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Modelling and Trading the Greek Stock Market with Mixed Neural Network Models

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Working Paper Series

No 15/11

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Abstract

In this paper, a mixed methodology that combines both the ARMA and NNR models is proposed to take advantage of the unique strength of ARMA and NNR models in linear and nonlinear modelling. Experimental results with real data sets indicate that the combined model can be an effective way to improve forecasting accuracy achieved by either of the models used separately. The motivation for this paper is to investigate the use of alternative novel neural network architectures when applied to the task of forecasting and trading the ASE 20 Greek Index using only autoregressive terms as inputs. This is done by benchmarking the forecasting performance of six different neural network designs representing a Higher Order Neural Network (HONN), a Recurrent Network (RNN), a classic Multilayer Percepton (MLP), a Mixed Higher Order Neural Network, a Mixed Recurrent Neural Network and a Mixed Multilayer Percepton Neural Network with some traditional techniques, either statistical such as a an autoregressive moving average model (ARMA), or technical such as a moving average convergence/divergence model (MACD), plus a naïve trading strategy. More specifically, the trading performance of all models is investigated in a forecast and trading simulation on ASE 20 fixing time series over the period 2001-2008 using the last one and a half year for out-of-sample testing. We use the ASE 20 daily fixing as many financial institutions are ready to trade at this level and it is therefore possible to leave orders with a bank for business to be transacted on that basis.

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1. INTRODUCTION

The use of intelligent systems for market predictions has been widely established. This paper deals with the application of mixed computing techniques for forecasting the Greek stock market. The development of accurate techniques is critical to economists, investors and analysts. This task is getting more and more complex as financial markets are getting increasingly interconnected and interdependent. The traditional statistical methods, on which forecasters were reliant in recent years, seem to fail to capture the interrelationship between market variables. This paper investigates methods capable of identifying and capturing all the discontinuities, the nonlinearities and the high frequency multipolynomial components characterizing the financial series today. A model category that promises such effective results is the combination of autoregressive models such as ARMA model with Neural Networks named Mixed-Neural Network model. Many researchers have argued that combining several models for forecasting gives better estimates by taking advantage of each model's capabilities when comparing them with single time series models.

The motivation for this paper is to investigate the use of several new neural networks techniques combined with ARMA model in order to overcome these limitations using autoregressive terms as inputs. This is done by benchmarking six different neural network architectures representing a Multilayer Percepton (MLP), a Higher Order Neural Network (HONN), a Recurrent Neural Network (RNN), a Mixed Higher order Neural Network, a Mixed Recurrent Neural Network and a Mixed Multilayer Percepton Neural Network Their trading performance on the ASE 20 time series is investigated and is compared with some traditional statistical or technical methods such as an autoregressive moving average (ARMA) model or a moving average convergence/divergence (MACD) model, and a naïve trading strategy.

As it turns out, the Mixed-HONN demonstrates a remarkable performance and outperforms all other models in a simple trading simulation exercise. On the other hand, when more sophisticated trading strategies using confirmation filters and leverage are applied, Mixed MLPs outperform all models in terms of annualised return. Our conclusion colloborates those of Lindemann *et al.* (2004) and Dunis *et al.* (2008b) where HONNs also demonstrate a forecasting superiority on the EUR/USD series over more traditional techniques such as a MACD and a naïve strategy. However, the RNN which performed remarkably well, show a disappointing performance in this research: this may be due to their inability to provide good enough results when only autoregressive terms are used as inputs.

The rest of the paper is organised as follows. In section 2, we present the literature relevant to the Mixed Neural Networks, the Recurrent Neural Network, the Higher Order

Neural Networks and the Multilayer Percepton. Section 3 describes the dataset used for this research and its characteristics. An overview of the different neural network models and statistical techniques is given in section 4. Section 5 gives the empirical results of all the models considered and investigates the possibility of improving their performance with the application of more sophisticated trading strategies. Section 6 provides some concluding remarks.

2. LITERATURE REVIEW

Stock market analysis is an area of financial application. Detecting trends of stock market data is a difficult task as they have complex, nonlinear, dynamic and chaotic behaviour. Time series methods such as ARMA model and autoregressive conditional heteroskedasticity models are not capable of accurately forecasting the time series as they are based on the theory of stationary stochastic processes. Empirical studies prove that artificial Neural Networks models perform better than these time series models. Ghiassi et al. (2005) compare forecasting performance of a dynamic Neural Network with traditional neural networks and ARMA models. Greg and Sarah (1999) use Neural Networks for GDP growth and determined whether the forecasting performance of financial and monetary variables can be improved using Neural Networks. Fatima and Hussain (2008) propose a Hybrid financial system that in terms of forecasting behaves better compared to standard models.

The motivation for this paper is to apply some of the most promising new Neural Networks architectures combining them with autoregressive models (in our case ARMA model) which have been developed recently with the purpose to overcome the numerous limitations of the more classic neural architectures and to assess whether they can achieve a higher performance in a trading simulation using only autoregressive series as inputs.

Combining different models can increase the chance to capture different patterns in the data and improve forecasting performance. Several empirical studies have already suggested that by combining several different models, forecasting accuracy can often be improved over an individual model. Using hybrid models or combining several models has become a common practice to improve the forecasting accuracy since the well-known M-competition (Makridakis *et al.*(1982)) in which combinations of forecasts from more than one model often led to improved forecasting performance. The basic idea of the model combination in forecasting is to use each model's unique feature to capture different patterns in the data. Both theoretical and empirical findings suggest that combining different methods can be an effective and efficient way to improve forecasts

(Makridakis (1989), Newbold *et al*. (1974) Palm *et al*. (1992), Winkler (1989)). Research in time series forecasting argues that predictive performance improves the combined models. (Bishop (1994), Clemen (1989), Hansen *et al*. (2003), Hibbert *et al*. (2000), Terui *et al*. (2002), Tseng *et al*. (2002), Zhang, (2003), Zhang *et al*. (2005)).

The reason for combining models comes from the assumption that either one cannot identify the true data generating process (Terui and Von Dyke. (2002)) or that a single model may not be sufficient to identify all the characteristics of the time series (Zhang (2003)). Moreover the use of hybrid neural network has not been used until the moment that scientists started to investigate not only the benefits of Hybrid Neural Networks against other statistical methods but also the differences between different combinations of Hybrid Neural Networks with other statistical models following the Hybrid GARCH-NN approach Wang (2007) and the Hybrid ARIMA/ ARCH-NN of Fatima and Hussain (2008). Abraham *et al.* (2002) analysed the 24-month stock data for NASDAQ-100 main indices. Their hybrid system is Neuro-Fuzzy, a combination of neural network and fuzzy logic system. Lastly Andreou *et al.* (2006) propose knowledge-oriented neural network models combining nonparametric with parametric models (Black –Scholes) for option price data.

RNNs have an activation feedback which embodies short-term memory allowing them to learn extremely complex temporal patterns. Their superiority against feedfoward networks when performing nonlinear time series prediction is well documented in Connor *et al.* (1993) and Adam *et al.* (1994). In financial applications, Kamijo *et al*. (1990) applied them successfully to the recognition of stock patterns of the Tokyo stock exchange while Tenti (1996) achieved remarkable results using RNNs to forecast the exchange rate of the Deutsche Mark. Tino *et al.* (2001) use them to trade successfully the volatility of the DAX and the FTSE 100 using straddles while Dunis and Huang (2002), using continuous implied volatility data from the currency options market, obtain remarkable results for their GBP/USD and USD/JPY exchange rate volatility trading simulation.

HONNs were first introduced by introduced by Giles and Maxwell (1987) as a fast learning network with increased learning capabilities. Although their function approximation superiority over the more traditional architectures is well documented in the literature (see among others Redding *et al*. (1993), Kosmatopoulos *et al*. (1995) and Psaltis *et al*. (1998)), their use in finance so far has been limited. This has changed when scientists started to investigate not only the benefits of Neural Networks (NNs) against the more traditional statistical techniques but also the differences between the different NNs model architectures. Practical applications have now verified the theoretical advantages of HONNs by demonstrating their superior forecasting ability and put them in the front line of research in financial forecasting. For example Dunis *et al.* (2006b) use them to forecast successfully the gasoline crack spread while Fultcher *et al.* (2006) apply HONNs to forecast the AUD/USD exchange rate, achieving a 90% accuracy. However, Dunis *et al.* (2006a) show that, in the case of the futures spreads and for the period under review, the MLPs performed better compared with HONNs and recurrent neural networks. Moreover, Dunis *et al.* (2008a), who also study the EUR/USD series for a period of 10 years, demonstrate that when multivariate series are used as inputs the HONNs, RNN and MLP networks have a similar forecasting power. Finally, Dunis *et al.* (2008b) in a paper with a methodology identical to that used in this research, demonstrate that HONN and the MLP networks are superior in forecasting the EUR/USD ECB fixing until the end of 2007, compared to the RNN networks, an ARMA model, a MACD and a naïve strategy.

3. THE ASE 20 GREEK INDEX AND RELATED FINANCIAL DATA

For Futures on the FTSE/ASE-20 that are traded in derivatives markets the underlying asset is the blue chip index FTSE/ASE-20. The FTSE/ASE-20 index is based on the 20 largest ASE stocks. It was developed in 1997 by the partnership of ASE with FTSE International and is already established benchmark. It represents over 50% of ASE's total capitalisation and currently has a heavier weight on banking, telecommunication and energy stocks.

The futures contract on the index FTSE/ASE-20 is cash settled in the sense that the difference between the traded price of the contract and the closing price of the index on the expiration day of the contract is settled between the counterparties in cash. As a matter of fact, as the price of the contract changes daily, it is cash settled on a daily basis, up until the expiration of the contract.
The futures contract is traded in index points, while the monetary value of the contract is calculated by multiplying the futures price by the multiplier 5 EUR per point. For example, a contract trading at 1,400 points has a value of 7,000 EUR.

The ASE 20 Futures is therefore a tradable level which makes our application more realistic and this is the series that we investigate in this paper[1].

Name of Period	Trading Days	Beginning	End
Total Dataset	2087	21 January 2001	31 December 2008
Training Dataset	1719	29 January 2001	30 August 2007
Out- of- sample Dataset(Validation Set)	349	31 August /2007	31 December 2008

[1] We examine the ASE 20 since its first trading day on 21 January 2001, and until 31 December 2008, using the continuous data available from datastream.

Table 1: The ASE 20 dataset

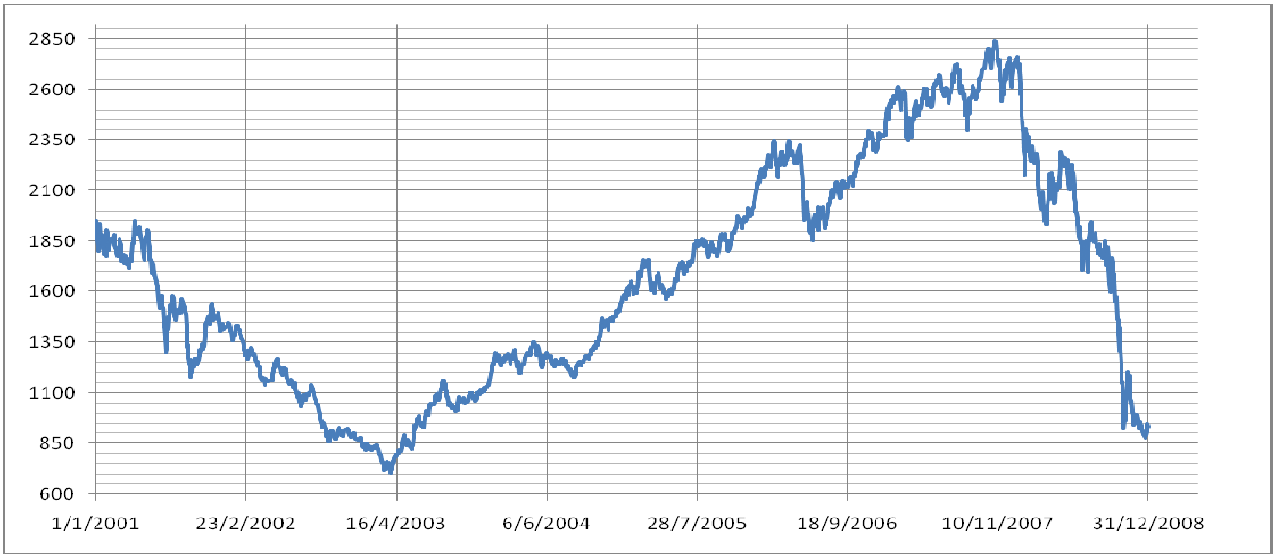


Fig. 1: ASE 20 fixing prices (total dataset).

The observed ASE 20 time series is non-normal (Jarque-Bera statistics confirms this at the 99% confidence interval) containing slight skewness and high kurtosis. It is also non-stationary and we decided to transform the ASE 20 series into stationary series of rates of return[2].

Given the price level P_1, P_2,…,P_t, the rate of return at time t is formed by:

$$R_t = \left(\frac{P_t}{P_{t-1}}\right) - 1 \qquad [1]$$

[2] Confirmation of its stationary property is obtained at the 1% significance level by both the Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) test statistics.

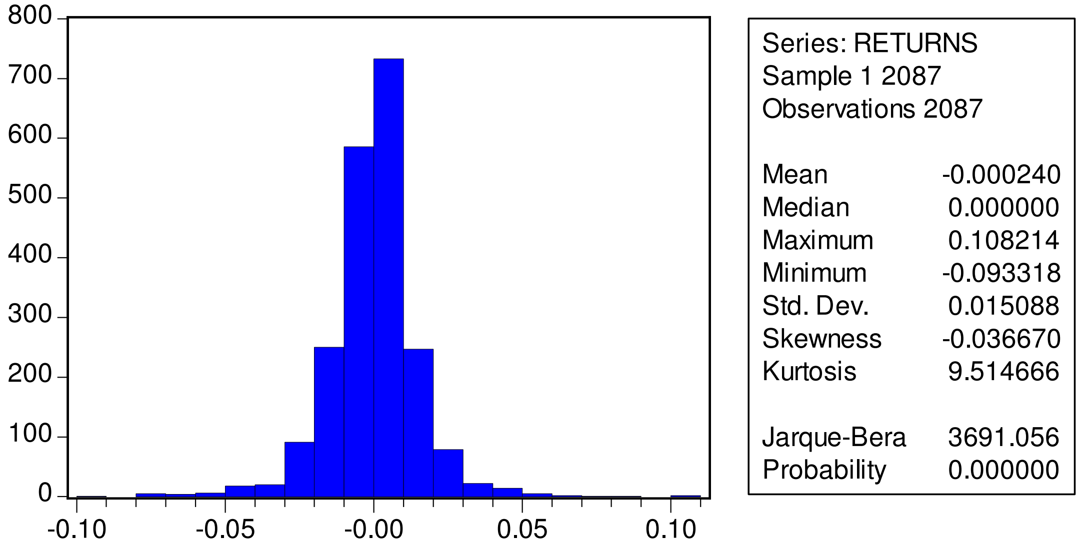


Fig. 2: ASE 20 returns summary statistics (total dataset).

As inputs to our networks and based on the autocorrelation function and some ARMA experiments we selected 2 sets of autoregressive and moving average terms of the ASE 20 returns.

Number	Variable	Lag
1	Athens Composite all share return	1
2	Athens Composite all share return	3
3	Athens Composite all share return	6
4	Athens Composite all share return	8
5	Athens Composite all share return	10
6	Athens Composite all share return	13
7	Athens Composite all share return	14
8	Moving Average of the Athens Composite all share return	15
9	Athens Composite all share return	16

10	Athens Composite all share return	18
11	Moving Average of the Athens Composite all share return	19

Table 2: Explanatory variables for traditional Neural Networks

Number	Variable	Lag
1	Athens Composite all share return	1
2	Athens Composite all share return	2
3	Athens Composite all share return	4
4	Athens Composite all share return	5
5	Athens Composite all share return	7
6	Athens Composite all share return	9
7	Moving Average of the Athens Composite all share return	10
8	Athens Composite all share return	13
9	Athens Composite all share return	14
10	Athens Composite all share return	15
11	Moving Average of the Athens Composite all share return	16
12	Athens Composite all share return	17

Table 3: Explanatory variables for Mixed Neural Networks

In order to train the neural networks we further divided our dataset as follows:

Name of Period	Trading Days	Beginning	End
Total Dataset	2087	21 January 2001	31 December 2008
Training Dataset	1373	29 January 2001	03 May2006
Test Dataset	346	04 May 2006	30 August 2007
Out-of- sample Dataset (Validation Set)	349	31 August 2007	31 December 2008

Table 4: The Neural Networks datasets

4. FORECASTING MODELS

4.1 Benchmark Models

In this paper, we benchmark our neural network models with 3 traditional strategies, namely an autoregressive moving average model (ARMA), a moving average convergence/divergence technical model (MACD) and a naïve strategy.

4.1.1 Naïve strategy

The naïve strategy simply takes the most recent period change as the best prediction of the future change. The model is defined by:

$$\hat{Y}_{t+1} = Y_t \qquad [2]$$

Where Y_t is the actual rate of return at period *t*

$\hat{Y}_{t+1}$ is the forecast rate of return for the next period

The performance of the strategy is evaluated in terms of trading performance via a simulated trading strategy.

4.1.2 Moving Average

The moving average model is defined as:

$$M_t = \frac{\left(Y_t + Y_{t-1} + Y_{t-2} + \ldots + Y_{t-n+1}\right)}{n} \qquad [3]$$

Where M_t is the moving average at time *t*

n is the number of terms in the moving average

Y_t is the actual rate of return at period *t*

The MACD strategy used is quite simple. Two moving average series are created with different moving average lengths. The decision rule for taking positions in the market is straightforward. Positions are taken if the moving averages intersect. If the short-term moving average intersects the long-term moving average from below a 'long' position is taken. Conversely, if the long-term moving average is intersected from above a 'short' position is taken[3].

The forecaster must use judgement when determining the number of periods *n* on which to base the moving averages. The combination that performed best over the in-sample sub-period was retained for out-of-sample evaluation. The model selected was a combination of the ASE 20 and its 7-day moving average, namely *n* = 1 and 7 respectively or a (1, 7) combination. The performance of this strategy is evaluated solely in terms of trading performance.

[3]A 'long' ASE 20 position means buying the index at the current price, while a 'short' position means selling the index at the current price.

4.1.3 ARMA Model

Autoregressive moving average models (ARMA) assume that the value of a time series depends on its previous values (the autoregressive component) and on previous residual values (the moving average component)[4].

The ARMA model takes the form:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t - w_1 \varepsilon_{t-1} - w_2 \varepsilon_{t-2} - ... - w_q \varepsilon_{t-q} \quad [4]$$

where

where	Y_t	is the dependent variable at time *t*
	Y_{t-1}, Y_{t-2}, and Y_{t-p}	are the lagged dependent variable
	ϕ_0, ϕ_1, ϕ_2, and ϕ_p	are regression coefficients
	ε_t	is the residual term
	ε_{t-1}, ε_{t-2}, and ε_{t-p}	are previous values of the residual
	w_1, w_2, and w_q	are weights.

Using as a guide the correlogram in the training and the test sub periods we have chosen a restricted ARMA (7, 7) model. All of its coefficients are significant at the 99% confidence interval. The null hypothesis that all coefficients (except the constant) are not significantly different from zero is rejected at the 99% confidence interval (see Appendix A1).

The selected ARMA model takes the form:

$$Y_t = 2.90 \cdot 10^{-4} + 0.376 Y_{t-1} - 0.245 Y_{t-3} - 0.679 Y_{t-7} + 0.374 \varepsilon_{t-1} - 0.270 \varepsilon_{t-3} - 0.677 \varepsilon_{t-7} \quad [6]$$

The model selected was retained for out-of-sample estimation. The performance of the strategy is evaluated in terms of traditional forecasting accuracy and in terms of trading performance[5].

4.2 Neural Networks and Mixed Neural Networks

Neural networks exist in several forms in the literature. The most popular architecture is the Multi-Layer Percepton (MLP).

[4] For a full discussion on the procedure, refer to Box et al. (1994) or Pindyck and Rubinfeld (1998).
[5] Statistical measures are given in section 4.2.5 below.

A standard neural network has at least three layers. The first layer is called the input layer (the number of its nodes corresponds to the number of explanatory variables). The last layer is called the output layer (the number of its nodes corresponds to the number of response variables). An intermediary layer of nodes, the hidden layer, separates the input from the output layer. Its number of nodes defines the amount of complexity the model is capable of fitting. In addition, the input and hidden layer contain an extra node, called the bias node. This node has a fixed value of one and has the same function as the intercept in traditional regression models. Normally, each node of one layer has connections to all the other nodes of the next layer.

The network processes information as follows: the input nodes contain the value of the explanatory variables. Since each node connection represents a weight factor, the information reaches a single hidden layer node as the weighted sum of its inputs. Each node of the hidden layer passes the information through a nonlinear activation function and passes it on to the output layer if the calculated value is above a threshold.

The training of the network (which is the adjustment of its weights in the way that the network maps the input value of the training data to the corresponding output value) starts with randomly chosen weights and proceeds by applying a learning algorithm called backpropagation of errors[6] (Shapiro (2000)). The learning algorithm simply tries to find those weights which minimize an error function (normally the sum of all squared differences between target and actual values). Since networks with sufficient hidden nodes are able to learn the training data (as well as their outliers and their noise) by heart, it is crucial to stop the training procedure at the right time to prevent overfitting (this is called 'early stopping'). This can be achieved by dividing the dataset into 3 subsets respectively called the training and test sets used for simulating the data currently available to fit and tune the model and the validation set used for simulating future values. The network parameters are then estimated by fitting the training data using the above mentioned iterative procedure (backpropagation of errors). The iteration length is optimised by maximising the forecasting accuracy for the test dataset. Our networks, which are specially designed for financial purposes, will stop training when the profit of our forecasts in the test sub-period is maximized. Then the predictive value of the model is evaluated applying it to the validation dataset (out-of-sample dataset).

There is a range of combination techniques that can be applied to forecasting the attempt to overcome some deficiencies of single models. The combining method aims at reducing the risk of using an inappropriate model by combining several to reduce the

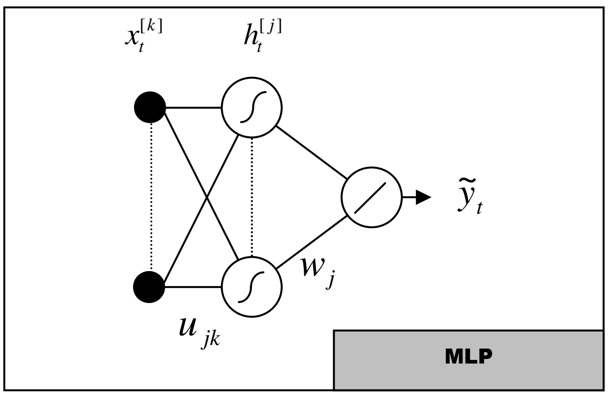
[6]Backpropagation networks are the most common multi-layer networks and are the most commonly used type in financial time series forecasting (Kaastra and Boyd (1996)).

risk of failure. Typically this is done because the underlying process cannot easily be determined (Hibon *et al*. (2005)).

Combining methods involves using several redundant models designed for the same function, where the diversity of the components is to be thought important (Brown *et al.* 2005). The procedure of making a mixed forecasting time series model can be achieved by combining an ARMA process in order to learn the linear component of the conditional mean pattern with an Artificial Neural Network process designed to learn its nonlinear elements. The construction of the Mixed ARMA-Neural Network model is detailed is in figure 6 below.

4.2.1 THE MULTI-LAYER PERCEPTON MODEL ARCHITECTURE

The network architecture of a 'standard' MLP looks as presented in figure 4[7]:



[7] The bias nodes are not shown here for the sake of simplicity.

Fig. 3: A single output, fully connected MLP model

Where:

$x_t^{[n]}\ (n = 1,2,\cdots,k+1)$ are the model inputs (including the input bias node) at time t

$h_t^{[m]}\ (m = 1,2,...,j+1)$ are the hidden nodes outputs (including the hidden bias node)

$\tilde{y}_t$ is the MLP model output

u_{jk} and w_j are the network weights

is the transfer sigmoid function: $S(x) = \dfrac{1}{1+e^{-x}}$, [6]

is a linear function: $F(x) = \sum_i x_i$ [7

The error function to be minimised is:

$E(u_{jk}, w_j) = \dfrac{1}{T}\sum_{t=1}^{T}\left(y_t - \tilde{y}_t(u_{jk}, w_j)\right)^2$, with y_t being the target value

4.2.2 THE RECURRENT NETWORK ARCHITECTURE

Our next model is the recurrent neural network. Whil
models is beyond the scope of this paper, we pres
significant differences between RNN and MLP a
of the recurrent network, see Elman (1990).

A simple recurrent network has activati
memory. The advantages of using recur
modelling non-linear time series, has
described in Tenti (1996) "the m
substantially more connections,
backpropagation networks", thu

time. However having said this RNNs can yield better results in comparison to simple MLPs due to the additional memory inputs.

A simple illustration of the architecture of an Elman RNN is presented below.

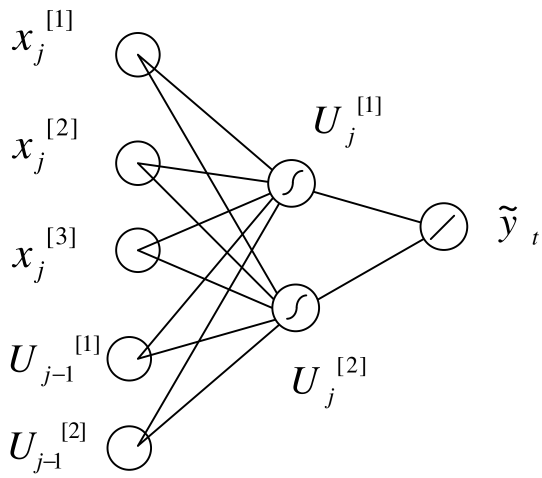


Fig. 4: Elman Recurrent neural network architecture with two nodes on the hidden layer

Where:

$x_t^{[n]}\ (n = 1,2,\cdots,k+1)$, $u_t^{[1]}, u_t^{[2]}$ are the model inputs (including the input bias node) at time t

$\tilde{y}_t$ is the recurrent model output

$d_t^{[f]}\ (f = 1,2)$ and $w_t^{[n]}\ (n = 1,2,\cdots,k+1)$ are the network weights

$U_t^{[f]}\ (f = 1,2)$ is the output of the hidden nodes at time t

$\circ$ is the transfer sigmoid function: $S(x) = \dfrac{1}{1+e^{-x}}$, [9]

$\oslash$ is the linear output function: $F(x) = \sum_i x_i$ [1

The error function to be minimised is:

$$E(d_t, w_t) = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \tilde{y}_t(d_t, w_t)\right)^2 \qquad [11]$$

In short, the RNN architecture can provide more accurate outputs because the inputs are (potentially) taken from all previous values (see inputs $U_{j-1}^{[1]}$ and $U_{j-1}^{[2]}$ in the figure above).

4.2.3 THE HIGHER ORDER NEURAL NETWORK ARCHITECTURE

Higher Order Neural Networks (HONNs) were first introduced by Giles and Maxwell (1987) and were called "Tensor Networks". Although the extent of their use in finance has so far been limited, Knowles *et al*. (2009) show that, with shorter computational times and limited input variables, "the best HONN models show a profit increase over the MLP of around 8%" on the EUR/USD time series (p. 7). For Zhang *et al.* (2002), a significant advantage of HONNs is that "HONN models are able to provide some rationale for the simulations they produce and thus can be regarded as "open box" rather then "black box". HONNs are able to simulate higher frequency, higher order non-linear data, and consequently provide superior simulations compared to those produced by ANNs (Artificial Neural Networks)" (p. 188). Furthermore HONNs clearly outperform in terms of annualised return and this enables Dunis et al. (2008) to conclude with confidence over their forecasting superiority and their stability and robustness through time.

While they have already experienced some success in the field of pattern recognition and associative recall[8], HONNs have only started recently to be used in finance. The architecture of a three input second order HONN is shown below:

[8] Associative recall is the act of associating two seemingly unrelated entities, such as smell and colour. For more information see Karayiannis *et al.* (1994).

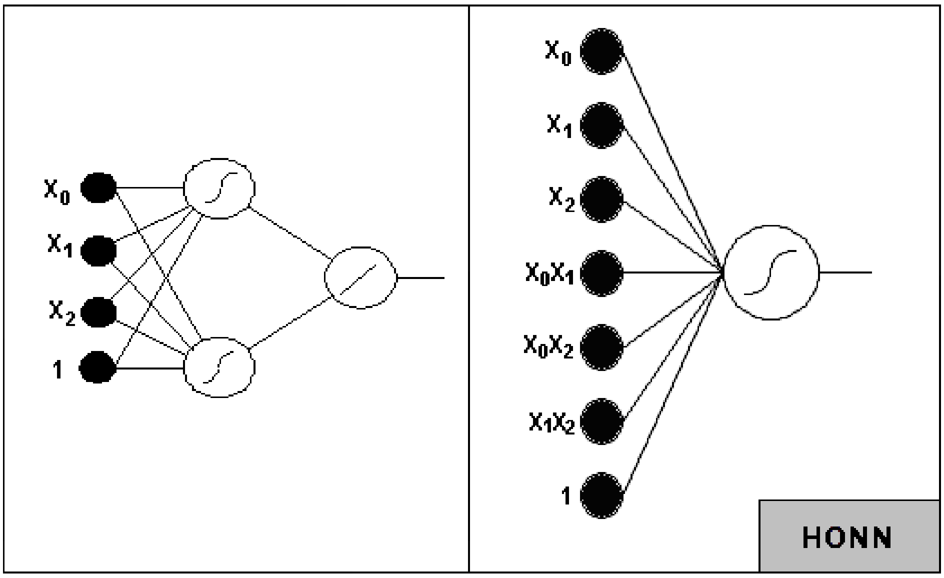


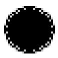
Fig. 5: Left, MLP with three inputs and two hidden nodes; right, second order HONN with three inputs

Where:

$x_t^{[n]}\ (n = 1,2,\cdots,k+1)$ are the model inputs (including the input bias node) at time t

$\tilde{y}_t$ is the HONNs model output

u_{jk} are the network weights



are the model inputs.

is the transfer sigmoid function: $S(x) = \dfrac{1}{1+e^{-x}}$, [12]

is a linear function: $F(x) = \sum_i x_i$ [13]

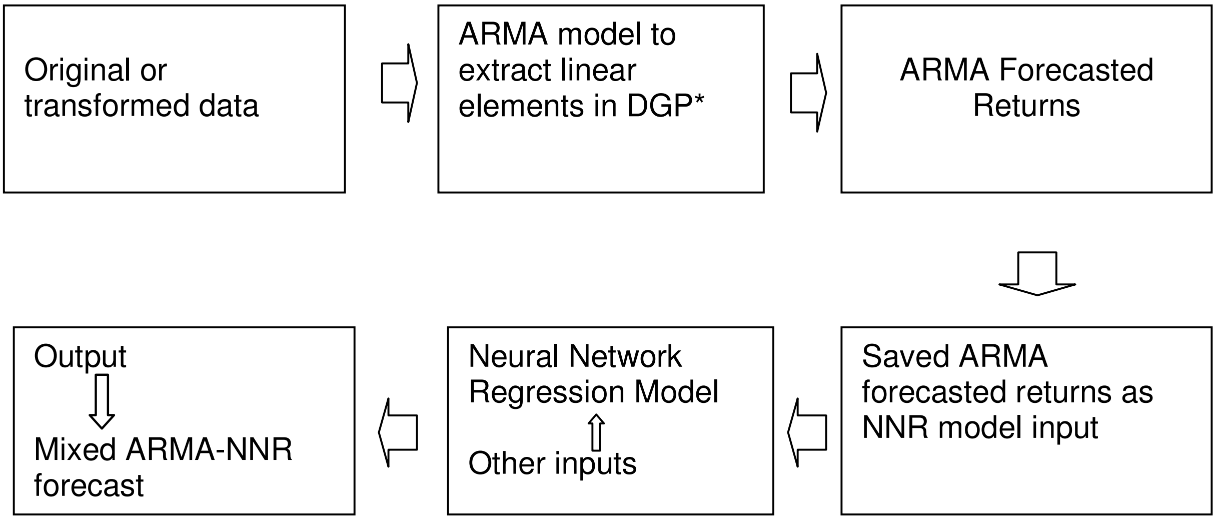
The error function to be minimised is:

$E(u_{jk}, w_j) = \dfrac{1}{T}\sum_{t=1}^{T}(y_t - \tilde{y}_t(u_{jk},))^2$, with y_t being the target value

HONNs use joint activation functions; this technique reduces the n relationships between inputs when training. Furthermore this redu weights and means that HONNS are faster to train than even the number of inputs can be very large for higher order ar over are rarely used.

Another advantage of the reduction of free weights means that the problems of overfitting and local optima affecting the results of neural networks can be largely avoided. For a complete description of HONNs see Knowles *et al.* (2005).

4.2.4 THE Mixed HONN, MLP, RNN, ARCHITECTURE



*DGP= Data Generating Process

<u>Fig. 6:</u> The architecture of Mixed Neural Network Model

The methodology we follow to construct the Mixed ARMA-NNR model is divided into 2 steps. In the first step the ASE 20 index is modelled with a traditional ARMA model. In the second step the forecasted returns of the ARMA model are used as an input to the neural networks for forecasting the selected time series.

4.3 FORECASTING ACCURACY MEASURES

As it is standard in the literature, in order to evaluate statistically our forecasts, the RMSE, the MAE, the MAPE and the Theil-U statistics are computed. The RMSE and MAE statistics are scale-dependent measures but give a basis to compare volatility forecasts with the realised volatility while the MAPE and the Theil-U statistics are independent of the scale of the variables. In particular, the Theil-U statistic is constructed in such a way that it necessarily lies between zero and one, with zero indicating a perfect fit. A more detailed description of these measures can be found on Pindyck and Rubinfeld (1998), Theil (1966) and Dunis and Chen (2005) while their mathematical formulae are in Appendix A.2. For all four error statistics retained (RMSE, MAE, MAPE and Theil-U) the lower the output, the better the forecasting accuracy of the model concerned. In the table below we present our results for the out-of-sample period.

	NAIVE	MACD	ARMA	MLP	RNN	HONN
RMSE	0.0329	0.0254	0.0239	0.0470	0.0241	0.0240
MAE	0.0234	0.0174	0.0161	0.0163	0.0170	0.0299
MAPE	811.13%	393.44%	115.00%	106.97%	275.23%	679.96%
THEIL-U	0.6863	0.7534	0.9446	0.9661	0.8287	0.7289

	Mixed MLP	Mixed RNN	Mixed HONN
RMSE	0.0240	0.0512	0.0240
MAE	0.0162	0.0189	0.0163
MAPE	107.06%	135.13%	103.56%
THEIL-U	0.9762	0.7318	0.9826

Table 5: Out-of-sample statistical performance

As can be seen from tables 5 and A.3 in the Appendix for the in-sample period, Mixed-HONNs seems to outperform all other models and present the most accurate forecasts in statistical terms in both in and out-of-sample periods. It seems that their ability to capture higher order correlations gives them an considerable advantage compared to the other models. Mixed-MLPs come second and Mixed-RNNs come third in our statistical evaluation in both periods. Furthermore, it is worth noting that the time that we need to train our HONNs was less than the time needed for the RNNs and the MLPs.

4.4 EMPIRICAL TRADING SIMULATION RESULTS

The trading performance of all the models considered in the validation subset is presented in the table below. We select the ARMA model with the higher profit in the in-sample period and choose the network with the higher profit in the test sub-period. Our trading strategy applied is simple and identical for all the models: go or stay long when the forecast return is above zero and go or stay short when the forecast return is below zero**.** Appendix A.4 provides the performance of all the NNs in the training and the test sub-periods while Appendix A.5 and A.2 provides the characteristics of our networks and the performance measures. The Mixed-RNNs are trained with gradient descent as for the Mixed-MLPs. However, the increase in the number of weights, as mentioned before, makes the training process extremely slow: to derive our results, we needed for the mixed-RNNs about ten times the time needed with the Mixed-MLPs. As shown in table 6 below, the Mixed-RNN has a lower performance compared to the Mixed-MLP model and Mixed-HONN.

	NAIVE	MACD	ARMA	MLP	RNN	HONN
Information Ratio (excluding costs)	0.32	0.46	0.20	0.60	0.59	0.70
Annualised Volatility (excluding costs)	36.70%	38.12%	38.13%	38.11%	38.11%	38.10%
Annualised Return (excluding costs)	11.42%	17.63%	7.68%	22.99%	22.51%	26.75%
Maximum Drawdown (excluding costs)	-49.41%	-50.63%	-36.50%	-36.26%	-36.22%	-38.71%
Positions Taken (annualised)	119	38	72	105	147	98

	Mixed MLP	Mixed RNN	Mixed HONN
Information Ratio (excluding costs)	0.83	0.78	0.91
Annualised Volatility (excluding costs)	38.08%	38.09%	38.07%
Annualised Return (excluding costs)	31.79%	29.63%	34.75%
Maximum Drawdown (excluding costs)	-26.29%	-27.94%	-28.20%
Positions Taken (annualised)	41	57	65

Table 6: Trading performance results

We can see that Mixed-HONNs perform significantly better than the Mixed-MLPs and the Mixed-RNNs and significantly better than the standard neural network architectures. Learning first the linear component of the data generating process before applying a neural network to learn its nonlinear elements definitely appears to add value in this application. Comparing the recent paper of Dunis et al. (2010a) we notice that Hybrid-NNR models outperform in terms of information ratio Mixed-NNR models. However much higher drawdowns, possibly linked to the higher trading frequency of the Hybrid models compared with the mixed models presented here.

5. TRADING COSTS AND LEVERAGE

Up to now, we have presented the trading results of all our models without considering transaction costs. Since some of our models trade quite often, taking transaction costs into account might change the whole picture. Following Dunis et al. (2008a), we check for potentional improvements to our models through the application of confirmation filters. Confirmation filters are trading strategies devised to filter out those trades with expected returns below a threshold d around zero. They suggest to go long when the forecast is above d and to go short when the forecast is below d. It just so happens that the Mixed ARMA-Neural Network models perform best without any filter. This is also the case of the MLP and HONN models. Still, the application of confirmation filters to the benchmark models and the RNN model could have led to these models outperforming the Mixed, MLP HONN models. This is not the case in order to conserve space, these results are not shown here but they are available from the authors.

5.1 TRANSACTION COSTS

According to the Athens Stock Exchange, transaction costs for financial institutions and fund managers dealing a minimum of 143 contracts or 1 million Euros is 10 Euros per contract (round trip). Dividing this transaction cost of the 143 contracts by average size deal (1 million Euros) gives us an average transaction cost for large players of 14 basis points (1 base point=1/100 of 1%) or 0.14% per position.

	NAIVE	MACD	ARMA	MLP	RNN	HONN
Information Ratio (excluding costs)	0.32	0.46	0.20	0.60	0.59	0.70
Annualised Volatility (excluding costs)	36.70%	38.12%	38.13%	38.11%	38.11%	38.10%
Annualised Return (excluding costs)	11.42%	17.63%	7.68%	22.99%	22.51%	26.75%
Maximum Drawdown (excluding costs)	-49.41%	-50.63%	-36.50%	-36.26%	-36.22%	-38.71%
Positions Taken (annualised)	119	38	72	105	147	98
Transaction costs	16.66%	5.32%	10.08%	14.7%	20.58%	13.72%

Annualised Return (including costs)	-5.24%	12.31%	-2.4%	8.29%	1.93%	13.03%

	Mixed MLP	Mixed RNN	Mixed HONN
Information Ratio (excluding costs)	0.83	0.78	0.91
Annualised Volatility (excluding costs)	38.08%	38.09%	38.07%
Annualised Return (excluding costs)	31.79%	29.63%	34.75%
Maximum Drawdown (excluding costs)	-26.29%	-27.94%	-28.20%
Positions Taken (annualised)	41	57	65
Transaction costs	5.74%	7.98%	9.1%
Annualised Return (including costs)	26.05%	21.65%	25.65%

Table 7: Out-of-sample results with transaction costs

We can see that, after transaction costs, the Mixed-MLP network outperforms all the other strategies based on the annualised return closely followed by the Mixed-HONN strategy. On the other hand, the naïve strategy and the ARMA model produce negative results after transaction costs are taken into account. The HONN and MACD achieve decent returns, yet well below those produced by our mixed ARMA- NNR models.

5.3 LEVERAGE TO EXPLOIT HIGH INFORMATION RATIOS

In order to further improve the trading performance of our models we introduce a “level of confidence” to our forecasts, i.e. a leverage based on the test sub-period. For the naïve model, which presents a negative return we do not apply leverage. The leverage factors applied are calculated in such a way that each model has a common volatility of 20%[9] on the test data set.

The transaction costs are calculated by taking 0.14% per position into account, while the cost of leverage (interest payments for the additional capital) is calculated at 4% p.a. (that is 0.016% per trading day[10]). Our final results are presented in table 8 below.

	NAIVE	MACD	ARMA	MLP	RNN	HONN
Information Ratio (excluding costs)	0.32	0.70	0.20	0.60	0.59	0.70
Annualised Volatility	36.70%	40.03%	38.13%	40.28%	40.21%	40.31%

[9] Since most of the models have a volatility of about 20%, we have chosen this level as our basis. The leverage factors retained are given in table 8 below.

[10] The interest costs are calculated by considering a 4% interest rate p.a. divided by 252 trading days. In reality, leverage costs also apply during non-trading days so that we should calculate the interest costs using 360 days per year. But for the sake of simplicity, we use the approximation of 252 trading days to spread the leverage costs of non-trading days equally over the trading days. This approximation prevents us from keeping track of how many non-trading days we hold a position.

(excluding costs)						
Annualised Return (excluding costs)	11.42%	18.51%	7.68%	24.30%	23.75%	28.30%
Maximum Drawdown (excluding costs)	-49.41%	-53.16%	-36.50%	-38.32%	-38.21%	-40.96%
Leverage Factor	-	1.050	-	1.057	1.055	1.058
Positions Taken (annualised)	119	38	72	105	147	98
Transaction and leverage costs	16.66%	5.60%	10.08%	15.02%	20.88%	14.04%
Annualised Return (including costs)	-5.24%	12.90%	-2.40%	9.28%	2.87%	14.26%

	Mixed-MLP	Mixed-RNN	Mixed-HONN
Information Ratio (excluding costs)	0.83	0.78	0.91
Annualised Volatility (excluding costs)	40.22%	40.22%	40.17%
Annualised Return (excluding costs)	33.57%	31.29%	36.67%
Maximum Drawdown (excluding costs)	-27.76%	-29.50%	-29.75%
Leverage Factor	1.056	1.056	1.055
Positions Taken (annualised)	41	57	65
Transaction and leverage costs	6.052%	8.30%	9.40%
Annualised Return (including costs)	27.51%	23.00%	27.27%

Table 8: Trading performance - final results

As can be seen from table 8, Mixed-MLPs continue to demonstrate a superior trading performance despite significant drawdowns. The Mixed-HONN strategy also performs well and presents the second highest annualised return. In general, we observe that all models are able to gain extra profits from the leverage as the increased costs are outweighed by the benefits of trading somewhat higher volumes. Again it is worth mentioning, that the time needed to train the HONN and the Mixed-HONN network was considerably shorter compared with that needed for the MLP, Mixed-MLP, RNN and the Mixed-RNN networks.

6. CONCLUDING REMARKS

In this paper, we apply Multi-layer Percepton, Recurrent, Higher Order, Mixed-Multilayer Percepton, Mixed-Recurrent and Mixed-Higher Order neural networks to a one-day-ahead forecasting and trading task of the ASE 20 fixing series with only autoregressive terms as inputs. We use a naïve strategy, a MACD and an ARMA model as benchmarks. We develop these different prediction models over the period January 2001 - August 2007 and validate their out-of-sample trading efficiency over the following period from September 2007 through December 2008.

The Mixed-HONNs demonstrates a higher trading performance in terms of annualised return and information ratio before transaction costs and more elaborate trading strategies are applied. When refined trading strategies are applied and transaction costs are considered the Mixed-MLPs manage to outperform all other models achieving the highest annualised return. The Mixed-HONNs and the Mixed-RNNs models perform remarkably as well and seem to have an ability in providing good forecasts when autoregressive series are only used as inputs.

It is also important to note that the Mixed-HONN network which presents a very close second best performance needs less training time than Mixed-RNN and Mixed-MLP network architectures, a much desirable feature in a real-life quantitative investment and trading environment: in the circumstances, our results should go some way towards convincing a growing number of quantitative fund managers to experiment beyond the bounds of traditional statistical and neural network models. In particular, the strategy consisting of modelling in a first stage the linear component of a financial time series and then applying a neural network to learn its nonlinear elements appears quite promising.

APPENDIX

A.1 ARMA Model

The output of the ARMA model used in this paper is presented below.

Dependent Variable: RETURNS
Method: Least Squares
Date: 03/17/09 Time: 22:18
Sample (adjusted): 8 1738
Included observations: 1731 after adjustments
Convergence achieved after 37 iterations
Backcast: 1 7

Variable	Coefficient	Std. Error	t-Statistic	Prob.

C	0.000290	0.000303	0.956602	0.3389
AR(1)	0.375505	0.052705	7.124626	0.0000
AR(3)	-0.244662	0.024991	-9.789999	0.0000
AR(7)	-0.678906	0.044902	-15.11958	0.0000
MA(1)	-0.374290	0.053055	-7.054702	0.0000
MA(3)	0.269470	0.026409	10.20353	0.0000
MA(7)	0.677169	0.044295	15.28785	0.0000

R-squared	0.026582	Mean dependent var	0.000288
Adjusted R-squared	0.023194	S.D. dependent var	0.012549
S.E. of regression	0.012403	Akaike info criterion	-5.937710
Sum squared resid	0.265213	Schwarz criterion	-5.915645
Log likelihood	5146.088	F-statistic	7.846483
Durbin-Watson stat	1.856760	Prob(F-statistic)	0.000000

Inverted AR Roots	.89-.44i	.89+.44i	.31-.92i	.31+.92i
	-.54+.70i	-.54-.70i	-.93	
Inverted MA Roots	.88-.45i	.88+.45i	.31-.92i	.31+.92i
	-.54+.70i	-.54-.70i	-.94	

A.2 Performance Measures

The performance measures are calculated as follows:

Performance Measure	Description	
Annualised Return	$$R^A = 252 * \frac{1}{N} \sum_{t=1}^{N} R_t$$ *with* R_t *being the daily return*	[14]
Cumulative Return	$$R^C = \sum_{t=1}^{N} R_t$$	[15]
Annualised Volatility	$$\sigma^A = \sqrt{252} * \sqrt{\frac{1}{N-1} * \sum_{t=1}^{N} (R_t - \overline{R})^2}$$	[16]

Information Ratio	$$IR = \frac{R^A}{\sigma^A}$$	[17]
Maximum Drawdown	*Maximum negative value of* $\sum(R_t)$ *over the period* $$MD = \underset{i=1,\cdots,t;t=1,\cdots,N}{Min}\left(\sum_{j=i}^{t} R_j\right)$$	[18]

Table 9: Trading simulation performance measures

A.3 Statistical Results in the Training and Test Sub-Periods

	NAIVE	MACD	ARMA	MLP	RNN	HONN
RMSE	0.0125	0.0131	0.0124	0.0153	0.0237	0.0141
MAE	0.0125	0.0097	0.0090	0.0111	0.0119	0.0103
MAPE	456.56%	235.17%	117.82%	371.57%	329.88%	234.72%
THEIL-U	0.6781	0.7459	0.8643	0.6842	0.7174	0.6938

	Mixed MLP	Mixed RNN	Mixed HONN
RMSE	0.0127	0.0205	0.0189
MAE	0.0091	0.0116	0.0126
MAPE	111.34%	285.99%	355.95%
THEIL-U	0.7881	0.7105	0.6989

Table 10: in sample statistical performance

A.4 Empirical Results in the Training and Test Sub-Periods

		NAIVE	MACD	ARMA	MLP	RNN	HONN
Information Ratio	***(excluding costs)***	1.55	1.24	1.24	1.57	1.53	1.61
Annualised Volatility	**(excluding costs)**	19.32%	19.49%	19.83%	19.60%	19.60%	19.59%
Annualised Return	***(excluding costs)***	29.86%	24.29%	24.66%	30.72%	30.02%	31.56%
Maximum Drawdown	***(excluding costs)***	-23.39%	-25.42%	-26.70%	-27.52%	-34.66%	-39.70%
Positions Taken	***(annualised)***	114	34	50	86	81	108

		Mixed-MLP	Mixed-RNN	Mixed-HONN
Information Ratio	***(excluding costs)***	2.07	1.93	2.11
Annualised Volatility	**(excluding costs)**	19.45%	19.47%	19.44%
Annualised Return	***(excluding costs)***	40.17%	37.57%	41.12%
Maximum Drawdown	***(excluding costs)***	-37.89%	-41.47%	-37.52

Positions Taken	***(annualised)***	46	68	47

Table 11: In-sample trading performance

A.5 Networks Characteristics

We present below the characteristics of the networks with the best trading performance on the test sub-period for the different architectures.

Parameters	MLP	RNN	HONN
Learning algorithm	***Gradient descent***	***Gradient descent***	***Gradient descent***
Learning rate	0.001	0.001	0.001
Momentum	0.003	0.003	0.003
Iteration steps	1500	1500	1000
Initialisation of weights	N(0,1)	N(0,1)	N(0,1)
Input nodes	11	11	11
Hidden nodes (1layer)	7	6	0
Output node	1	1	1

Table 12: Network Characteristics for Traditional Neural Networks

Parameters	Mixed-MLP	Mixed-RNN	Mixed-HONN
Learning algorithm	***Gradient descent***	***Gradient descent***	***Gradient descent***
Learning rate	*0.001*	*0.001*	*0.001*
Momentum	*0.003*	*0.003*	*0.003*
Iteration steps	*1500*	*1500*	*1000*
Initialisation of weights	*N(0,1)*	*N(0,1)*	*N(0,1)*
Input nodes	*13*	*13*	*13*
Hidden nodes (1layer)	*6*	*7*	*0*
Output node	*1*	*1*	*1*

Table 13: Network characteristics for Mixed Neural Networks

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