prices of non-tradables, and provide a detailed analysis of the effects of fundamentals. The fundamentals that affect the long-run equilibrium real exchange rate and the relative price of non-tradables include terms of trade, total and net factor productivity, rural transformation, dependency ratio, financial liberalization, relative unit labour cost, relative rate of return to capital, government investment, tax rate and the foreign real interest rate.

*Key Words:* Extended NATREX model; Equilibrium exchange rate; Relative price of non-tradables; China; Dynamic stability; Steady state

*JEL classification:* F31, F32, F41, F43
1. Introduction

The NATREX model, introduced by Stein (1995a), is the “natural real exchange rate” that would prevail if speculative and cyclical factors could be removed whilst unemployment is at its natural rate. In his framework, the medium run equilibrium conditions determining the NATREX are the basic balance of payments which is in equilibrium and the portfolio balance between the holdings of assets denominated in the home and in the foreign currency. In the long run, the fundamentals are defined as disturbances to productivity and social thrift at home and abroad. They affect the evolution of capital and foreign debt via the investment function and the current account. When capital and foreign debt converge to their steady state the NATREX becomes a function of economic fundamentals. The distinction between the medium and long run is an essential feature of the NATREX model. The NATREX is a moving equilibrium real exchange rate responding to continuous changes in exogenous and endogenous real fundamentals.

Stein’s model was developed for studying the equilibrium US dollar and was therefore designed to capture the features of advanced industrial countries. This paper is the first attempt to extend Stein’s (1995a) NATREX model to China. We incorporate fundamentals that have rarely been studied by the existing literature into the framework of the NATREX model to capture the unique characteristics of the Chinese economy. Based on dynamic stability analysis, we derive the medium-run and long-run real equilibrium exchange rates that are delivered by these dynamic fundamentals.

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1 This extended NATREX model is relevant to all emerging markets which have some of the characteristics of China’s economy.
We extend the original NATREX model of Stein (1995a) in a number of crucial ways that allow us to shed light on the determinants of equilibrium real exchange rates in China and other similar emerging market economies:

First, the two state variables in Stein’s model are capital per effective labour and foreign debt per effective labour. As China is a net creditor\(^2\), the two state variables for China are capital per effective labour and net foreign assets per effective labour.

Second, instead of using approximations for productivity, we consider a production function to derive total factor productivity. Furthermore, rural transformation is incorporated into the production function to reflect the effect of China’s rural-urban migration and rural industrialization on the real exchange rate.

Third, time preference is regarded as exogenous in Stein’s model. Following Modigliani and Cao (2004), we treat time preference as an endogenous variable that is determined by fundamentals such as demographic factors and liquidity constraints.

Fourth, aggregate investment is decomposed into domestic private investment, government investment and foreign direct investment (FDI). This enables us to analyse, for the first time, the effects on the real exchange rate of such fundamentals as relative unit labour cost, relative rate of return to capital, taxation and country risk, all of which play an important role in the emerging market economies.

Fifth, as the uncovered interest parity (UIP) does not seem to hold for China, the country risk premium is introduced in the portfolio balance equation to explain the divergence from UIP.

Sixth, following Lim and Stein (1995), we regard the terms of trade for China as an exogenous fundamental, which is a more realistic assumption for emerging market economies.

\(^2\) This also applies to other South East Asian countries.
economies. Based on the exogenous terms of trade, the goods market clearing condition is equivalent to non-tradable goods market equilibrium.

The remaining of the paper is organised as follows. Section 2 outlines the structure of the model and the specification of the individual components. Section 3 examines the dynamic stability of the model and analyses the medium-run and steady state equilibrium. Section 4 analyses in detail the effects of the economic fundamentals on the relative price of non-tradables and on the real exchange rate in the medium-run and the long-run. The final section summarises the main findings and contributions of the paper.

2. The Structure of the Model

2.1. Consumption

Consumption, $C$, is proportional at rate $g$ to the current wealth, $W: C(t) = gW(t)$, where $g$ is the social time preference. Wealth is a function of capital per effective labour, $k(t)$, and real foreign assets per effective labour, $F(t)^3: W(t) = k(t) + F(t)$. Therefore, the consumption function can be written as $C = C(k, F; g)$ with $\partial C / \partial g > 0$. Following Modigliani and Cao (2004), the social time preference $g$ is modeled as an endogenous rather than exogenous variable, dependent on demographic factors and financial liberalization.

As argued by Modigliani and Cao (2004), for China the relation between the number of minors and employed population is the crucial demographic variable. In their study of Chinese savings, Modigliani and Cao (2004) find that One-Child policy has led to a

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3 Capital per effective labour, $k(t)$, foreign assets per effective labour, $F(t)$ and other quantity variables are all measured per unit of effective labour in the entire economy. We refer to capital and foreign assets as capital stock per effective labour and net foreign assets per effective labour for simplicity.
gradual reduction in the ratio of minors (under 15) to employment and thereby has reduced the consumption-to-income ratio. Therefore, dependency ratio ($DEP$), the ratio of minors to the employed population will be incorporated into the consumption function to capture the demographic effects.

Existing literature studying consumption in China shows the insignificance of interest rate effect and the importance of liquidity constraints on consumption (e.g., Li, 1999; Wang et al., 2000; Yang and Li, 1997; Zhang, 1997, Zhang and Wan, 2002). The consumers’ behaviour in developing countries could be dominated by liquidity constraints that affect the ability to substitute consumption intertemporally (Rossi, 1988). On the other hand, as argued by Prasad (2004), China’s transformation into a dynamic private-sector-led economy and its integration into the global economy have been among the most dramatic economic developments of the recent decades. Therefore, under an imperfect financial market, the effectiveness of financial liberalisation in relaxing the liquidity constraints is an important determinant of consumption in China. Following Kose et al (2006), we incorporate the level of financial market development, measured by the ratio of total credit to the private sector to GDP ($CREP$), into the consumption function.

Therefore, the consumption function for China could be expressed as:

$$C = C(k, F; g) = C(k, F; DEP, CREP)$$

2.2. Total Factor Productivity

Various approximations of productivity have been used in the existing NATREX studies. In Stein’s (1995a, productivity is approximated using twelve-quarter moving average of the growth rate of the real GDP in the US and G-10. Lim and Stein (1995) use the average product of labour in their study of NATREX for Australia. Connolly
and Devereux (1995) employ relative income per capita in terms of the US to analyze the NATREX for France and Germany. Crouhy-Veyrac and Marc (1995) use the ratio of business capital to employment in their study of Latin America. Stein (1995b) employs the $q$-ratio as an approximation of productivity, where $q = \text{industrial share prices (GR62)/prices of industrial products (GR63)}$ in his study of NATREX for Germany. For China, instead of using approximations, we will derive total factor productivity ($TFP$) from the production function\(^4\) ($y = y(k; TFP)$) and incorporate it as a key fundamental into the extended NATREX model.

Since the reform and opening up policy was implemented in 1978, two forms of rural transformation have taken place: rural-urban migration and rural industrialization. Rural urban migration has been reducing China’s rural population through migration from countryside to cities. Rural industrialization has shifted farmers from working in their fields to working in labour-intensive rural enterprises (i.e. Town and Village Enterprises). Rural transformation is particular relevant for China as China’s economic growth benefits greatly from its unlimited labour supply which is generated from rural transformation. Different from transition countries, China’s transformation from central-planned to market oriented economy is characterized by shifting of labour from lower productive primary sector to more productive secondary and tertiary sectors\(^5\).

The link between China’s rural transformation and economic growth has been analyzed by Woo (1998). He found that labour reallocation accounts for 1.1% and

\(^4\) We use $TFP$ because productivity approximated by output per labour or the growth rate of output may lead to measurement errors. For instance, conventionally output per labour is a function of capital per labour and total factor productivity. Using output per labour as an approximation of total factor productivity implies capital per labour must expand at the same speed of output per labour and total factor productivity. Otherwise, total factor productivity will be over valued or under valued by using output per labour as an approximation.

\(^5\) Chow (1993) found the marginal value product of labour in 1978 to be 63 yuan in agriculture, 1027 yuan in industry, 452 yuan in construction, 739 yuan in transportation and 1809 yuan in commerce.
1.3% of average economic growth in China during the periods 1979-1993 and 1985-1993, respectively. Following Woo (1998), we decompose TFP into net factor productivity (NFP) and rural transformation (RT). Therefore, the production function for China takes the form of:

\[ y = \left( k; NFP, RT \right) \]

2.3. Savings

Savings can be expressed as Gross National Income (GNI) less consumption:

\[ s = y(k; NFP, RT) + r'F - C(k, F; CREP, DEP) = S(k, F; NFP, RT, r', CREP, DEP) \]

where \( r' \) denotes the world’s real interest rate.

2.4. Investment

Stein (1995a) derives the investment function from Tobin’s \( q \) ratio. However, as argued by Song et al. (2001), Tobin’s \( q \) ratio does not seem to be applicable to China. First, firms’ capital assets are valued in the financial markets in Tobin’s model. China’s financial markets have a development history of merely fifteen years. Not only the scale of financial markets is relatively small but also there are restrictions on the transactions in the financial markets imposed by the government. Furthermore, the assumption of perfect competitive market, a crucial assumption of Tobin’s model, does not hold for China.

Recent studies have tried to explain China’s aggregate investment using different models (e.g. Sun, 1998, Zhu and Liang, 1999, Shen, 1999, 2000, Song et al., 2001,

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6Shenzhen and Shanghai Stock Exchange Market, the first two Chinese stock markets were established in 1992.
Qin and Song, 2003, He and Qin, 2004). Among these studies, Song et al. (2001) and He and Qin (2004) employ the neoclassical investment model. In Song et al. (2001), aggregate investment is modelled as a function of the user cost of capital and expected output. He and Qin (2004) apply the neoclassical investment model to the business sector investment. The business sector investment is modelled as a function of the cost of capital, output and government investment. In He and Qin (2004), the user cost of capital is defined as follows:

\[
c = \frac{p_k (r + \delta)}{p(1 - \tau)}
\]  

(4)

where \(c\), \(p_k\), \(p\), \(r\), \(\delta\) and \(\tau\) are the user cost of capital, price of capital goods, output price, real interest rate, rate of economic depreciation and the composite tax rate.

Following Song et al. (2001) and He and Qin (2004), we model domestic private investment using the neoclassical model. However, compared with Song et al. (2001) which apply the neoclassical model to China’s aggregate investment, we first decompose aggregate investment into domestic investment and foreign direct investment (FDI), with the former further being decomposed into domestic private investment and government investment. Similar decomposition has been implemented by He and Qin (2004) where they decompose domestic aggregate investment into business sector investment and government investment and each investment is modeled individually.

Following He and Qin (2004), domestic private investment per effective labour for China (\(I_{DPL}\)) can be modelled as:

\[
I_{DPL} = f(y, c)
\]

7 For an review of these papers, please refer to He and Qin (2004).
8 For reasons of why neoclassical model is more applicable to model China’s investment, refer to Song et al. (2001).
It can be further written as:

\[ I_{DPI} = f(y,c) = f(y(k; NFP, RT), c) = f(k; NFP, RT, c) \]

Government investment is treated as exogenous. Before the launch of the national policy of “reform and opening up” in 1978, State-owned Enterprises (SOEs) were fully administrated by Chinese government under the central-planned economy. Restructuring and privatization have reduced the share of SOEs. However, according to data from Chinese Statistics Yearbook, a considerable proportion of investment still flows to (SOEs). For instance, 35.5% of total investment of fixed assets flew to SOEs in 2004. Investment to SOEs is clearly affected by government investment policies. According to Xiang (1999), one of the major roles of government investment is to finance state-prioritized investment projects. Zhu and Liang (1999) and Shen (1999, 2000) include government investment as an explanatory variable of aggregate investment and find it significant. Therefore, the ratio of government investment to total fixed assets investment \((GI)\) is incorporated into the investment function as an exogenous variable to catch the effects of government behaviour.

Foreign direct investment (FDI) is an important component of aggregate investment in China. The reduction of barriers to FDI and implementation of policies to improve the investment environment have played a key role in attracting FDI in China. Special Economic Zones (SEZs), open coastal cities (OCCs) and FDI favourable policies are among the most successful measures of China’s economic reform since 1978. There are extensive studies analysing the determinants of FDI to China\(^9\). Among them, wage has been widely employed as a crucial determinant of FDI to China (i.e. Dees, 1998; Coughlin and Segev, 2002; Fung et al, 2002; Shan, 2002; Sun et al, 2002; Zhang,

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\(^9\) For a literature review of recent study of determinants of FDI in China, please refer to Ho (2004).
2000, 2001; Ho, 2004). As the US is regarded as the foreign country in this study, relative unit labour cost of China to the US will be employed rather than China’s wage. Considerable amount of literature show that country risk has a significant impact on foreign investment decisions. Some recent studies include Nordal (2001), Bevan and Estrin (2004), and Janicki and Wunnava (2004). Some studies on the determinants of FDI in China try to incorporate country risk related variables as determinants due to data limitation on country risk. For instance, Ng and Tuan (2003) incorporate trade constraints and both Zhang (2000) and Zhang (2001) incorporate trade barriers and political stability as determinants of FDI to China. In our study, use net foreign assets \( F \) as an approximate of country risk (as in Lim and Stein, 1995) and incorporate it into the FDI function. Furthermore, we introduce the relative return to capital of China as an important determinant of FDI. Therefore, FDI is a function of:

\[
FDI = f(RULC, RRC, F)
\]

(6)

where \( RULC \) and \( RRC \) are relative unit labour cost of China and relative return to capital of China (relatively to its competitors) respectively.

Therefore, the aggregate investment function can be expressed as:

\[
I = I(I_{dp}, GI, FDI) = I(k, F; NFP, RT, c, GI, RULC, RRC) + + + - - + +
\]

(7)

2.5. Goods Market and Current Account

Following Lim and Stein (1995), we assume that the economy produces an exportable good 1 and a non-tradable good \( n \). The foreign country does likewise where the export good is good 2. \( R_n \) denotes the relative price of non-traded good \( n \) (\( p_n \)) to the exported good (\( p_1 \)):

\[
R_n = \frac{p_n}{p_1}.
\]

(8)
The terms of trade ($T$) is the relative price of exported good 1 ($p_1$) to imported good 2 ($p'_2$) measured in a common currency:

$$T = N \frac{p_1}{p'_2},$$

where $N$ is the nominal exchange rate defined as foreign currency per Chinese Yuan, CNY.

The real exchange rate of China, $R$, is a function of terms of trade $T$ and the relative price of non-tradables $R_n$:

$$R = T(R_n)^a$$

where $a$ denotes the weight given to the non-tradable sector in the GDP deflator. The relationship between nominal and real exchange rate is defined as:

$$N = R \frac{p'}{p},$$

where $p$ and $p'$ the Chinese GDP price deflator and the foreign GDP price deflator (which is exogenous) respectively.

We regard China’s terms of trade as exogenous. China’s export share in the world has increased considerably from for 0.75% of total world exports in 1978 to 7.3% in 2005. However, Kamin et al (2006) evaluate the question of whether China’s buoyant export growth has led to significant changes in the import prices. They find that the impact of Chinese exports on global import prices has been, while non-negligible, fairly modest. In terms of the China-US trade relationship, they identify a statistically significant effect of US imports from China on US import prices, but given the size of this effect and the relatively low share of imports in US GDP, the ultimate impact on US consumer prices has likely been quite small. Furthermore, using multi-country database of trade transaction, they find that since 1993 Chinese exports lower annual
import inflation in a large set of economies by 0.25% or less on average. Therefore, the terms of trade are regarded as exogenous in this study given that the influence of China in the world trade is still limited despite the relative increase of its importance.

Aggregate consumption can be decomposed into consumption of non-traded good $n$ ($C_n$) and consumption of imported good 2 ($C_2$). The relative price of non-traded good $n$ ($p_n$) to the imported good 2 ($p'_2$) can be expressed as:

$$\frac{p_n}{p'_2} = \left( \frac{Np_1}{p'_2} \right) \frac{p_n}{p_1} = TR_n$$  \hspace{1cm} (12)

This relative price affects shares of $C_n$ and $C_2$ within the aggregate consumption $C$.

For instance, an increase in the relative price of non-traded good $n$ ($p_n$) to the imported good 2 ($p'_2$) will decrease demand for non-traded good and increase demand for imported good. Therefore, consumption of non-traded good $n$ ($C_n$) and consumption of imported good 2 ($C_2$) can be expressed as:

$$C_n = C_n(R_n, k, F; DEP, CREP, T)$$  \hspace{1cm} (1a)

$$C_2 = C_2(R_n, k, F; DEP, CREP, T)$$  \hspace{1cm} (1b)

Production can be decomposed into production of non-traded good $n$ ($y_n$) and exportable good ($y_1$). The relative price of non-traded good $n$ ($p_n$) to the exportable good 1 ($p_1$), $R_n = p_n / p_1$, affects allocation of supply of $y_n$ and $y_1$. For instance, an increase in $R_n$ will increase the supply of non-traded good, $y_n$, and decrease the supply of exportable good, $y_1$. Therefore, $y_n$ and $y_1$ can be expressed as:

$$y_n = y_n(R_n, k; NFP, RT)$$  \hspace{1cm} (2a)

$$y_1 = y_1(R_n, k; NFP, RT)$$  \hspace{1cm} (2b)
Capital is used to produce non-tradable good \( n \) and exportable good 1, while capital good consists of both non-tradable good \( n \) and imported good 2. Relative price of non-tradables to imported goods, \( TR_n \), affects shares of \( I_n \) and \( I_2 \) within the aggregate investment \( I \). For instance, a higher relative price of non-tradables discourages investment using non-tradables, \( I_n \), and encourages investment using imported goods, \( I_2 \).

Recall the definition of user cost of capital, equation (4). If the investment good consists of fraction \( m \) of imported good 2 and fraction \((1-m)\) of non-traded good \( n \), then \( I = I_2^m I_n^{(1-m)} \) and therefore the price of capital is \( p_k = (p_2'/N)^m (p_n)^{(1-m)} \). If capital is used to produce fraction \( a \) of exportable good 1 and fraction \((1-a)\) of non-tradable good \( n \), then the output price is \( p = (p_1)^a (p_n)^{(1-a)} \). The relative price \( \frac{p_k}{p} \) in equation (4) can be rewritten as:

\[
\frac{p_k}{p} = \frac{(p_2'/N)^m (p_n)^{(1-m)}}{(p_1)^a (p_n)^{(1-a)}} = T^{-m} R_n^{a-m} \tag{13}
\]

The user cost of capital can be rewritten as:

\[
c = \frac{p_k (r + \delta)}{p(1-\tau)} = T^{-m} R_n^{a-m} d \quad \text{or} \quad c = c(T, R_n, d) \tag{14}
\]

where \( d = \frac{r + \delta}{1-\tau} \). As we assume the depreciation rate, \( \delta \), is a constant, \( d \) is a function of \( r \) and \( \tau \) : \( d = d(r, \tau) \). Therefore equation (14) can be rewritten as:

\[
c = c(T, R_n, r, \tau) \tag{15}
\]

An increase in terms of trade \( T \) decreases the user cost of capital and increases investment. Higher \( r \) and \( \tau \) raise user cost of capital and discourage investment. The effect of relative price of non-tradable goods \( R_n \) is ambiguous, depending on the sign...
of \((a - m)\). Compared with its main effect of allocating investment using non-tradables and imported goods within aggregate investment, the ambiguous effect of \(R_n\) on user cost of capital is negligible. As mentioned above, an increase in \(R_n\) will discourage demand for investment using non-tradables, \(I_n\) and encourage demand for investment using imported goods, \(I_2\).

Therefore, the investment using non-tradables and imported goods can be expressed as:

\[
I_2 = I_2(R_n, k, F; NFP, RT, c(R_n, T, r, \tau), GI, RULC, RRC, T) \\
+ + + + + + +/- + - - + - + + \\
= I_2(R_n, k, F; NFP, RT, r, \tau, GI, RULC, RRC, T) \\
+ + + + + + +/- + - - + - + + \\
(7a)
\]

\[
I_n = I_n(R_n, k; NFP, RT, c(R_n, T, r, \tau), GI, T) \\
- + + + +/- + - - + - - + - \\
= I_n(R_n, k; NFP, RT, r, \tau, GI, T) \\
- + + + +/- + - - + - - + - \\
(7b)
\]

\[
I = I_2 + I_n = I(R_n, k, F; NFP, RT, c(R_n, T, r, \tau), GI, RULC, RRC, T) \\
+/+ + +/- +/- + - - + + + +/- \\
= I(R_n, k, F; NFP, RT, r, \tau, GI, RULC, RRC, T) \\
+/+ + +/- +/- + - - + + + \\
(7c)
\]

Capital accumulation is given by

\[
dk / dt = I - nk
\]

(16)

where \(n\) is the population growth rate.

Based on the exogenous terms of trade, the equilibrium condition for the good market is the market clearing condition for the non-traded good:

\[10\] On one hand, higher terms of trade implies a higher relative price of non-tradables to imported goods, which discourages \(I_n\) and encourages \(I_2\), so its total effect on aggregate investment \(I\) becomes ambiguous. One the other hand, higher terms of trade implies a lower user cost of capital, which stimulates both segments of investment \((I_n\) and \(I_2\)) and hence aggregate investment \(I\). Compared to its positive effect on aggregate investment, the effect of terms of trade on allocating demand of investment using non-tradables and imports is negligible, so we are assuming that the sign of the terms of trade is positive.
\[(I - S) + CA = 0\]

\[C_n(R_n, k, F; DEP, CREP, T) + I_n(R_n, k; NFP, RT, r, \tau, GI, T) - y_n(R_n, k; NFP, RT) = 0\]  

(17)

The market clearing equation (17) implies that the demand for the non-traded good, which consists of consumption \(C_n\) and investment using non-tradables \(I_n\), equals the supply of the non-traded good \(y_n\).

The current account is the trade balance plus the interest rate income on foreign assets, \(r'F\). The trade balance is the value of exported good 1 \((y_1)\) less the value of imported good 2, which consists of consumption and investment that uses imported goods \((C_2\) and \(I_2\)).

\[CA = y_1(R_n, k; NFP, RT) - I_2(R_n, k, F; NFP, RT, r, \tau, GI, RULC, RRC, T) - C_2(R_n, k, F; DEP, CREP, T) + r'F\]  

(18)

2.6. Accumulation of Foreign Assets

Rate of change of foreign assets is savings less investment and minus \(nF\):

\[\frac{dF}{dt} = S - I - nF = CA - nF\]  

(19)

where \(n\) is the growth rate of effective labour.

2.7. Portfolio Balance

In his study of NATREX of the US dollar, Stein (1995a) finds that the real long-term bond yields of US and G-10 converge and it implies acceptance of the uncovered interest parity (UIP) hypothesis. In their study for the small open economy Australia, Lim and Stein (1995) find that there are some significant deviations from UIP. Deviations from both covered and uncovered interest rate parity conditions capture transaction costs, including political risks, exchange rate risk (market pressure), and
transaction costs—which Frankel (1991) calls “the country premium”. With regards to China, Ma et al (2004) found that though onshore and offshore interest rate differentials have been shrinking over time, China’s capital controls are still effective as these interest rate differentials still remain large. Liu and Otani (2005) show that deviations from the uncovered interest rate parity condition for China exhibit strong non-stationarity and persistency. Therefore, for a typical developing country like China, UIP is unlikely to hold due to the existence of the country premium. Lane and Milesi-Ferretti (2001) suggest that a country’s steady state risk premium (in their case measured as the real interest rates differential) is inversely and linearly related to net foreign asset position in their study of long term capital movement for a group of developed and developing counties including China. Other studies which relate the deviations from UIP to net foreign assets include Selaive and Tuesta (2003a, 2003b), Cavallo and Ghironi (2002), and Benczúr et al (2006). Therefore, the portfolio balance is expressed as:

\[ r = r' + h(F) = (r', F) \]

where foreign assets \( F \) is used to approximate the country risk premium of China.

### 3. Analysis of the Model

The model consists of equations 3, 7c, 10, 16, 17, 18, 19, and 20, where \( c \) in equations 7c and 17 has been replaced by equation 15\(^1\).
3.1. Medium-Run Equilibrium

The medium-run is defined as the period in which the capital intensity and foreign assets are taken as predetermined variables. The terms of trade are exogenous for China, which implies that the equilibrium condition for the goods market is equivalent to the market clearing for non-tradables:

\[
C_n(R_n,k,F;DEP,CREP,T) + I_n(R_n,k,F;NFP,RT,r',\tau,GI,T) = y_n(R_n,k;NFP,RT) \tag{17}
\]

The first two items on the left-hand side are consumption and investment of non-traded goods, the sum of which is the demand for non-traded goods \( D_n \). The right hand side of the equation (17) gives the supply of non-traded goods \( S_n \).

The relative price of non-tradables, \( R_n \), equilibrates the market of non-traded goods. Solving explicitly for \( R_n \) in equation (17) yields:

\[
R_n(t) = R_n(k(t),F(t);Z(t)), \tag{21}
\]

\[
Z = [DEP,CREP,NFP,RT,r',\tau,GI,T] \tag{21a}
\]

where \( Z \) denotes the fundamentals that determine the relative price of non-tradables.

Based on equations (10) and (21), the medium-run equilibrium real exchange rate is given by:

\[
R(t) = T[R_n(k(t),F(t);Z(t))]^u = R(k(t),F(t);Z) \tag{22}
\]

In the medium-run, \( k \) and \( F \) are exogenous. Therefore, any disturbance to the exogenous variables will shift the demand and/or supply curve of non-tradables and generate a new \( R_n \) to maintain the goods market equilibrium. The effects of changes in exogenous variables on \( R_n \) in the medium-run are obtained from equations (17) and (21) and listed in Appendix A.
3.2. Dynamic Adjustment

The long-run dynamics involve endogenous movements of the capital and foreign assets. Combining the change of capital equation (16), investment equation (7c) and portfolio balance (20) yields the equation for the evolution of capital:

\[ \frac{dk}{dt} = J(k, F; Z), \quad J_k < 0, \quad J_F > 0 \]

(23)

Based on portfolio balance equation (20) and savings equation (3), we obtain:

\[ s = S(k, F; Z), \quad S_k > 0, \quad S_F < 0 \]

(24)

From equations (23), (24) and (19) we obtain the equation for the evolution of foreign assets:

\[ \frac{dF}{dt} = S - J = L(k, F; Z), \quad L_k > 0, \quad L_F < 0 \]

(25)

Equations (4.23) and (4.25) describe the dynamic system concerning the evolution of capital and foreign assets. Now we are going to analyse the dynamic stability of capital and foreign assets in a phase diagram (Figures 1 and 2).

(1) \( J = 0 \) is the locus of points of capital and foreign assets at which the rate of investment is zero. It is positive sloped because of \( \frac{dF}{dk} \mid_{J=0} = -\frac{J_k}{J_F} > 0 \) given \( J_k < 0 \) and \( J_F > 0 \). An increase in capital decreases marginal productivity of capital and decreases further investment: \( J_k < 0 \). An increase in the foreign assets reduces country risk and real domestic interest rate, hence generates higher investment: \( J_F > 0 \). To the left of \( J = 0 \) where marginal productivity exceeds the user cost of capital \( \left( \frac{\partial Y}{\partial K} > c \right) \) and \( k < k^* \), capital rises. To the right of \( J = 0 \) where marginal

---

12 See Appendix B for the signs of the derivatives. Following Stein (1995a) and Lim and Stein (1995), we assume the population growth \( n \) is zero for mathematical convenience.
productivity is lower than the user cost of capital \( \frac{\partial Y}{\partial K} < c \) and \( k > k^* \), capital declines.

(2) \( L = 0 \) is the locus of the points of capital and foreign assets where there are no capital outflows since investment equals savings. On any points of \( L = 0 \) curve there is zero current account: \( CA = 0 \). The \( L = 0 \) curve is positive sloped because of

\[
\frac{dF}{dk}\bigg|_{L=0} = -\frac{L_k}{L_F} > 0 \quad \text{given} \quad L_k > 0 \quad \text{and} \quad L_F < 0.
\]

An increase in capital lowers investment \( (J_k < 0) \) and raises savings \( (S_k > 0) \) and hence increases savings less investment: \( L_k > 0 \). Higher foreign assets increase wealth and hence consumption rises. Higher consumption means lower savings \( (S_F < 0) \) and higher investment \( (J_F > 0) \) and therefore \( (S - J) \) declines: \( L_F < 0 \). Above \( L = 0 \) curve where foreign assets exceed their steady state value \( (F > F^*) \), investment exceeds savings and there is current account deficit \( (CA < 0) \). Thus foreign assets decline towards their steady state. Below \( L = 0 \) curve where foreign assets are lower than their steady state value \( (F < F^*) \), savings exceed investment and there is current account surplus \( (CA > 0) \). Thus foreign assets rise towards their steady state.

To ensure the stability of the model, the slope of \( J = 0 \) has to be greater than that of \( L = 0 \). This is explained in detail as follows.

Given
\[
\frac{\partial S}{\partial k} > 0, \quad \frac{\partial S}{\partial F} < 0, \quad \frac{\partial J}{\partial k} < 0, \quad \frac{\partial J}{\partial F} > 0, \quad \frac{\partial L}{\partial k} > 0, \quad \frac{\partial L}{\partial F} < 0,
\]
the sign of
\[
-\frac{J_k}{J_F} - \left( -\frac{L_k}{L_F} \right)
\]
is ambiguous. The two possibilities are:

\[
-\frac{J_k}{J_F} - \left( -\frac{L_k}{L_F} \right) \Rightarrow G = J_kL_F - L_kJ_F > 0 \quad \text{(case 1)}
\]
\[
\frac{J_k}{J_F} - \left( -\frac{L_k}{L_F} \right) \Rightarrow G = J_k L_F - L_k J_F < 0 \quad \text{(case 2)}
\]

Case 1 implies that \( J = 0 \) has a greater slope than \( L = 0 \) and is illustrated in Figure 1. All streamlines in this phase diagram flow noncyclically towards the equilibrium point E. Some streamlines stay in a single region and others cross from one region to another. When a streamline crosses over, it must have either an infinite slope (crossing \( L = 0 \)) or a zero slope (crossing \( J = 0 \)) as suggested by the dotted line attached to it. This is due to the fact that, along \( L = 0 \) (or \( J = 0 \)) curve, \( L \) (or \( J \)) is stationary over time, so the streamline must not have any horizontal (vertical) movement while crossing that curve. The equilibrium point (E) on this diagram is a stable node as all streamlines associated to it lead noncyclically towards it. Such a stable node E under case 1 ensures the stability of the model.

Case 2 implies the slope of \( J = 0 \) is smaller than \( L = 0 \) and is illustrated in Figure 2. The equilibrium point (E) on this diagram is a saddle point - it is stable in some direction but unstable in others. A saddle point has one pair of streamlines, the stable branches of the saddle point that flow directly and consistently toward the equilibrium, and one pair of streamlines, the unstable branches of the saddle point that flow directly and consistently away from it. All the other trajectories head toward the saddle point initially but sooner or later turn away from it. Since stability is observed only on the stable branches, a saddle point is generically classified as an unstable equilibrium (Chiang, 1987). Since the equilibrium point E in case 2 is a saddle point, it can not ensure the stability of the model.

Therefore, the stability condition \( G > 0 \) must hold to ensure the stability of the model, which is described by Figure 1. The stability condition \( G = J_k L_F - L_k J_F > 0 \) holds as long as (a) the impact of capital stock on investment is greater than the impact of
net foreign assets on investment \((-J_k > J_F)\) along \(J = 0\) and (b) the impact of net foreign assets on current account is greater than the impact of capital on current account \((-L_F > L_k)\) along \(L = 0\).

3.3. The Steady-State

The long-run steady state is reached when capital and foreign assets converge to sustainable constants \(k^*\) and \(F^*\):

\[
J(k^*, F^*; Z) = 0 \tag{26}
\]

\[
L(k^*, F^*; Z) = S(k^*, F^*; Z) - J(k^*, F^*; Z) = 0 \tag{27}
\]

Solving equations (26) and (27) we can obtain the steady states:

\[
k^* = k(Z) \tag{28}
\]

\[
F^* = F(Z) \tag{29}
\]

Changes in \(k^*\) and \(F^*\) will affect the equilibrium condition in the goods market. The goods market equilibrium is equivalent to the non-tradables equilibrium. Therefore, relative price of non-tradables will adjust to its steady state \(R_n^*\) to equilibrate the non-tradables market while capital and foreign assets are at their steady states. Therefore, the non-tradables market equilibrium under steady state can be described as:

\[
C_n(R_n^*, k^*, F^*; DEP, CREP, T) + I_n(R_n^*, k^*, F^*; NFP, RT, r', \tau, GI, T) = y_n(R_n^*, k^*; NFP, RT) \tag{30}
\]

Solving equation (30) we can get the expression for the steady state relative price of non-tradables (equation (31a)) and derive \(\frac{dR_n^*}{dZ}\) (equation (31b)):

\[
R_n^* = R_n(k(Z), F(Z); Z) = R_n^*(Z) \tag{31a}
\]
\[
\frac{dR_n^*}{dZ} = \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dZ} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dZ} + \frac{\partial R_n}{\partial Z}
\]  
\[ (31b) \]

\[
R^* = T(R_n^*)^\omega = R^*(Z)
\]  
\[ (32) \]

The last item on the right hand side of equation (31b) catches the direct effect of disturbance in fundamentals on \( R_n \) in the medium-run. The signs of this item are derived and explained in Appendix A. The first two items catch the indirect effect of disturbance in fundamentals on \( R_n \) through changes in \( k^* \) and \( F^* \) in the long-run. Details of the derivation of \( \frac{dk^*}{dZ} \), \( \frac{dF^*}{dZ} \) and mathematical computation of their signs are shown in Appendix B. Derivation of \( \frac{dR_n^*}{dZ} \) is discussed in Appendix C. All signs are summarised in Appendix D. According to equation (32), the fundamentals which affect the relative price of non-tradables, \( R_n^* \), affect the long-run real exchange rate, \( R^* \), in a similar way. The only exception is the terms of trade. As equation (10) indicates, changes in the terms of trade affect the real exchange rate directly and indirectly via changes in \( R_n \). These direct and indirect effects will be explained in details in the following section.

4. The Relative Price of Non-Tradables and the Real Exchange Rate in the Medium-Run and Long-Run

Now we are going to analyze the sign of \( \frac{dR_n^*}{dZ} \), combining the phase diagram (Figure 3) which indicates the trajectories of capital and foreign assets to their steady state when there are changes in the fundamentals, and Figure 4 which describes the goods market equilibrium (equation 17)\(^{13} \). The signs of \( \frac{\partial R_n}{\partial Z} \) are determined by the

\(^{13}\) The demand curve, \( D_n \), is downward sloping due to the fact that an increase in the relative price of non-tradables decreases the demand for consumption of non-tradables and investment using non-
effect of changes in fundamentals on $R_n$ in the medium-run. The signs of $dk^*/dZ$ and $dF^*/dZ$ are determined by the effect of changes in fundamentals on steady state capital and foreign assets in the long-run.

DependencyRatio

An increase in dependency ratio raises consumption of non-tradables, which shifts the demand curve from $D_{n0}$ to $D_{n2}$ and appreciates the relative price of non-tradables from A to C. Therefore, the direct effect of a higher dependency ratio on the price of non-tradables is positive: $\partial R_n/\partial DEP > 0$.

In the long-run, a rise in the dependency ratio reduces foreign assets and capital. High consumption increases borrowing from foreign countries and leads to net long-term capital inflows. The capital decreases due to the higher risk premium generated by lower foreign assets. Therefore, a rise in the dependency ratio leads the economy to stabilize at lower foreign assets and lower capital: $dF^*/dDEP < 0$, $dk^*/dDEP < 0$.

The trajectories of capital and foreign assets are described as $E1 - E$.

With lower foreign assets, wealth reduces unambiguously. Consumption gradually declines and savings gradually rise. Demand for non-tradables reduces, say, from $D_{n2}$ to $D_{n0}$, which reduces the relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dDEP < 0$.

The non-tradable sector in China is regarded as labour intensive. Thus a decline in capital increases supply of non-tradables from $S_{n0}$ to $S_{n2}$ and depreciates the tradables. The opposite applies to the upward sloping supply curve of non-tradables, $S_n$. The medium-run equilibrium of the goods market is at point A ($D_n = S_n$) where the real exchange rate is $R_{n0}$.

---

14 Higher capital stock will draw resources away from the non-tradables sector to the tradables sector as non-tradables sector is labour intensive. Therefore, there is a negative relationship between capital and supply of non-tradables.
relative price of non-tradables to point $P$ : $(\partial R_n/\partial k)dk^*/dDEP < 0$. The relative price of non-tradables is lower than at the initial point A. This is due to the fact that desired capital inflows\textsuperscript{15} decline and interest income from foreign countries reduces or there will be interest payment to foreign countries if the economy changes from net creditor to net debtor. To produce the trade surplus needed to offset lower interest income from or higher interest payments to foreign countries, the relative price of non-tradables must depreciate below its initial level, and so does the real exchange rate.

\[
\frac{dR_n}{dDEP} = \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dDEP} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dDEP} + \frac{\partial R_n}{\partial DEP} < 0 \tag{33}
\]

Therefore, an increase in dependency ratio $DEP$ first appreciates the relative price of non-tradables and then depreciates it in long-run equilibrium.

\textit{Financial Liberalisation}

A higher degree of financial liberalisation relaxes liquidity constraints on consumption and enables current consumption to be repaid by future income. A higher consumption, financed by borrowing, shifts the demand for non-tradables from $D_{n0}$ to $D_{n2}$ and appreciates the relative price of non-tradables: $\partial R_n/\partial CREP > 0$.

In the long-run, a rise in financial liberalization reduces foreign assets and capital. An increase in consumption financed by borrowing generates capital inflows and drives the interest rate higher. Capital decreases not only because of higher user cost of capital generated by higher domestic interest rate but also because of higher risk premium generated by lower foreign assets. Therefore, a higher financial

\textsuperscript{15} The NATREX adjusts to produce whatever current account balances are needed to match changing long-term capital flows.
liberalization leads the economy to stabilize at lower foreign assets and lower capital: \( \frac{dF^*}{dCREP} < 0, \frac{dk^*}{dCREP} < 0 \). The trajectories of capital and foreign assets are described as \( E1 - E \).

Lower foreign assets reduce consumption as wealth declines. Demand for non-tradables reduces, say, from \( D_{n2} \) to \( D_{n0} \), and relative price of non-tradables depreciates: \( \frac{\partial R_n}{\partial F} \frac{dF^*}{dCREP} < 0 \). Lower capital increases supply of non-tradables from \( S_{n0} \) to \( S_{n2} \) and depreciates the relative price of non-tradables to point \( P \). Therefore, \( \frac{\partial R_n}{\partial k} \frac{dk^*}{dCREP} < 0 \). The relative price of non-tradables is lower than initial point A.

\[
\frac{dR_n^*}{dCREP} = \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dCREP} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dCREP} + \frac{\partial R_n}{\partial CREP} < 0 \tag{34}
\]

An increase in financial liberalization has similar effects on the relative price of non-tradables as an increase in the dependency ratio: appreciates \( R_n \) in the medium-run and depreciates it in the steady state.

**Net Factor Productivity in the Tradable Sector**

The effect of a rise in net factor productivity on the real exchange rate allows us to analyse the Balassa-Samuelson effect (Balassa, 1964). A productivity increase in tradables sector increases investment and hence increases demand for investment using non-tradables from \( D_{n0} \) to \( D_{n2} \): \( \frac{\partial I_n}{\partial NFP} > 0 \), where \( NFP \) denotes net factor productivity in the tradables sector. A higher productivity in the tradables sector shifts resources from the non-tradables sector to the tradables sector and hence decreases supply of non-tradables from \( S_{n0} \) to \( S_{n1} \): \( \frac{\partial y_n}{\partial NFP} < 0 \). \( R_n \) increases from point A to F: \( \frac{\partial R_n}{\partial NFP} > 0 \). On one hand, capital formation leads to current
account deficit. On the other hand, higher output of tradables given higher productivity in tradables sector generates current account surplus. Hence there is current account deficit or surplus and capital formation in the medium-run, which is described by $E2$ or $E3$.

In the long-run, an increase of productivity in tradables sector raises foreign assets and capital. A higher $NFP_t$ generates current account surplus due to a higher supply of tradables. Investment in tradables sector further increases output of tradables and current account surplus. Therefore: $dk^*/dNFP_t > 0$ and $dF^*/dNFP_t > 0$. If the starting point is $E2$, at point $N$ current account deficit turns into surplus. Current account surplus raises foreign assets, which reduces savings and raises consumption. The current account converges to balance while foreign assets are increasing towards their steady state. The trajectories of steady state capital and foreign assets are described as: $E2 – N – E$. If the starting point is $E3$, the trajectory is $E3 – E$.

An increase in output due to a higher $NFP_t$ raises income and consumption, creating an excess demand for non-tradables and appreciating relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dNFP_t > 0^{16}$. Capital formation in tradables sector decreases supply of non-tradables and increases $R_n$: $(\partial R_n/\partial k)dk^*/dNFP_t > 0$. Hence there is a further appreciation of $R_n$ in the long-run.

$$
\frac{dR_n^*}{dNFP_t} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{dNFP_t} + \left(\frac{\partial R_n}{\partial F}\right)\frac{dF^*}{dNFP_t} + \frac{\partial R_n}{\partial NFP_t} > 0 \quad (35)
$$

---

16 If the real wage in tradables sector is bid up due to higher productivity, prices in non-tradables will also be forced up. Due to China’s great labour surplus, we ignore the effect of higher productivity of tradables sector on real wage.
Therefore, an increase in productivity in the tradables sector, given exogenous terms of trade, generates a steady appreciation of the relative price of non-tradables in long-run equilibrium.

As a developing country, we expect the productivity increase in China occurs in the tradables sector, which is the situation described by equation (35). By estimating the effect of productivity increase on the real exchange rate, we will be able to test the existence of the Balassa-Samuelson effect.

**Net Factor Productivity in the Non-Tradables Sector**

If the productivity increase occurs in the non-tradables sector, investment increases and so does demand for investment using non-tradables: \( \partial I_n / \partial NFP_n > 0 \), where \( NFP_n \) denotes net factor productivity in non-tradables sector. A higher productivity in non-tradables increases supply of non-tradables: \( \partial y_n / \partial NFP_n > 0 \). The former increases demand for non-tradables from \( D_{n0} \) to \( D_{n2} \) and the latter increases supply of non-tradables from \( S_{n0} \) to \( S_{n2} \), which shifts \( R_n \) from A to U. The total direct on \( R_n \) is negative as we assume the supply effect dominates the investment effect: \( \partial R_n / \partial NFP_n < 0 \).

In the long-run, an increase in productivity in non-tradables sector raises capital and foreign assets. A shift of resources from tradables sector to non-tradables sector due to higher \( NFP_n \) decreases the supply of tradables and capital accumulation generates current account deficit. This is captured by point \( E2 \). As capital accumulates output rises gradually and savings rise relative to investment. At point N savings equals investment; after that savings exceed investment and there is current account surplus.
Along trajectory $E2 - N - E$, capital and foreign assets increase: $dk^*/dNFP_n > 0$ and $dF^*/dNFP_n > 0$.

A rise in wealth increases demand for non-tradables and appreciates relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dNFP_n > 0$. Higher capital in non-tradable sector increases supply of non-tradables which depreciates relative price of non-tradables: $(\partial R_n/\partial k)dk^*/dNFP_n < 0$. Due to the fact that there is not only a rise in $NFP_n$ but also capital accumulation in non-tradables sector, the rise in the supply of non-tradables is much higher than that of the demand for non-tradables. Therefore, an increase in $NFP_n$ has a total effect of depreciating the relative price of non-tradables.

\[
\frac{dR_n^*}{dNFP_n} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{dNFP_n} + \left(\frac{\partial R_n}{\partial F}\right)\frac{dF^*}{dNFP_n} + \frac{\partial R_n}{\partial NFP_n} < 0 \quad (36)
\]

An increase in $NFP_n$ depreciates the relative price of non-tradables in the long-run.

**Rural Transformation**

Rural transformation takes the form of rural-urban migration and rural urbanization. Both shift labour force from lower productivity agriculture sector to higher other sectors that have higher productivity. While rural transformation leads to more labour in other sectors, it does not reduce labour in agriculture. It is a feasible argument for China since its enormous labour supply surplus (most of them are rural residents) will fill in the position of labour that is shifted to other sectors. Rural transformation shifts resources from agriculture to other sectors, but it does not alter the net factor productivity (technological progress) in each individual sector. It increases the total factor productivity by increasing the weights of higher productivity sectors and reducing the weight of lower productivity sector (agriculture).
The directions in which the labour shifts affect the trajectory of the real exchange rate. If more labour is allocated to tradables sectors \((RT_t)\), which implies the tradables sector is more productive, the trajectories of foreign assets and capital are the same as when there is an increase in net factor productivity in tradables sector: \(E3 - E\) or \(E2 - N - E\). If more labour is allocated to non-tradable sectors \((RT_n)\), which implies the non-tradables sector is more productive, the trajectories of foreign assets and capital are the same as when there is an increase in net factor productivity in non-tradables sector: \(E2 - N - E\). Hence an increase in rural transformation which allocates more labour to non-tradables/tradables has similar direct and indirect effect with an increase in productivity in non-tradables/tradables sector:

\[
\begin{align*}
\frac{dR^*_n}{dRT_t} &= \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dRT_t} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dRT_t} + \frac{\partial R_n}{\partial RT_t} > 0 \\
\frac{dR^*_n}{dRT_n} &= \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dRT_n} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dRT_n} + \frac{\partial R_n}{\partial RT_n} < 0
\end{align*}
\]

For China we expect the rural transformation is occurring with labour shifting from non-tradables to tradables sector, which is the situation described by equation \((37a)\).

**Terms of Trade**

For China the terms of trade are exogenous. According to equation \((10)\), the terms of trade influence the real exchange rate directly and through its effects on \(R_n\). The direct effect is always positive. Now we are going to analyse the indirect effect.

In the medium-run, increase in terms of trade imply an increase in the relative price of non-tradables to imports, \(TR_n = p_n \div p_i\). The non-tradables become relative expensive compared with imports. The consumption demand for non-tradables and investment demand using non-tradables decrease. On the other hand, increase in the terms of
trade will decrease the user cost of capital and stimulate investment demand in the non-tradables component. The total direct effect of higher terms of trade on the demand for non-tradables is ambiguous. As we assume the consumption effect dominates the investment effect, the total demand for non-tradables will decrease and the demand curve will shift from $D_{n0}$ to $D_{n1}$. The relative price of non-tradables will decrease to point $B: \frac{\partial R_n}{\partial T} < 0$. However, this indirect effect is rather small compared with the direct effect of the terms of trade on the real exchange rate (see equation (10)). Therefore, we expect higher terms of trade to cause appreciation of the real exchange rate in medium-run equilibrium.

In the long-run, increase in the terms of trade will increase capital and foreign assets. Improvements in the terms of trade increase the current account due to the price effect: domestic exports are sold to the world market at a relative higher price and goods are imported from the world market at a relative lower price. Lower user cost of capital stimulates capital formation. The higher investment may exceed the savings and generates current account deficit. Therefore, under the capital formation, there might be current account surplus ($E3$) or deficit ($E2$). In the first case, the trajectory is $E3 - E$, while in the second case the trajectory is $E2 - N - E$. In both cases, the capital formation generates higher capital and higher foreign assets: $d k^* / dT > 0$ and $d F^* / dT > 0$.

Since the non-tradable sector is labour intensive, an increase in capital reduces supply of non-tradables from $S_{n0}$ to $S_{n1}$ and increases relative price of non-tradables: $(\partial R_n / \partial k) dk^* / dT > 0$. Furthermore, the increase in wealth due to higher foreign assets raises demand for non-tradables, say, from $D_{n1}$ to $D_{n0}$ and therefore raises the relative
price of non-tradables: \((\partial R_n/\partial F) dF^*/dT > 0\). Eventually the relative price of non-tradables will rise from point \(A\) to point \(G\).

\[
\frac{dR^*_n}{dT} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{dT} + \left(\frac{\partial R_n}{\partial F}\right)\frac{dF^*}{dT} + \frac{\partial R_n}{\partial T} > 0
\]  

(38)

According to equation (10), higher terms of trade have a practically one to one positive direct effect on the real exchange rate. On the other hand, higher terms of trade first depreciate the relative price of non-tradables in the medium-run and appreciate it in the steady state. \(R^*_n\) increases from point \(A\) to \(G\) and further appreciates the real exchange rate, \(R^*_n\), — this is the indirect effect of terms of trade on the real exchange rate via \(R^*_n\), which reinforces the positive direct effect. Therefore, higher term of trade will appreciate the real exchange rate, both in the medium-run and the long-run.

*Foreign Real Interest Rate*

An increase in the world’s real interest rate \(r^*\) raises the user cost of capital and hence reduces demand for investment. Since some of the investment uses non-tradables, the demand for non-tradables declines, which shifts the demand curve of non-tradables from \(D_{n0}\) to \(D_{n1}\). The relative price of non-tradables depreciates from \(A\) to \(B\) : \(\partial R_n/\partial r^* < 0\).

As the domestic economy is a net creditor, an increase in \(r^*\) increases interest income from foreign countries and produces current account surplus. A lower demand for investment also helps to generate the current account surplus. In the long-run, output declines gradually due to lower capital. If the extra interest income is insufficient to compensate the decline in output, the current account will turn from surplus to deficit.
in the long-run. There will be a decline in both capital and net foreign assets. Such a trajectory can be described by $E4 - M - E$, where current account turns to deficit at point $M$. If the extra interest income is greater than the decline in output, there will be a continuous current account surplus and thus higher net foreign assets, even though capital is lower. Higher foreign assets lower country risk and encourage FDI. If FDI inflows are insufficient, there will be a lower capital eventually. Such a trajectory can be described by $E4' - E$. If FDI inflows are sufficient to offset the decline of capital due to higher interest rate, there will be a higher capital. The trajectory is described by $E5 - X - E$, where FDI inflows offset the decline of capital at point $X$.

Along $E4 - M - E$, a lower capital raises supply of non-tradables and reduces its relative price: $(\partial R_n/\partial k)dk^*/dr' < 0$. Lower net foreign assets reduce wealth and therefore reduces demand for non-tradables: $(\partial R_n/\partial F)dF^*/dr' < 0$. The total effect on relative price of non-tradables is depreciation.

Along $E4' - E$, the decline in capital increases the supply of non-tradables and depreciates the relative price of non-tradables: $(\partial R_n/\partial k)dk^*/dr' < 0$. Demand for non-tradables increases due to a higher wealth, which appreciates the relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dr' > 0$. The total effect is ambiguous.

Along $E5 - X - E$, since the non-tradable sector is labour intensive a higher capital decreases supply of non-tradables from $S_{n0}$ to $S_{n1}$ and raises its relative price: $(\partial R_n/\partial k)dk^*/dr' > 0$. Higher net foreign assets appreciate the relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dr' > 0$. The total effect on relative price of non-tradables is appreciation.
Relative Unit Labour Cost of China

Most of FDI to China flows to tradable sector due to government’s policy of encouraging export oriented industry and relative cheap labour supported by enormous labour supply. Since the terms of trade are exogenous for China, an increase in the unit labour cost of China relative to its competitors (RULC) makes it less profitable to sell tradables at the exogenous world prices and attracts less FDI. The relative price of non-tradables is left unaffected. Therefore, there is no direct effect of a higher relative unit labour cost on the relative price of non-tradables in the medium-run: \( \frac{\partial R_n}{\partial RULC} = 0 \)

In the long-run, a higher RULC decreases capital and foreign assets. Initially, investment falls below savings and generates current account surplus. Hence there is capital decumulation and current account surplus (E4). The output gradually declines and so do the savings. At point M savings are equivalent to investment. Along trajectory M – E , as output continues to decline, savings fall below investment and there is current account deficit. Hence the economy is stabilized at point E with lower capital and foreign assets: \( dk^*/dRULC < 0 \) and \( dF^*/dRULC < 0 \).

Lower output of tradables reduces foreign assets and wealth, which reduces demand for non-tradables from \( D_{n0} \) to \( D_{n1} \): \( (\partial R_n/\partial F) dF^*/dRULC < 0 \). Since the capital in
non-tradable sector remains unchanged as the destination of FDI is the tradables sector, the supply of non-tradables is not altered: \((\partial R_n/\partial k)dk^*/dRULC = 0\). Therefore a higher \(RULC\) depreciates the relative price on non-tradables in long-run equilibrium.

\[
\frac{dR_n^*}{dRULC} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{dRULC} + \left(\frac{\partial R_n}{\partial F}\right)\frac{dF^*}{dRULC} + \frac{\partial R_n}{\partial RULC} < 0 \quad (40)
\]

Relative Rate of Return to Capital of China

An increase in the relative rate of return to capital in China makes China’s market more attractive to FDI and generates capital inflows. As we assume the destination of FDI is the tradables sector, capital inflows occur in the tradables sector. Since FDI is imported investment, the demand for non-tradables investment is not affected, nor does the supply of non-tradables. Therefore, an increase in \(RRC\) does not have direct effects on the relative price of non-tradables in the medium-run: \(\partial R_n/\partial RRC = 0\)

In the long-run, a higher \(RRC\) increases capital and foreign assets. Originally, capital inflows in tradables sector raise investment relative to savings and generate current account deficit, as described by point E2. However, in the long-run, the output of tradables increases gradually due to higher capital in tradables sector, and so do savings. At point \(N\), savings equal investment. Along \(N-E\) savings exceed investment and there is current account surplus. Thus, the economy stabilizes with higher capital and foreign assets: \(dk^*/dRRC > 0\) and \(dF^*/dRRC > 0\).

A higher capital in tradables does not affect the supply of non-tradables sector. Thus it does not affect relative price of non-tradables: \((\partial R_n/\partial k)dk^*/dRRC = 0\). As foreign assets increase, wealth increases. Consequently consumption of non-tradables
increases from $D_{n0}$ to $D_{n2}$, which appreciates relative price of non-tradables: $(\partial R_n/\partial F)dF^*/dRRC > 0$. Therefore, a higher relative rate of return to capital appreciates the relative price of tradables in long-run equilibrium.

$$\frac{dR_n^*}{dRRC} = \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dRRC} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dRRC} + \frac{\partial R_n}{\partial RRC} > 0 \quad (41)$$

**Government Investment**

A higher $GI$ raises demand for investment using non-tradables and appreciates the relative price of non-tradables: $\partial R_n/\partial GI > 0$. The demand curve shifts from $D_{n0}$ to $D_{n2}$. There are capital formation and current account deficit (see point E2).

In the long-run, a higher $GI$ increases capital and foreign assets. After the government investment is put into place, output starts to increase and so do savings. At point $N$ savings equal to investment; after point $N$ savings exceed investment and there is current account surplus. Therefore, there are higher capital and foreign assets: $\partial k/\partial GI > 0$ and $\partial F/\partial GI > 0$. The trajectory is described by $E2 - N - E$.

Along the trajectory $E2 - N - E$, higher capital tends to reduce supply of non-tradables and appreciates the price of it: $(\partial R_n/\partial k)dk^*/dGI > 0$. The supply curve shifts from $S_{n0}$ to $S_{n1}$. Higher foreign assets raise wealth and increase consumption of non-tradables: $(\partial R_n/\partial F)dF^*/dGI > 0$. The demand curve shifts from $D_{n2}$ to $D_{n4}$ and there is a long-run steady appreciation of the relative price of non-tradables.

$$\frac{dR_n^*}{dGI} = \left( \frac{\partial R_n}{\partial k} \right) \frac{dk^*}{dGI} + \left( \frac{\partial R_n}{\partial F} \right) \frac{dF^*}{dGI} + \frac{\partial R_n}{\partial GI} > 0 \quad (42)$$
However, if GI crowds out domestic private investment and given that it has lower efficiency as its main purpose is to sustain SOEs and public services, the output of GI may not be sufficient to turn current account from deficit to surplus. Thus there will be decline in foreign assets in the long-run. Lower foreign assets imply higher country risk premium and higher user cost of capital both discourage investment. Therefore, capital declines in the long-run. Under such a scenario, a higher GI will depreciate the relative price of non-tradables in the long-run.

**Taxation**

A higher taxation ($\tau$) increases user cost of capital and discourages investment. A lower demand for investment decreases investment using non-tradables and shifts demand curve from $D_{n0}$ to $D_{n1}$. The direct effect of an increase in $\tau$ depreciates relative price of non-tradables: $dR_n/d\tau<0$.

In the long-run, a higher $\tau$ reduces capital and foreign assets: $dk^*/d\tau<0$, $dF^*/d\tau<0$. Originally, a lower investment generates current account surplus, as described by point $E4$. However, output gradually declines so do savings. At point $M$, savings equal investment and current account is in balance. Along $M-E$, savings are lower than investment and there is current account deficit.

A lower capital raises output of non-tradables and depreciates relative price of non-tradables: $(\partial R_n/\partial k)dk^*/d\tau<0$. The supply curve shifts from $S_{n0}$ to $S_{n2}$. Lower foreign assets reduce wealth and the consumption declines. Demand curve shifts from $D_{n1}$ to $D_{n3}$ and depreciates relative price of non-tradables: $(\partial R_n/\partial F)dF^*/\tau<0$.

$$\frac{dR_n^*}{d\tau} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{d\tau} + \left(\frac{\partial R_n}{\partial F}\right)\frac{dF^*}{d\tau} + \frac{\partial R_n}{\partial \tau} < 0$$  \hspace{1cm} (43)
5. Conclusions

In this paper we extend Stein’s (1995a) NATREX model to China. This is a dynamic model which investigates the determinants of the real exchange rate and of the relative price of non-tradables in the medium-run and the long-run, when short-run shocks are stripped out. The two state variables are capital and net foreign assets. In the medium-run, these two state variables are predetermined. Changes in fundamentals and levels of capital and net foreign assets determine the medium-run equilibrium real exchange rate and relative price of non-tradables. In the long-run, capital and net foreign assets converge to the new steady states delivered by changes in the fundamentals. Hence the long-run equilibrium real exchange rate and the relative price of non-tradables are entirely determined by the fundamentals. The dynamic stability of the model requires (a) the impact of capital stock on investment to be greater than the impact of net foreign assets on investment when investment is zero and (b) the impact of net foreign assets on current account to be greater than the impact of capital on current account when current account is in balance.

The fundamentals that affect the long-run equilibrium value of the real exchange rate include terms of trade, total and net factor productivity, rural transformation, dependency ratio, financial liberalization, relative unit labour cost, relative rate of return to capital, government investment, tax rate and the world’s real interest rate. According to the model, higher terms of trade, total and net factor productivity (in the tradables sector), rural transformation, relative rate of return to capital and government investment appreciate the equilibrium real exchange rate in the long-run. On the other hand, higher relative unit labour cost, dependency ratio, financial liberalization and tax rate depreciate the long-run equilibrium real exchange rate.
We make a number of contributions to the literature. Instead of using net foreign debt (Stein, 1995a), we use net foreign assets as one of the state variables since we regard China as a net creditor. We incorporate fundamentals that reflect the unique characteristics of the Chinese economy but have not been studied by the existing literature into the NATREX framework, i.e. relative unit labour cost, relative return to capital, rural transformation, demographic factors, liquidity constraints. Instead of using GDP growth rate and other approximations of productivity as in most of the NATREX applications, we employ total and net factor productivity as an important determinant of the long-run equilibrium real exchange rate. In particular, we introduce rural transformation in the production function, which has rarely been implemented in existing studies of China’s production function. Time preference, an exogenous variable in Stein’s (1995a) original NATREX model, is endogenized as a function of dependency ratio and financial liberalization to reflect the unique consumption pattern in China. Aggregate investment is decomposed into three components, private domestic investment, government investment and foreign direct investment, and each component is modelled individually. Private domestic investment is modelled on the neoclassical model and government investment is regarded as exogenous (capturing aspects of the planned economy). In particular, foreign direct investment is determined by fundamentals such as relative unit labour cost and relative rate of return to capital which have played a crucial role in the Chinese economy. Finally, we provide a detailed mathematical and economic analysis of the predictions of the model in both the medium-run and the long-run.
Appendix A. Medium-Run Equilibrium

In the medium-run, capital stock and foreign assets are predetermined. In other words, $k$ and $F$ do not alter in the medium-run. The equilibrium of the goods market is described by equation (17):

$$C_n(R_n, k, F; DEP, CREP, T) + I_n(R_n, k, F; NFP, RT, r', \tau, GI, T) - y_n(R_n, k; NFP, RT) = 0$$

(17)

where

$$\frac{\partial C_n}{\partial R_n} < 0, \quad \frac{\partial C_n}{\partial DEP} > 0, \quad \frac{\partial C_n}{\partial CREP} > 0 \quad (\frac{\partial I_n}{\partial g} > 0, \quad \frac{\partial I_n}{\partial T} < 0, \quad \frac{\partial I_n}{\partial NFP} > 0$$

$$\frac{\partial I_n}{\partial GI} > 0, \quad \frac{\partial I_n}{\partial T} < 0, \quad \frac{\partial I_n}{\partial r'} < 0, \quad \frac{\partial I_n}{\partial \tau} < 0$$

Total differentiate $C_n$, $I_n$ and $y_n$ separately:

$$dC_n = \frac{\partial C_n}{\partial R_n} dR_n + \frac{\partial C_n}{\partial DEP} dDEP + \frac{\partial C_n}{\partial CREP} dCREP + \frac{\partial C_n}{\partial T} dT$$

$$dI_n = \frac{\partial I_n}{\partial R_n} dR_n + \frac{\partial I_n}{\partial NFP} dNFP + \frac{\partial I_n}{\partial GI} dGI + \frac{\partial I_n}{\partial T} dT$$

$$dy_n = \frac{\partial y_n}{\partial R_n} dR_n + \frac{\partial y_n}{\partial NFP} dNFP + \frac{\partial y_n}{\partial GI} dGI + \frac{\partial y_n}{\partial T} dT$$

where $Z = [DEP, CREP, NFP, RT, r', \tau, GI, T]$

(44)

Therefore, total differentiation of equation (17) can be rewritten as:

$$dC_n + dI_n - dy_n = 0$$

(47)

$$\Rightarrow \left( \frac{\partial C_n}{\partial R_n} dR_n + \frac{\partial C_n}{\partial Z} dZ \right) + \left( \frac{\partial I_n}{\partial R_n} dR_n + \frac{\partial I_n}{\partial Z} dZ \right) - \left( \frac{\partial y_n}{\partial R_n} dR_n + \frac{\partial y_n}{\partial Z} dZ \right) = 0$$

(48)

$$\Rightarrow dR_n = \frac{\frac{\partial C_n}{\partial Z} dZ + \frac{\partial I_n}{\partial Z} dZ - \frac{\partial y_n}{\partial Z} dZ}{M} \quad \text{where} \quad M = \left( \frac{\partial C_n}{\partial R_n} + \frac{\partial I_n}{\partial R_n} - \frac{\partial y_n}{\partial R_n} \right) > 0$$

(49)

Now we are going to analyse the sign of $dR_n/dZ$ when there is a change in $Z$:
\[
\frac{dR_n}{d\text{DEP}} = \left( \frac{\partial C_n}{\partial \text{DEP}} \right)_M > 0; \quad \frac{dR_n}{d\text{CREP}} = \left( \frac{\partial C_n}{\partial \text{CREP}} \right)_M > 0; \quad \frac{dR_n}{dg} = \left( \frac{\partial C_n}{\partial g} \right)_M > 0;
\]

\[
\frac{dR_n}{d\text{NFP}} = \left( \frac{\partial I_n - \partial y_n}{\partial \text{NFP}} \right)_M < 0; \quad \frac{dR_n}{d\text{RT}} = \left( \frac{\partial I_n - \partial y_n}{\partial \text{RT}} \right)_M > 0; \quad \frac{dR_n}{d\tau} = \left( \frac{\partial I_n}{\partial \tau} \right)_M < 0;
\]

\[
\frac{dR_n}{d\text{GI}} = \left( \frac{\partial I_n}{\partial \text{GI}} \right)_M > 0; \quad \frac{dR_n}{dT} = \left( \frac{\partial I_n}{\partial T} + \frac{\partial C_n}{\partial T} \right)_M < 0.
\]
Appendix B. Long-Run Equilibrium

In the long-run, the dynamic system concerns the evolution of the capital stock and real foreign assets, which is described by equations (23) and (25).

\[
\frac{dk}{dt} = J(k, F; Z) \quad (23)
\]

\[
\frac{dF}{dt} = S(k, F; Z) - J(k, F; Z) = L(k, F; Z) \quad (25)
\]

The steady state is described by equation (4.26) and (4.27):

\[
J(k^*, F^*; Z) = 0 \quad (26)
\]

\[
L(k^*, F^*; Z) = S(k^*, F^*; Z) - J(k^*, F^*; Z) = 0 \quad (27)
\]

Now we are going to solve the steady state equation. Total differentiate equations (26) and (27) and solve for \( \frac{dk^*}{dZ} \) and \( \frac{dF^*}{dZ} \):

\[
dJ = \frac{\partial J}{\partial k} dk^* + \frac{\partial J}{\partial F} dF^* + \frac{\partial J}{\partial Z} dZ = 0 \quad (50)
\]

\[
dL = \left( \frac{\partial S}{\partial k} - \frac{\partial J}{\partial k} \right) dk^* + \left( \frac{\partial S}{\partial F} - \frac{\partial J}{\partial F} \right) dF^* + \left( \frac{\partial S}{\partial Z} - \frac{\partial J}{\partial Z} \right) dZ = 0 \quad (51)
\]

\[
\begin{bmatrix}
\frac{\partial J}{\partial k} & \frac{\partial J}{\partial F} \\
\frac{\partial S}{\partial k} & \frac{\partial S}{\partial F}
\end{bmatrix}
\begin{bmatrix}
\frac{dk^*}{dZ} \\
\frac{dF^*}{dZ}
\end{bmatrix}
= \begin{bmatrix}
-\frac{\partial J}{\partial Z} \\
-\left( \frac{\partial S}{\partial Z} - \frac{\partial J}{\partial Z} \right)
\end{bmatrix}
dZ
\]

\[
\Rightarrow \begin{bmatrix}
\frac{dk^*}{dF^*}
\end{bmatrix} = \frac{1}{G} \begin{bmatrix}
\frac{\partial S}{\partial F} & \frac{\partial J}{\partial F} \\
\frac{\partial J}{\partial k} & \frac{\partial S}{\partial k} & \frac{\partial J}{\partial k} & \frac{\partial J}{\partial Z} & \left( \frac{\partial S}{\partial Z} - \frac{\partial J}{\partial Z} \right)
\end{bmatrix}
dZ \quad (52)
\]

\[
\Rightarrow \frac{dk^*}{dZ} = \frac{\left( \frac{\partial S}{\partial F} - \frac{\partial J}{\partial F} \right) \frac{\partial J}{\partial Z} + \frac{\partial J}{\partial k} \left( \frac{\partial S}{\partial k} - \frac{\partial J}{\partial k} \right) - \frac{\partial J}{\partial Z} \left( \frac{\partial S}{\partial Z} - \frac{\partial J}{\partial Z} \right)}{G} = \frac{\partial J \frac{\partial S}{\partial F} - \frac{\partial J}{\partial F} \frac{\partial S}{\partial k}}{G} \quad (53)
\]

\[
\Rightarrow \frac{dF^*}{dZ} = \frac{\left( \frac{\partial S}{\partial F} - \frac{\partial J}{\partial F} \right) \frac{\partial J}{\partial Z} + \frac{\partial J}{\partial k} \left( \frac{\partial S}{\partial k} - \frac{\partial J}{\partial k} \right) - \frac{\partial J}{\partial Z} \left( \frac{\partial S}{\partial Z} - \frac{\partial J}{\partial Z} \right)}{G} = \frac{\partial J \frac{\partial S}{\partial k} - \frac{\partial J}{\partial k} \frac{\partial S}{\partial Z}}{G} \quad (54)
\]
where

\[
G = \begin{bmatrix}
\frac{\partial J}{\partial k} & \frac{\partial J}{\partial F} & \frac{\partial J}{\partial k} & \frac{\partial J}{\partial F} \\
\frac{\partial S}{\partial k} & \frac{\partial S}{\partial F} & \frac{\partial S}{\partial k} & \frac{\partial S}{\partial F}
\end{bmatrix} = \frac{\partial J}{\partial k} \left( \frac{\partial S}{\partial F} - \frac{\partial J}{\partial F} \right) - \frac{\partial J}{\partial F} \left( \frac{\partial S}{\partial k} - \frac{\partial J}{\partial k} \right) = \frac{\partial J}{\partial k} \frac{\partial L}{\partial F} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial F} > 0
\]

Equations (53) and (54) represent the effect of changes in the fundamentals on capital and foreign assets in the steady state. Now we are going to determine the signs of equations (53) and (54) given \( \frac{\partial S}{\partial k} > 0, \frac{\partial S}{\partial F} < 0, \frac{\partial J}{\partial k} < 0, \frac{\partial J}{\partial F} > 0, \frac{\partial L}{\partial k} > 0, \frac{\partial L}{\partial F} < 0 \) and \( G > 0 \).

The signs of \( \frac{dk^*}{dZ} \) and \( \frac{dF^*}{dZ} \) are determined as follows:

\[
\frac{dk^*}{dNFP} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial NFP} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial NFP}}{G} > 0; \quad \frac{dF^*}{dNFP} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial NFP} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial NFP}}{G} > 0;
\]

\[
\frac{dk^*}{dNFP_n} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial NFP_n} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial NFP_n}}{G} > 0; \quad \frac{dF^*}{dNFP_n} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial NFP_n} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial NFP_n}}{G} > 0;
\]

\[
\frac{dk^*}{dNFP_i} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial NFP_i} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial NFP_i}}{G} > 0; \quad \frac{dF^*}{dNFP_i} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial NFP_i} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial NFP_i}}{G} > 0;
\]

\[
\frac{dk^*}{dRT} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial RT} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial RT}}{G} > 0; \quad \frac{dF^*}{dRT} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial RT} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial RT}}{G} > 0;
\]

\[
\frac{dk^*}{dRT_n} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial RT_n} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial RT_n}}{G} > 0; \quad \frac{dF^*}{dRT_n} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial RT_n} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial RT_n}}{G} > 0;
\]

\[
\frac{dk^*}{dRT_i} = \frac{\frac{\partial J}{\partial F} \frac{\partial k}{\partial RT_i} - \frac{\partial S}{\partial F} \frac{\partial J}{\partial RT_i}}{G} > 0; \quad \frac{dF^*}{dRT_i} = \frac{\frac{\partial S}{\partial k} \frac{\partial J}{\partial RT_i} - \frac{\partial L}{\partial k} \frac{\partial J}{\partial RT_i}}{G} > 0;
\]
\[
\frac{d k^*}{d \text{CREP}} = \frac{\partial J \partial S - \partial S \partial J}{G} < 0; \quad \frac{d F^*}{d \text{CREP}} = \frac{\partial S \partial J - \partial J \partial S}{G} < 0;
\]

\[
\frac{d k^*}{d \text{DEP}} = \frac{\partial J \partial S - \partial S \partial J}{G} < 0; \quad \frac{d F^*}{d \text{DEP}} = \frac{\partial S \partial J - \partial J \partial S}{G} < 0;
\]

\[
\left(\frac{d k^*}{d g}\right) = \frac{\partial J \partial S - \partial S \partial J}{G} < 0; \quad \frac{d F^*}{d g} = \frac{\partial S \partial J - \partial J \partial S}{G} < 0;
\]

\[
\frac{d k^*}{d \text{RULC}} = \frac{\partial J \partial S - \partial S \partial J}{G} < 0; \quad \frac{d F^*}{d \text{RULC}} = \frac{\partial S \partial J - \partial J \partial S}{G} < 0;
\]

\[
\frac{d k^*}{d \text{RRC}} = \frac{\partial J \partial S - \partial S \partial J}{G} > 0; \quad \frac{d F^*}{d \text{RRC}} = \frac{\partial S \partial J - \partial J \partial S}{G} > 0;
\]

\[
\frac{d k^*}{d \text{T}} = \frac{\partial J \partial S - \partial S \partial J}{G} > 0; \quad \frac{d F^*}{d \text{T}} = \frac{\partial S \partial J - \partial J \partial S}{G} > 0;
\]

\[
\frac{d k^*}{d \text{GI}} = \frac{\partial J \partial S - \partial S \partial J}{G} > 0; \quad \frac{d F^*}{d \text{GI}} = \frac{\partial S \partial J - \partial J \partial S}{G} > 0;
\]

\[
\frac{d k^*}{d r'} = \frac{\partial J \partial S - \partial S \partial J}{G} > 0; \quad \frac{d F^*}{d r'} = \frac{\partial S \partial J - \partial J \partial S}{G} > 0;
\]

\[
\frac{d k^*}{d \tau} = \frac{\partial J \partial S - \partial S \partial J}{G} < 0; \quad \frac{d F^*}{d \tau} = \frac{\partial S \partial J - \partial J \partial S}{G} < 0.
\]
Appendix C. The Long-Run Equilibrium Exchange Rate and Relative Price of Non-Tradables

In Appendix 4.B we derived the direct effect of fundamentals $Z$ on $R_n$ in the medium-run when $k$ and $F$ are predetermined. In this section we will derive the total effect of of fundamentals $Z$ on $R_n$ in the long-run, when $k$ and $F$ converge to their steady states. Recall the non-tradables market equilibrium (equation 17):

$$C_n(R_n,k,F;DEP,CREP,T)+I_n(R_n,k,F;NFP,RT,r',	au,GI,T)−y_n(R_n,k;NFP,RT)=0$$

(17)

The total effect of fundamentals $Z$ on $R_n$ in the medium-run is derived as in Appendix 4.B but now $k$ and $F$ change. Total differentiate the goods market equilibrium function we can get:

$$\left(\frac{\partial C_n}{\partial R_n}dR_n^* + \frac{\partial C_n}{\partial k}dk^* + \frac{\partial C_n}{\partial F}dF^* + \frac{\partial C_n}{\partial Z}dZ\right) + \left(\frac{\partial I_n}{\partial R_n}dR_n^* + \frac{\partial I_n}{\partial k}dk^* + \frac{\partial I_n}{\partial F}dF^* + \frac{\partial I_n}{\partial Z}dZ\right)$$

(56)

$$\Rightarrow dR_n^* = \left(\frac{\partial C_n}{\partial k} + \frac{\partial I_n}{\partial k} - \frac{\partial y_n}{\partial k}\right)\frac{dk^*}{M} + \left(\frac{\partial C_n}{\partial F} + \frac{\partial I_n}{\partial F}\right)\frac{dF^*}{M} + \left(\frac{\partial C_n}{\partial Z} + \frac{\partial I_n}{\partial Z} - \frac{\partial y_n}{\partial Z}\right)dZ$$

(57)

$$\Rightarrow \frac{dR_n^*}{dZ} = \left(\frac{\partial C_n}{\partial k} + \frac{\partial I_n}{\partial k} - \frac{\partial y_n}{\partial k}\right)\frac{dk^*}{dZ} + \left(\frac{\partial C_n}{\partial F} + \frac{\partial I_n}{\partial F}\right)dF^* + \left(\frac{\partial C_n}{\partial Z} + \frac{\partial I_n}{\partial Z} - \frac{\partial y_n}{\partial Z}\right)$$

(58)

$$\Rightarrow \frac{dR_n^*}{dZ} = \left(\frac{\partial R_n}{\partial k}\right)\frac{dk^*}{dZ} + \left(\frac{\partial R_n}{\partial F}\right)dF^* + \left(\frac{\partial R_n}{\partial Z}\right)$$

(31b)

Equation (31b) describes the total effect of fundamentals $Z$ on $R_n$ in long-run equilibrium. The last item is the direct effect of fundamentals $Z$ on $R_n$ in the medium-run. The signs of this item have been analyzed in Appendix A. The first two
items in equation (31b) catch the indirect effect of fundamentals $Z$ on $R_n$ in the long-run when $k$ and $F$ reach new steady states. The signs of $\frac{dk^*}{dZ}$ and $\frac{dF^*}{dZ}$ have been analyzed in Appendix 4.C. The signs of the first two items are explained in the main text. The long run equilibrium exchange rate can be written as:

$$R^* = T(R_n^*)^a = R^*(Z)$$ (32)

According to equation (32), the fundamentals which affect the relative price of non-tradables, $R_n^*$, affect the real exchange rate, $R^*$, in a similar way. One exception are the terms of trade which have a direct and indirect (via $R_n^*$) effect on the real exchange rate (these are analyzed in Section 4).
### Appendix D. Equilibrium Effects

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<th>Medium-run</th>
<th>Long-run</th>
<th>Trajectories</th>
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<tr>
<td>Z</td>
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<td>T</td>
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<tr>
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<td>+</td>
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<td>GI</td>
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<td>(net creditor)</td>
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<tr>
<td>τ</td>
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</table>

**Note:** The fundamentals affect the relative price of non-tradables and affect the real exchange rate in a similar way, both in the medium run and long run, and hence are not repeated here. The only exception is the terms of trade. Based on equation (10), the terms of trade affect the real exchange rate directly and indirectly through their effect on $R_u$. As the indirect effect is rather small compared with the direct effect, higher terms of trade appreciate the real exchange rate regardless of whether there is appreciation or depreciation of the relative price of non-tradables. This implies that in the medium run and long run $dR^*/dT$ is always positive.
References


Rossi, N., 1988, “Government Spending, the Real Interest Rate and the Behaviour of Liquidity-Constrained Consumers in Developing Countries”, IMF Staff Paper, 35, 104-140.


Figure 1. Dynamic Adjustment—Case 1: $G > 0$
Figure 2. Dynamic Adjustment—Case 2: $G < 0$
Figure 3. Trajectories of Capital and Foreign Assets to Their Steady States

\[ \frac{\partial Y}{\partial K} > c \]
\[ \frac{\partial Y}{\partial K} < c \]
\[ J = 0 \]
\[ L = 0 \]
Figure 4. (Non-tradable) Goods Market Equilibrium