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## Mean Reversion, Long Memory and Fractional Integration in African Stock Market Prices

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# MEAN REVERSION, LONG MEMORY AND FRACTIONAL INTEGRATION IN AFRICAN STOCK MARKET PRICES

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## ABSTRACT

We examine the behavior of stock market prices in several African countries by means of fractionally integrated techniques. In doing so, we can test for mean reversion in these markets. Our results can be summarized as follows: we cannot find evidence of mean reversion in any single market, and evidence of long memory returns (i.e., orders of integration above 1 in the logged stock prices) is obtained in the cases of Egypt and Nigeria, and, in a lesser extent in Tunisia, Morocco and Kenya. Permitting the existence of a structural change, the break dates take place in the earlier 2000s in the majority of the cases, and evidence of mean reversion seems to take place in the periods before the breaks in most of the countries. If we focus on the absolute and squared returns, evidence of long memory is obtained in Nigeria and Egypt. Thus, for these two countries, a long memory model incorporating positive fractional degrees of integration in both the level and the volatility process should be considered.

**Keywords:** Long memory; Fractional integration; Stock market returns.
**JEL Classification:** C4; F3; G15

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## 1. Introduction

According to the Efficient Market Hypothesis (EMH) stock market prices should follow a random walk process since both asset prices and returns are determined by the outcome of supply and demand in a competitive rational market (Fama, 1970, Summers, 1986). This is based on the idea that it should not be possible to make systematic profits above transaction costs and risk premia, and therefore returns should be unpredictable. On the other hand, several authors have found evidence of mean reversion in stock market prices (see, for example, Poterba and Summers, 1988 and Fama and French, 1988). The standard econometric approach to settle this issue empirically relies on establishing the order of integration of the series by carrying out unit root tests. Standard methods are the ADF-test (Dickey and Fuller, 1979), PP (Phillips and Perron, 1988), KPSS (Kwiatkowski et al., 1992), etc. According to these methods, the series is either nonstationary I(1) or stationary I(0). In the former case, that includes the random walk as a particular case, shocks are of a permanent nature while in the latter case shocks are transitory and therefore, mean reverting, disappearing in the long run.

More recently, the possibility of fractional orders of integration with a slow hyperbolic rate of decay has also been taken into account. Thus, the number of differences required to render a series stationary I(0) may not necessarily be an integer value (usually 1) and may be a value between 0 and 1, or even above 1. This approach has been widely employed to analyze financial data in developed countries. Examples are the papers of Crato (1994), Cheung and Lai (1995), Barkoulas and Baum (1996), Barkoulas, Baum, and Travlos (2000), Sadique and Silvapulle (2001), Henry (2002), Tolvi (2003) and Gil-Alana (2006) among many others. In this context, if stock market prices are I(0) they are mean reverting, with shocks disappearing relatively fast; if they are fractionally integrated (I(d)) with d above

0 but smaller than 1 (i.e. $0 < d < 1$), prices still display mean reversion though the adjustment takes a longer time, in fact, longer as higher is the value of d. On the other hand, if stock prices are I(1) the EMH is satisfied under the random walk model and no mean reversion is obtained. Lack of mean reversion is also obtained if the series is I(d) with $d > 1$.[3]

This paper examines the existence of mean reversion in the stock market prices in several African countries, namely, Egypt, Morocco, Tunisia, Nigeria, Mauritius, Kenya, South Africa, Zimbabwe, Botswana and Namibia, by means of fractionally integrated techniques. Like developed financial markets, African emerging capital markets provide investors with opportunities to diversify their portfolios. In addition, the macroeconomic dynamics that govern stock market returns in African economies are unarguably different from those of developed countries. Capital markets in emerging economies tend to be underdeveloped and most likely inefficient. Market thinness and non-synchronous trading are among other factors that may affect stock returns in emerging capital markets of Africa. Harvey (1995) points out that emerging capital markets tend to have higher expected returns and display more volatility than those of developed capital markets. We examine these markets by means of long range dependence techniques. To the best of our knowledge there are no applications involving these techniques in African countries.

The structure of the paper is as follows: Section 2 briefly reviews the literature on stock market prices in developing countries. Section 3 presents the data and the empirical results based on fractional integration, extending also the model to allow for a structural break in the data. Section 4 contains some concluding comments.

## 2. Brief literature review

[3] In a close-related literature Granger and Ding (1995a, b) focus on power transformations of the absolute value of the returns. They estimate a long memory process to study persistence in volatility, and establish some stylized facts (temporal and distributional properties).

A number of studies including Koong et al. (1997), Kilic (2004), Sadique and Silvapulle (2001), and Magnusson and Wydick (2002) have examined stock market prices in the context of long memory processes. For instance Koong et al. (1997) examined the properties of the returns of four Pacific Basin stock markets including those for Australia, Hong Kong, Singapore and Japan. They find no evidence of long memory for the four stock market index returns. Kilic (2004) also investigated long memory in the stock returns for Istanbul stock exchange using the FIGARCH framework. He finds evidence against long memory for the daily stock returns. However, he finds evidence of long memory for ISE 100 index volatility. Sadique and Silvapulle (2001) examined the long memory properties of stock market returns for Japan, Korea, New Zealand, Malaysia, Singapore, the USA and Australia. They find evidence of long memory in stock market returns for Korea, Malaysia, New Zealand and Singapore. Based on this finding, they show that stock prices are I(d) with d > 1 and thus, conclude that the stock markets for these four countries are not efficient. The finding that the markets are inefficient indicates the inability of the four markets to channel financial capital in their respective economies. It also suggests that the stock markets for these countries are not well- diversified.

Magnusson and Wydick (2002) examined the efficiency of eight emerging African stock markets. They find that the markets are random walk processes, though they conclude that similar to the emerging stock markets in South-East Asia and Latin America, the African stock markets are inefficient.

Lim (2009) uses non-linear models to examine the returns in several Middle East and African countries using non-linear models. He finds that the returns contain predictable components, and therefore are inefficient. Assaf (2006) investigated the stock market returns and the volatility for countries of MENA region including — Egypt, Jordan, Morocco, and Turkey. Using long range dependence techniques, he found long memory in the stock returns for Egypt and Morocco, while evidence of anti-persistence for Jordan and Turkey. He also found evidence of long memory in the volatility in all countries examined. Alagidede and Panagiotidis (2009) investigated the stock market returns for Egypt, Kenya, Morocco, Nigeria, South Africa, Tunisia and Zimbabwe, finding evidence against the random walk in all cases. However, using smooth transition and conditional volatility models they found evidence of volatility clustering, leptokurtosis and leverage effect in the African data.

From the preceding literature review, it is clear that only a handful of studies have investigated the issue of long memory in the emerging African capital markets. To fill the void, this paper investigates the existence of fractional integration for Egypt, Morocco, Tunisia, Nigeria, Mauritius, Kenya, South Africa, Zimbabwe, Botswana and Namibia using long range dependence techniques. We examine the returns and the volatility (measured in terms of absolute and squared returns) using a parametric approach of long memory which is the most efficient one in the context of fractional integration.

## 3. Data and empirical results

The data analyzed in this paper correspond to daily closing prices of CASE 30 (Egypt); MASI (Morocco), TUNINDEX (Tunisia) and NSE All Share (Nigeria). Additionally, we use monthly data corresponding to SEM (Mauritius), NSE 20 (Kenya), JSE All Share (South Africa), ZSE Industrials (Zimbabwe), BSE (Botswana) and JSE All Share (Namibia), obtained from Datastream.[2] A summary of the time series and their corresponding sample periods are displayed in Table 1.

**[Insert Table 1 about here]**

[2] We thank Paul Alagidede and Theodore Panagiotidis for kindly providing the dataset.

We consider the following model,

$$y_t \; = \; \alpha \; + \; \beta t \; + \; x_t, \qquad t \; = \; 1, 2, ..., \tag{1}$$

$$(1 - L)^d \, x_t \; = \; u_t \, , \qquad t \; = \; 1, 2, ... \, , \tag{2}$$

where $y_t$ corresponds to the return series, obtained by taking first differences on the log-transformed series; α and β are the coefficients corresponding to the intercept and the time trend respectively; d may be a real value, and $u_t$ is supposed to be I(0). In what follows we consider the three standard cases of no regressors in the undifferenced regression model (1) (i.e, α = β = 0 a priori); an intercept (α unknown and β = 0 a priori), and an intercept with a linear time trend (α and β unknown). We use an estimation procedure based on the Whittle function in the frequency domain (Dahlhaus, 1989) along with a testing procedure developed by Robinson (1994). The latter is a Lagrange Multiplier (LM) procedure that is supposed to be the most efficient method in the context of fractional integration. It tests the null hypothesis $H_o$: $d = d_o$ for any real value $d_o$, in a model given by (1) and (2), and given its asymptotic N(0,1) distribution we can easily build up confidence bands for the non-rejection values.[3]

**[Insert Table 2 about here]**

Table 2 displays the estimates of d in (2) under the assumption that the underlying disturbances $u_t$ are white noise. Thus, in this case, all the time dependence is described throughout the fractional differencing parameter d, and the non-rejection of the null hypothesis of d = 0 implies that the stock prices follows a random walk model which is consistent with the EMH. Starting with the daily data (Egypt, Morocco, Tunisia and Nigeria), the first thing we observe is that the results are very similar for the three cases of no

[3] Empirical applications based on this procedure can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000) among many others. A brief discussion of this method is provided in the Appendix.

regressors, an intercept, and an intercept with a linear trend. In all cases, the estimates of d are statistically significantly positive implying that the return series display long memory behavior, and thus rejecting the EMH. The highest estimate is obtained for Nigeria (0.342), followed by Morocco (0.243), Tunisia (0.215) and finally Egypt (0.116).

However, a very different picture emerges if we focus on the countries where only monthly data are available. Here, we cannot reject the I(0) hypothesis in any single case with the exception of Kenya (d = 0.136). For Mauritius, the estimates are also positive though statistically insignificant. In all the other cases (South Africa, Zimbabwe, Botswana and Namibia) the estimates are negative though insignificant. Thus, the results presented so far seem to indicate that stock market prices data are very sensitive to the frequency-data used in the analysis.

**[Insert Table 3 about here]**

In Table 3 we assume that the disturbances are AR(1). Higher AR orders were also examined and the results were very similar to those reported here. Note that given the fractional nature of the d-differencing polynomial in (2) the process can itself be expressed in terms of an AR(∞) process, and thus, the contribution of the short run AR(k) dynamics only affects the first k terms. Starting again with the daily data, the estimates are positive in all cases though smaller than in Table 1 and they are now insignificantly different from 0 in the case of Morocco. For the monthly data, the I(0) hypothesis cannot be rejected in any country, and the estimates are positive for Kenya and Botswana, and negative for Mauritius, South Africa, Zimbabwe and Namibia.

According to the above results, stock market prices in Nigeria, Egypt and Tunisia (and in some cases in Morocco and Kenya) present orders of integration which are strictly above

1, implying that their corresponding returns are long memory (d > 0). We also observe substantial differences depending on the countries analyzed and in particular, if the frequency employed is daily or monthly. Thus, we also computed the results for the daily data on a monthly frequency in order to check that our results are not sensitive to the frequency of the data employed.

**[Insert Table 4 about here]**

Table 4 displays the estimates for the four countries where daily data were available and long memory was found, but using now the last day of the month on a monthly basis. The results indicate positive and significant orders of integration (and thus evidence of long memory) for Nigeria and Egypt, but evidence of I(0) in the cases of Morocco and Tunisia.[4]

In what follows we examine the possibility of a structural break in the data. It is well known that fractional integration may be related with the presence of breaks (see, e.g., Diebold and Inoue, 2001; Granger and Hyung, 2004). Given the short number of observations employed in this work, we suppose that there is just a single break in the data. Following Gil-Alana (2008) we assume that $y_t$ is the observed time series, generated by the model

$$y_t = \alpha_1 + \beta_1 t + x_t; \quad (1 - L)^{d_1} x_t = u_t, \quad t = 1, ..., T_b \tag{3}$$

$$y_t = \alpha_2 + \beta_2 t + x_t; \quad (1 - L)^{d_2} x_t = u_t, \quad t = T_b + 1, ..., T, \tag{4}$$

where the $\alpha$'s and the $\beta$'s are the coefficients corresponding respectively to the intercepts and the linear trends; $d_1$ and $d_2$ may be real values, $u_t$ is I(0), and $T_b$ is the time of the break that is supposed to be unknown. The model in equations (3) and (4) can be written as:

$$(1 - L)^{d_1} y_t = \alpha_1 \tilde{1}_t(d_1) + \beta_1 \tilde{t}_t(d_1) + u_t, \quad t = 1, ..., T_b, \tag{5}$$

[4] Though we do not report the results in the paper, we also performed other parametric (Sowell, 1992) and semiparametric (Robinson, 1995) methods on the same daily and monthly data, and the results, available from the authors upon request, lead essentially to the same results as those reported here.

$$(1-L)^{d_2} y_t \;=\; \alpha_2 \, \tilde{1}_t(d_2) \;+\; \beta_2 \, \tilde{t}_t(d_2) \;+\; u_t, \quad t \;=\; T_b+1, \ldots, T, \qquad (6)$$

where $\tilde{1}_t(d_i) = (1-L)^{d_i} 1$, and $\tilde{t}_t(d_i) = (1-L)^{d_i} t$, i = 1, 2.[5]

The approach adopted here is based on the least square principle. First, we choose a grid for the values of the fractionally differencing parameters $d_1$ and $d_2$, for example, $d_{io}$ = 0, 0.01, 0.02, …, 2, i = 1, 2. Then, for a given partition $\{T_b\}$ and given $d_1$, $d_2$-values, $(d_{1o}^{(j)}, d_{2o}^{(j)})$, we estimate the α's and the β's by minimising the sum of squared residuals,

$$\min_{w.r.t.\{\alpha_1,\alpha_2,\beta_1,\beta_2\}} \sum_{t=1}^{T_b} \left[ (1-L)^{d_{1o}^{(j)}} y_t - \alpha_1 \, \tilde{1}_t(d_{1o}^{(j)}) - \beta_1 \, \tilde{t}_t(d_{1o}^{(j)}) \right]^2 +$$

$$\sum_{t=T_b+1}^{T} \left[ (1-L)^{d_{2o}^{(j)}} y_t - \alpha_2 \, \tilde{1}_t(d_{2o}^{(j)}) - \beta_2 \, \tilde{t}_t(d_{2o}^{(j)}) \right]^2$$

in case of uncorrelated $u_t$, or, alternatively, using GLS for weakly autocorrelated disturbances. Let $\hat{\alpha}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for partition $\{T_b\}$ and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values in the objective function, we obtain RSS($T_b$; $d_{1o}^{(1)}$, $d_{2o}^{(1)}$), and minimising this expression for all values of $d_{1o}$ and $d_{2o}$ in the grid we obtain: $RSS(T_b) \;=\; \arg\min_{\{i,j\}} \; RSS(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$. Then, the estimated break date, $\hat{T}_k$, is such that $\hat{T}_k \;=\; \arg\min_{i=1,\ldots,m} RSS(T_i)$, where the minimisation is over all partitions $T_1$, $T_2$, …, $T_m$, such that $T_i - T_{i-1} \geq |\varepsilon T|$. The regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., $\hat{\alpha}_i \;=\; \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i \;=\; \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i \;=\; \hat{d}_i(\{\hat{T}_k\})$, for i = 1 and 2. Several Monte Carlo experiments conducted in Gil-Alana (2008) show that the procedure performs well even in relatively small samples.

[5] In what follows, we assume that $(1-L)^{d_i} y_t = \tilde{1}_t(d_i) = \tilde{t}(d_i) = 0$, for $t \leq 0$. This is a standard assumption in the applied work, and is related with the Type II definition as opposed to the Type I definition of fractional integration. (See, Robinson and Marinucci, 2001).

Table 5 displays the results based on a model with an intercept, while Table 6 refers to the case of a linear time trend. In both cases we employ white noise and AR(1) disturbances. Starting with the model with an intercept, we observe that the break date takes place at 1999 for Botswana; at 2001 for Namibia and South Africa; at 2002 for Kenya, and at 2003 for Mauritius and Zimbawe. These results are robust for the two types of disturbances. If we focus on the orders of integration, we observe several negative values, especially if the disturbances are autocorrelated, and the only evidence of long memory is obtained for the cases of Kenya, Botswana and Namibia during the second subsamples. Generally, the orders of integration are higher during the second subsamples, implying an increase in the degree of dependence across time.

**[Insert Tables 5 and 6 about here]**

Including a linear time trend (in Table 6) we see that the break dates take place at roughly the same periods as in the previous table, being generally in the earlier 2000s; the orders of integration are also very similar to those based on a model with an intercept, and evidence of long memory is now obtained in the cases of Mauritius and Kenya in a model with white noise $u_t$, and Botswana with autocorrelated errors.

In the final part of this article, we also investigate if the volatility processes, proxied by the squared and the absolute returns, display some degree of strong dependence. Thus, we perform the same type of analysis as before but on the squared and absolute return series. The results for the original data (i.e., Egypt, Morocco, Tunisia and Nigeria on a daily basis, and Mauritius, South Africa, Zimbabwe, Botswana and Namibia on a monthly frequency) are displayed in Table 7.

**[Insert Tables 7 and 8 about here]**

Starting again with the daily data we observe a significant degree of long memory in the four countries examined, the values ranging from 0.164 (squared returns in Morocco with a linear trend) to 0.302 (absolute returns in Nigeria with a linear trend). However, using the countries with monthly data, the I(0) hypothesis cannot be rejected on the squared and absolute return series, the orders of integration being positive in some cases and negative in others. Thus, once more the results seem to be sensitive to the data-frequency used. We finally computed the same approach on the four countries with daily data but based on a monthly-frequency, (in Table 8), and the only evidence of long memory was obtained for Nigeria in all cases and for Egypt with the absolute returns.

## 4. Concluding comments

This paper has investigated the order of integration in a sample of ten African countries including Egypt, Morocco, Tunisia, Nigeria, Mauritius, Kenya, South Africa, Zimbabwe, Botswana, and Namibia using fractionally integrated techniques. The results from the various fractionally integrated techniques implemented in the study failed to find evidence of mean reversion for all of the stock market price series given that the orders of integration were found to be equal to or higher than 1 in all cases. We obtained evidence of long memory in the returns for Egypt, Morocco, Tunisia and Nigeria, and also in the absolute and squared returns of the same countries, implying a degree of predictability in the returns and inefficiencies in the markets. With respect to the remaining countries (Mauritius, Kenya, South Africa, Zimbabwe, Botswana and Namibia) the random walk model cannot be rejected in any of them with the exception of Kenya implying that the EMH cannot be rejected in these countries. These results are consistent with those of Magnusson and Wydick (2002) though the non-rejection of the random walk could be a consequence of the small number of observations available for these countries. Therefore, the results presented in this work should be considered as additional evidence in favour of the inefficiencies observed in the African stock markets, in particular in relation with the volatility processes. Finally, the fact that significantly positive orders of integration are observed in both the levels and the volatility processes in the cases of Nigeria and Egypt suggests that a richer model, incorporating these features should be taken into account to explain the dynamic behaviour of some African stock market returns.

## Appendix

The Lagrange Multiplier (LM) test of Robinson (1994) for testing $H_o$: d = $d_o$, in the model given by the equations (1) and (2) is

$$\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{-1/2} \hat{a},$$

where T is the sample size and:

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} = \frac{2}{T}\left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log\left| 2\sin\frac{\lambda_j}{2} \right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau} \log g(\lambda_j;\hat{\tau}); \qquad \lambda_j = \frac{2\pi\, j}{T}; \qquad \hat{\tau} = \arg\min \sigma^2(\tau).$$

$\hat{a}$ and $\hat{A}$ in the above expressions are obtained through the first and second derivatives of the log-likelihood function with respect to d (see Robinson, 1994, page 1422, for further details). $I(\lambda_j)$ is the periodogram of $u_t$ evaluated under the null, i.e.:

$$\hat{u}_t = (1 - L)^{d_o} y_t - \hat{\beta}' w_t; \quad \hat{\beta} = \left( \sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, (1 - L)^{d_o} y_t; \quad w_t = (1 - L)^{d_o} z_t,$$

where $z_t = (1, t)^T$, and g is a known function related to the spectral density function of $u_t$:

$$f(\lambda;\sigma^2;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \le \pi.$$

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**Table 1: Description of the dataset**

| Country | Frequency | Sample period | N. of observations |
|---|---|---|---|
| Egypt | Daily | Jan. 4th, 1993 - Jun. 16th, 2006 | 3509 |
| Morocco | Daily | Jan. 2nd, 2002 - Jun. 30th, 2006 | 1173 |
| Tunisia | Daily | Jan. 2nd, 1998 - Jun. 30th, 2006 | 2216 |
| Nigeria | Daily | Jun. 30th, 1995 - Jun. 30th, 2006 | 2871 |
| Mauritius | Monthly | January 1997 – February 2006 | 110 |
| Kenya | Monthly | January 1997 – February 2006 | 110 |
| South Africa | Monthly | January 1997 – February 2006 | 110 |
| Zimbabwe | Monthly | January 1997 – February 2006 | 110 |
| Botswana | Monthly | January 1997 – February 2006 | 110 |
| Namibia | Monthly | January 2000 – November 2006 | 83 |

**Table 2: Estimates of d Based on White Noise Disturbances**

| Country | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Egypt (D) | **0.116** **[0.095, 0.139]** | **0.116** **[0.095, 0.139]** | **0.116** **[0.095, 0.139]** |
| Morocco (D) | **0.243** **[0.195, 0.299]** | **0.243** **[0.195, 0.299]** | **0.243** **[0.194, 0.300]** |
| Tunisia (D) | **0.215** **[0.182, 0.252]** | **0.215** **[0.182, 0.252]** | **0.215** **[0.182, 0.252]** |
| Nigeria (D) | **0.342** **[0.308, 0.381]** | **0.342** **[0.307, 0.379]** | **0.342** **[0.308, 0.380]** |
| Mauritius (M) | 0.079 [-0.029, 0.253] | 0.079 [-0.029, 0.253] | 0.039 [-0.096, 0.242] |
| Kenya (M) | **0.136** **[0.044, 0.273]** | **0.136** **[0.044, 0.274]** | 0.082 [-0.040, 0.256] |
| South Africa (M) | -0.029 [-0.142, 0.140] | -0.029 [-0.143, 0.140] | -0.104 [-0.256, 0.105] |
| Zimbabwe (M) | -0.012 [-0.165, 0.200] | -0.012 [-0.165, 0.200] | -0.012 [-0.164, 0.199] |
| Botswana (M) | -0.028 [-0.219, 0.180] | -0.028 [-0.223, 0.175] | -0.045 [-0.225, 0.160] |
| Namibia (M) | -0.007 [-0.100, 0.133] | -0.007 [-0.101, 0.131] | -0.071 [-0.183, 0.090] |

In bold are the cases where the estimates are significantly above 0. In brackets the 95% confidence band. D = daily returns and M = monthly returns.

**Table 3: Estimates of d Based on AR(1) Disturbances**

| Country | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Egypt (D) | **0.081**<br>**[0.053, 0.114]** | **0.081**<br>**[0.053, 0.113]** | **0.082**<br>**[0.053, 0.114]** |
| Morocco (D) | 0.025<br>[-0.042, 0.104] | 0.025<br>[-0.042, 0.104] | 0.000<br>[-0.089, 0.094] |
| Tunisia (D) | **0.073**<br>**[0.023, 0.131]** | **0.073**<br>**[0.023, 0.131]** | **0.070**<br>**[0.020, 0.130]** |
| Nigeria (D) | **0.054**<br>**[0.005, 0.112]** | **0.054**<br>**[0.005, 0.112]** | **0.055**<br>**[0.005, 0.114]** |
| Mauritius (M) | -0.051<br>[-0.197, 0.137] | -0.051<br>[-0.198, 0.136] | -0.270<br>[-0.392, 0.059] |
| Kenya (M) | 0.130<br>[-0.024, 0.319] | 0.129<br>[-0.024, 0.323] | -0.084<br>[-0.197, 0.305] |
| South Africa (M) | -0.082<br>[-0.291, 0.089] | -0.083<br>[-0.292, 0.089] | -0.116<br>[-0.324, -0.074] |
| Zimbabwe (M) | -0.117<br>[-0.188, 0.023] | -0.116<br>[-0.188, 0.023] | -0.184<br>[-0.221, 0.035] |
| Botswana (M) | 0.098<br>[-0.336, 0.400] | 0.092<br>[-0.345, 0.343] | 0.076<br>[-0.312, 0.312] |
| Namibia (M) | 0.053<br>[-0.107, 0.274] | 0.053<br>[-0.109, 0.265] | -0.029<br>[-0.254, 0.240] |

In bold the cases where the estimates are significantly above 0. In brackets the 95% confidence band.

**Table 4: Estimates of d for the Four Annual Series at a Monthly Frequency**

| i) White noise disturbances | | | |
|---|---|---|---|
| Country | No regressors | An intercept | A linear time trend |
| Egypt (M) | **0.147**<br>**[0.076, 0.245]** | **0.147**<br>**[0.076, 0.245]** | **0.149**<br>**[0.075, 0.252]** |
| Morocco (M) | 0.000<br>[-0.128, 0.211] | 0.000<br>[-0.130, 0.206] | -0.125<br>[-0.295, 0.144] |
| Tunisia (M) | 0.003<br>[-0.100, 0.152] | 0.003<br>[-0.100, 0.151] | -0.003<br>[-0.108, 0.146] |
| Nigeria (M) | **0.171**<br>**[0.062, 0.322]** | **0.171**<br>**[0.062, 0.317]** | **0.183**<br>**[0.067, 0.342]** |
| ii) AR(1) disturbances | | | |
| Country | No regressors | An intercept | A linear time trend |
| Egypt (M) | **0.213**<br>**[0.096, 0.363]** | **0.214**<br>**[0.096, 0.369]** | **0.226**<br>**[0.096, 0.397]** |
| Morocco (M) | -0.071<br>[-0.209, 0.304] | -0.072<br>[-0.208, 0.285] | -0.094<br>[-0.157, 0.149] |
| Tunisia (M) | -0.012<br>[-0.247, 0.218] | -0.012<br>[-0.247, 0.218] | -0.009<br>[-0.251, 0.202] |
| Nigeria (M) | **0.139**<br>**[0.002, 0.440]** | **0.139**<br>**[0.002, 0.426]** | **0.182**<br>**[0.017, 0.510]** |

In bold the cases where the estimates are significantly above 0. In brackets the 95% confidence band.

**Table 5: Estimates of the d's in a model with a single break and an intercept**

| | White noise disturbances | | | AR (1) disturbances | | |
|---|---|---|---|---|---|---|
| | Bk. date | $d_1$ | $d_2$ | Bk. date | $d_1$ | $d_2$ |
| Mauritius | Oct-2003 | -0.01 | 0.08 | Jan-2003 | -0.09** | -0.08 |
| Kenya | Oct-2002 | -0.10** | **0.18*** | Dec-2002 | -0.06 | **0.12*** |
| South Africa | Dec-2001 | -0.09** | 0.07 | Aug-2001 | -0.11 | -0.09 |
| Zimbabwe | Nov-2003 | -0.11** | 0.09 | Feb-2003 | -0.09** | -0.12 |
| Botswana | Oct-1999 | 0.03 | **0.16*** | Sep-1999 | -0.07 | **0.11*** |
| Namibia | Mar-2001 | 0.08 | 0.09 | Mar-2001 | 0.02 | **0.19*** |

* and in bold, evidence of long memory at the 5% level. ** means evidence of mean reversion.

**Table 6: Estimates of the d's in a model with a single break and a linear trend**

| | White noise disturbances | | | AR (1) disturbances | | |
|---|---|---|---|---|---|---|
| | Bk. date | $d_1$ | $d_2$ | Bk. date | $d_1$ | $d_2$ |
| Mauritius | Oct-2000 | -0.12** | **0.11*** | Jan-2003 | -0.19** | -0.10 |
| Kenya | Mar-2001 | -0.11** | **0.16*** | Dec-2002 | -0.08 | -0.04 |
| South Africa | Aug-2000 | -0.10** | -0.09 | Aug-2001 | -0.11** | -0.03 |
| Zimbabwe | Feb-2003 | 0.00 | -0.02 | Feb-2003 | -0.17** | -0.10 |
| Botswana | Sep-1999 | 0.03 | **0.09*** | Aug-1999 | 0.03 | **0.08*** |
| Namibia | Mar-2001 | -0.10** | -0.07 | Mar-2001 | -0.14** | -0.11 |

* and in bold, evidence of long memory at the 5% level. ** means evidence of mean reversion.

**Table 7: Estimates of d in the volatility processes**

| Country | Squared returns | | | Absolute returns | | |
|---|---|---|---|---|---|---|
| | No regressors | An intercept | A linear time | No regressors | An intercept | A linear time |
| Egypt (D) | **0.265 [0.239, 0.295]** | **0.265 [0.239, 0.295]** | **0.264 [0.237, 0.294]** | **0.251 [0.232, 0.272]** | **0.251 [0.232, 0.272]** | **0.250 [0.230, 0.272]** |
| Morocco (D | **0.169 [0.139, 0.204]** | **0.169 [0.139, 0.204]** | **0.164 [0.133, 0.200]** | **0.255 [0.225, 0.289]** | **0.254 [0.225, 0.289]** | **0.253 [0.223, 0.288]** |
| Tunisia (D) | **0.177 [0.151, 0.205]** | **0.177 [0.151, 0.205]** | **0.174 [0.147, 0.203]** | **0.240 [0.215, 0.268]** | **0.240 [0.215, 0.267]** | **0.234 [0.209, 0.262]** |
| Nigeria (D) | **0.289 [0.267, 0.313]** | **0.289 [0.267, 0.313]** | **0.289 [0.267, 0.313]** | **0.301 [0.280, 0.324]** | **0.301 [0.280, 0.324]** | **0.302 [0.281, 0.325]** |
| Mauritius (M) | 0.013 [-0.112, 0.179] | 0.013 [-0.112, 0.178] | 0.001 [-0.117, 0.163] | 0.031 [-0.093, 0.232] | 0.031 [-0.094, 0.228] | 0.022 [-0.095, 0.197] |
| Kenya (M) | -0.025 [-.100, 0.095] | -0.025 [-0.100, 0.096] | -0.043 [-0.145, 0.101] | -0.081 [-0.133, 0.007] | -0.081 [-0.134, 0.007] | -0.106 [-0.181, 0.009] |
| South Africa (M) | 0.085 [-0.019, 0.213] | 0.085 [-0.019, 0.212] | 0.052 [-0.057, 0.188] | 0.098 [-0.017, 0.240] | 0.098 [-0.017, 0.237] | 0.072 [-0.023, 0.203] |
| Zimbabwe (M) | -0.005 [-0.092, 0.129] | -0.005 [-0.093, 0.129] | -0.042 [-0.154, 0.115] | 0.070 [-0.004, 0.188] | 0.069 [-0.004, 0.187] | 0.025 [-0.082, 0.174] |
| Botswana (M) | 0.019 [-0.133, 0.209] | 0.019 [-0.135, 0.204] | -0.065 [-0.239, 0.156] | 0.024 [-0.313, 0.244] | 0.023 [-0.314, 0.229] | -0.119 [-0.312, 0.144] |
| Namibia (M) | 0.130 [-0.053, 0.438] | 0.131 [-0.053, 0.434] | 0.129 [-0.062, 0.430] | 0.008 [-0.102, 0.387] | 0.008 [-0.102, 0.368] | 0.006 [-0.143, 0.332] |

In bold are the cases where the estimates are significantly above 0. In brackets are the 95% confidence bands.

**Table 8: Estimates of d in the volatility processes for the four annual series at a monthly frequency**

| Country | Squared returns | | | Absolute returns | | |
|---|---|---|---|---|---|---|
| | No regressors | An intercept | A linear time | No regressors | An intercept | A linear time |
| Egypt (M) | 0.025 [-0.051, 0.147] | 0.025 [-0.052, 0.147] | -0.008 [-0.126, 0.142] | **0.083 [0.014, 0.200]** | **0.083 [0.014, 0.200]** | 0.058 [-0.055, 0.200] |
| Morocco (M) | -0.018 [-0.130, 0.153] | -0.018 [-0.132, 0.151] | -0.225 [-0.443, 0.064] | -0.024 [-0.123, 0.143] | -0.024 [-0.124, 0.142] | -0.176 [-0.322, 0.101] |
| Tunisia (M) | -0.063 [-0.224, 0.138] | -0.063 [-0.224, 0.137] | -0.109 [-0.254, 0.101] | 0.000 [-0.190, 0.241] | 0.000 [-0.190, 0.236] | -0.042 [-0.178, 0.183] |
| Nigeria (M) | **0.243 [0.063, 0.502]** | **0.243 [0.063, 0.615]** | **0.290 [0.077, 0.719]** | **0.309 [0.115, 0.574]** | **0.302 [0.115, 0.638]** | **0.362 [0.137, 0.704]** |

In bold are the cases where the estimates are significantly above 0. In brackets are the 95% confidence bands.