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PORTFOLIO RISK ANALYSIS: CONDITIONAL ESTIMATES OF VALUE-AT-RISK AND INTERNATIONAL VOLATILITY SPILLOVERS

By

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I dedicate this book to my wife Monika.

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ABSTRACT

In this thesis, we are concerned with the establishment of more accurate and easily implemented methods of modelling portfolio Value-at-Risk (VaR). We establish this by taking the view that unconditional volatility estimates are inappropriate in VaR analysis.

To provide the motivation and the justification for forwarding an alternative model we examine three empirical issues. The first issue is whether the traditional approach based on the use of unconditional measures of volatility and correlation matrix of returns are inappropriate. This thesis forwards the argument that unconditional (historical) variances and covariance are based on rigorous assumptions which are not efficient, given the distributional properties of speculative price changes, conditional on the information set available, and therefore are not appropriate in estimating portfolio VaR.

Following this, the emphasis is placed in estimating variances and covariances as time-varying. Thereafter, we consider whether conditional time series models, of the variances and covariances of asset returns, provide a better indication of a portfolio's VaR. We then propose a "simplified" VaR approach that is based on historical returns of the current portfolio. This simplified VaR is faster to compute and offers flexibility in the econometric specification of the portfolio volatility. Once again, conditional volatility models are proposed to estimate portfolio VaR. The results indicate that the VaR estimates from the simplified model are more accurate than those obtained using time-varying correlations. "Stress" and other non-parametric analysis validate further our conclusions.

Finally, we use (conditional) systematic risk estimates to search for international volatility spillovers. This affects the VaR estimates through the introduction of time-

varying, possibly asynchronous components of portfolio volatility that are ignored in the original static framework of portfolio theory. Consequently, we put forward the notion that VaR estimates depends on the recent history of other markets. However, unlike previous studies, the analysis considers the effect of exchange rate movements on VaR estimates and the nature of the relationship between national stock markets. Our findings highlights the importance of considering the exchange rate in the estimation of VaR and in determining which national market plays the role of market leader.

We found that VaR models using exponential smoothing techniques are not inferior to those based on the more advanced multivariate GARCH volatility estimates. Furthermore, in this thesis we proposed a VaR methodology which overcomes many limitations of the above and other VaR models, *i.e.* dimentionality and stability of the correlation matrix, and unlike them does not requires a specification of the probability distribution of returns used in the calculation of the VaR and worst case scenarios. Our methodology uses past (historical) returns but still maintains the multivariate properties of the data. As stress analysis has shown, the model proposed here provides more efficient and unbiased VaR estimates.

Lastly, we provide a summary of the investigations along with the innovations provided in the thesis. Discussed in the conclusion are the implications of the thesis to both practitioners and academics.

Introduction

Over recent years, the number of different investment vehicles offered to professional and other investors has grown significantly. At the same time, the types of trading activities open to these investors has also grown. A number of these activities, such as dynamic asset allocation and hedging, market timing, arbitrage, as well as some forms of performance measurement, require that investors quantify and control their portfolio risk on a frequent basis. Investors need to know the potential losses their portfolio may incur in the course of a day or week, and the largest cumulative loss that it is likely to be incurred at any time. For example, banks need to evaluate their potential losses in order to set aside enough capital to cover them. Similarly, a company needs to track the value of its assets and any cash flows resulting from losses in the portfolio. A pension fund may want to understand potential losses on its portfolio, both to allocate its assets better and to fulfil its obligations and make the set of payments to investors. In addition, pension fund managers face a constant constraint, the value of their portfolios must never fall below a threshold level, *i.e.* fund liabilities (minus any reserves). Therefore, they need to know the maximum losses their portfolio may incur in the short run.

After some institutions reported heavy losses either because of inappropriate derivative pricing (i.e. Orange County, Procter and Gamble, NatWest) or of fraudulent operations (i.e. Barings Bank and Sumitomo) regulators and owners become concerned more than ever about "catastrophic" risks and the adequacy of capital of the financial institution to meet such risks. Regulators are now demanding that banks and other financial institutions quantify on a regular basis the amount of risk in their portfolios. For example, the Bank of International Settlement (BIS) in its 1994 and 1995 proposal stresses the need for banks to monitor the "market" or Value at risk (VaR) of their portfolios. VaR is a concept developed in the field of risk management that defines the minimum amount of money that one could expect to lose with a given probability over specific period of time. The VaR number applies to current portfolio holdings, so an implicit assumption underlying its computation is that the current portfolio will remain unchanged throughout the holding period. When the holding period is as short as one day, the VaR is referred to as portfolio daily earning at risk (DEaR). Throughout this thesis when we refer to one day periods we may use either term.

As it will be apparent in the next chapter, VaR is rapidly becoming a popular management tool. Its continuing success promises to increase the precision and the readability of risk management systems, therefore, reducing the chances of their failure in unusual market conditions. Historical estimates of asset returns, variances and covariances, traditionally used to estimate portfolio risk rely upon the crucial assumptions about the distributional properties of the data: mainly normality, stationarity and independence. However, the stylised facts about daily returns contradict these hypotheses and make the use of the historical estimates inappropriate in VaR analysis. The accurate modelling of daily portfolio volatility and its use in risk monitoring provides the main motivation of this thesis. Given the inappropriateness of unconditional estimates to measure portfolio risk, we will search for alternative methods in estimating volatility which use the information set more efficiently.

Essentially, this thesis is concerned with the examination of four empirical issues which are related to establishing an accurate method of modelling portfolio volatility on a daily basis and its use in monitoring risk. The first issue to be considered is whether the models that use unconditional measures of volatility and correlation are appropriate. Following this, the thesis will move on to establishing whether the VaR solution obtained from a time invariant variance covariance matrix of the portfolio assets is inefficient (see Markowitz (1952), (1956)). To elaborate, we will argue that unconditional (historical) variances and covariances are based on restrictive assumptions which do not comply with the distributional properties of speculative price changes and so are inappropriate in estimating portfolio VaR. Consequently, the issue to consider is whether alternative approaches can provide unbiased estimates of VaR which are computationally feasible and continue to utilise all available information efficiently.

The third issue to be considered is whether the results for VaR using conditional time series models provide a better indication of a portfolio's VaR. It is well documented that conditional volatility models are particularly suited to daily financial data series since they allow for fat tails and other non-linearities present on the (unconditional) distributions of the latter. We will investigate the ability of a type of conditional volatility model, known as Generalised Autoregressive Conditional Heteroskedasticity or GARCH, in estimating portfolio VaR. Our conclusions are based on "stress" analysis and other non-parametric tests. The final issue to be examined is to test the hypothesis that the volatility transmission mechanism and market linkages can be better explained using the conditional second moments. Understanding the way "news" is transmitted from one market to another will help to forecast volatility rises and protect portfolios from large (unexpected) losses.

To investigate these issues, the data-set employed throughout this thesis is daily closing prices on thirteen markets from the beginning of 1986 until the end of 1995. Each market is selected so that it matches the regional and individual market capitalisation of the world index. As a proxy to the world index, we use the Morgan Stanley Capital International or MSCI World Price Index. To isolate changes in the joint second moments, this thesis deploys a time-varying variance-covariance matrix through the use of the multivariate conditional time series. The time series analysis employed here is based upon the principles of the Autoregressive Conditional Heteroskedastic (ARCH) framework introduced by Engle (1982). We have chose a multivariate generalised ARCH (GARCH) over other types of conditional volatility models for three reasons. Its success in modelling variances and covariances of daily financial time series has been well documented. It is very flexible in the conditional mean and variance specification so it can be used to fit the distributional properties of a particular series of returns. Finally it is relatively easier to compute than other equivalent models, *i.e.* stochastic volatility (SV), while it is not inferior in performance.

As will be discussed in the subsequent chapters, there are computational problems in employing multivariate GARCH models even on small sized portfolios. In recognition of this, we will employ the Sharpe (1963) Single Index Model (SIM) algorithm to simplify calculations. Following Engle *et al.* (1984) and Bollerslev *et al.* (1988), the ARCH model has been extended to a bivariate specification to capture the path of both specific and systematic risk for each of the thirteen domestic portfolios in relation to the world market. This will overcome the dimensionality problem of estimating jointly the variance-covariance matrix, and yet still yield a reasonable, time-varying matrix.

This thesis makes an important contribution to the estimation of VaR risk management in general. It is known that correlations measured from daily returns are unstable. Even their sign is often ambiguous. For large portfolios, the number of pairwise correlations is unmanageable. The method proposed in this thesis does not require the estimation of correlations and hence can be applied to portfolios of virtually any size. We calculate past portfolio returns holding current weights constant. Historical returns contain all the necessary information about asset co-movement, hence, we fit a volatility model and forecast future portfolio volatility and VaR. Since we need to model a single time series of past portfolio returns, we have a lot of flexibility in selecting the volatility model. Such flexibility is missed in other studies that estimate portfolio VaR using a full correlation matrix.

A second important issue addressed by this thesis is the probabilistic calculation of the Worst Case Scenario (WCS). Risk managers have been worried about catastrophic risk; they need to know what losses they will face if uncommon events occurs. Two popular methodologies used today to calculate the WCS is the setting-up of hypothetical scenarios and Monte Carlo simulation. The first method relies on a arbitrary hypothesis made by the risk manager. The second one has also a number of drawbacks which are addressed in detail in chapter four. Our methodology is not based on the setting-up of ad-hoc hypothetical scenarios or arbitrary distributional assumptions of asset returns. Instead we estimate the WCS from the empirical distribution of past portfolio returns. Yet, our method provides rapid estimates because it does not require structural Monte Carlo simulations; furthermore, and most importantly, it can be applied to any size of portfolio. In this thesis we use a non-parametric method when estimating the WCS.

This thesis also sheds light on the issue regarding the impact on portfolio risk of any change in asset correlations. It is well documented that daily pairwise covariances and correlation coefficients of financial asset returns are changing through time. As portfolio risk depends on estimates of all pair-wise correlations included in the portfolio, any changes will affect the riskiness of the portfolio. We show that although individual correlations coefficients may vary substantially from one period to the next, the overall impact on the portfolio riskiness may be modest if the portfolio is well diversified.

This thesis also makes a significant contribution in discovering the existence of market inter-linkages. Unlike any other previous study, we are searching for market inter-dependencies in the changes the systematic risk of national stock prices. By understanding how any volatility shock started in one country will be transmitted to the rest it can help in the measurement and control of market risk in a globally diversified portfolio.

The rest of this thesis is organised as follows. In the first chapter, the theory surrounding the concept of VaR together with its relationship to modern portfolio theory is considered. The distributional assumption of portfolio theory is examined along with the consequences on portfolio VaR in the face of violation of these assumptions. This is followed by a theoretical discussion of the problems surrounding the computation of the variance-covariance matrix and the introduction of the SIM by Sharpe (1963) to be used in this thesis as the means of overcoming the limitations of constructing a 13x13 variance-covariance matrix. To justify the employment of GARCH models, (to be discussed in chapter two) we will examine the issues surrounding the distribution of speculative price changes where two issues of importance are discussed; first, the nature of the distribution itself and second, the issue of non-stationarity in the data.

Chapter two discusses the methodology to be employed in this thesis. It begins by outlining the empirical evidence about the distributional properties of speculative price changes and the methods proposed to model stylised facts, like volatility clusters and excess kurtosis. Its particular aim is to focus on the modelling of security betas. Following this, we investigate the various statistical methods used in previous studies for improving portfolio volatility modelling. These include beta adjustment procedures based on ordinary least squares (OLS), exponential smoothing (ES) and non-linear time series models of the GARCH family and stochastic volatility (SV) approaches. On the basis of the above discussion, we justify the employment of the bivariate GARCH model on the basis that it is computationally easier and it best 'fits' the variances.

In Chapter three we will use a dynamic specification of the SIM based on the bivariate GARCH in mean model to investigate whether such volatility modelling, to assess securities' risk, can be used to capture the time variation not only in the total risk of a security's return but also in its systematic and unsystematic components. The objective here is to construct a time-varying correlation matrix which can be used to estimate portfolio's market risk conditional on the information set available on the previous day; that is, the computing of a beta measure of security risk.

This chapter has three additional aims. First, it investigates whether the volatility of a portfolio based on time-varying variances and covariances estimated as above provides a better input for calculating the portfolio VaR than alternative simpler methods such as the ES. Next, it seeks to model as time-varying the systematic and unsystematic components of risk for portfolios diversified across a number of major stock markets all over the world. Finally, it applies a non-linear statistical technique for analysing the volatility components of returns.

Chapter four considers a "simplified" approach to the conditional estimation of the VaR by estimating portfolio volatility conditional on its historical returns. This method has an appealing property; it estimates current (and past) portfolio volatility without the need to calculate the asset components' variance-covariance matrix and still takes fully into consideration all current and past pairwise correlations. This approach provides a fast, and flexible way of modelling volatility. Theoretically, it will provide an accurate VaR measure for portfolios of any size. Furthermore, this chapter will analyse the issue of the worst case scenario (WCS). Worst case scenario goes beyond VaR since it tells us if the worst happens what the likely losses are. We estimate the portfolio's WCS by applying bootstrapping simulation on the portfolio scaled innovations. Hence, by using the empirical distribution (which contain any fat tails and other non-normalities) of the past portfolio returns we take into account, in our WCS estimation, the catastrophic risk. Finally we analyse the impact that the aggregated correlation instability has on the portfolio risk.

Chapter five investigates market interdependencies amongst national markets. Given the integration of international markets, volatility originating from any country will be transmitted (to an extent) to others. The way volatility is transmitted may be not instantaneous but will occur with a lag. By studying the volatility transmission mechanisms, managers may improve the way they manage their portfolio risk, in particular if the portfolio contains non-linear positions (*e.g.* options). Unlike previous studies in this area, we model market dependencies in the second moments of national market returns as opposed to their first moments to monitor the systematic risk as opposed to overall risk. Further, interdependence is explored through changes in the market betas with the world factor where the MSCI is used as a proxy for the world market. Market integration implies that excess volatility is transmitted from one national market to another. The most notorious example of volatility transmission across national markets is the October 1987 Crash. In this investigation, the GARCH methodology is employed to model national markets' time-varying volatility. This enables us to evaluate the volatility transmission mechanism and market linkages in a dynamic context. The betas generated through the use of GARCH are modelled using variance decomposition and impulse response function in a vector autoregression framework to analyse how "news" is transmitted across national indices. Further, this analysis provides us with a deeper understanding of the percentage error variance in a national market that is attributable to innovations in each of the other markets. It also aims to reveal the length of time that elapses before market volatilities return to their long run levels following a "news" shock. Our analysis goes further by allowing for asynchronous transmission of volatility between markets and secondly, by addressing volatility spillovers due to systematic volatility as opposed to total volatility.

Finally, chapter six summarises and concludes the thesis. The findings of all empirical chapters will be reviewed and a conclusion will be drawn on the basis of these results. This will ultimately be followed by a discussion of the implications of the thesis for future research and for practitioners in the market.

Chapter 1

Portfolio Values-at-Risk

The deregulation and lifting of capital movement restrictions that took place in the 1980's (i.e. Big-Bang) and has provided professional investors particularly in many industrialised countries, with the opportunity to diversify their portfolios globally. The measurement and management of risk on a portfolio diversified across international equities motivates the empirical investigation of Value-at-Risk or VaR techniques which is the focus in this thesis. VaR is a popular technique currently widely used to measure a portfolio's market risk. It determines the minimum amount a portfolio's value could decrease over a given period of time with a given probability as a result of changes in the market prices or rates of return. The concept of VaR is very appealing because it is consistent with the objective of shareholder wealth maximisation, a central tenet in Markowitz (1952, 1956) portfolio theory. VaR quantifies the potential loss in shareholder wealth with a given probability over a specific period of time. Using VaR calculations an institution can judge how it should reallocate the assets in its portfolio to achieve the risk level it desires.

This chapter introduces the concept of VaR and its relationship to portfolio theory. It first describes the distributional assumptions upon which modern portfolio theory is based. Thereafter it examines the consequences for portfolio efficiency and riskiness that follow from the violation of these assumptions. Finally, it discusses the issues surrounding the distribution of speculative price changes where it examines the form of the distribution and the non-stationary nature of price data. The motivation of this discussion is to justify the use of conditional volatility models to be examined in chapter two and the implementation of one of these, the Generalised Autoregressive Conditional Heteroskedastic (GARCH) in the subsequent empirical chapters.

1.1 Portfolio Values at Risk

Using a probability level of v and a holding period of t days, the portfolio's market risk, or Value-at-Risk (VaR), is the loss that is expected to be exceeded during the next t day holding period with probability v[1]. VaR is rapidly becoming a popular management tool given that it summarises in a single statistical measure all possible portfolio losses over a short period of time due to "normal" market movements. Losses greater than the VaR are suffered only with a specified small probability. The level of probability that this loss in portfolio value has to be incurred is chosen to fit the investor's particular circumstances. The length of the time over

[1] For risk management purposes we are only concerned with potential losses, not gains. However, as we will see later the VaR can also be extended to estimating the likely gains.

which VaR is measured is set equal to the time period needed to close or neutralise all portfolio positions[2].

The method for calculating VaR depends not only on the horizon chosen but also on the kinds of assets in the portfolio. The method we consider here is suitable for portfolios consisting of stocks, bonds and currencies over a short horizon. If the portfolio contains non-linear positions (derivatives) either these positions need to be linearised, *e.g.* by multiplying their delta by the volatility of the underlying asset, or a different VaR method may be employed, *e.g.* see Barone-Adesi *et al.* (1997).

In statistical terms, the VaR is the lower sided confidence interval for the change in portfolio value over a specified time horizon. Thus, given a probability level v and a time horizon t, the portfolio VaR is :

$$Prob\left\{ \mathrm{VaR}_T^{t,(1-v)} < P_{T+t} - P_T \right\} \leq (1-v) \qquad (1.1.a)$$

where P is the value of the portfolio holdings at time T. The portfolio's expected daily loss, also known as Daily-Earnings-at-Risk (DEaR), describes the magnitude of the daily losses on the portfolio for a given probability. For example the 1% probability DEaR for a portfolio with value P will be

$$\mathrm{DEaR} = P\,\sigma_p\, 2.33 \qquad (1.1.b)$$

[2] In the 1996 amendment of the 1995 proposal the BIS required banks to compute the VaR on a daily basis with a horizon of 10 trading days and set the confidence level to 99%.

where σ_p is the daily volatility (standard deviation) of portfolios returns and 2.33 is the number of standard deviations which gives the one-tailed probability of 99%.

VaR uses standard statistical methods to look at portfolio risk over time. The most popular method today is the "variance-covariance" VaR approach. It is so named because it is derived from the variance-covariance of the relevant underlying market rates of return. If W denotes the Nx1 vector of current portfolio weights and Ω is the variance-covariance matrix of their returns, the portfolio variance σ_p^2, is given by:

$$\sigma_p^2 = W^T \Omega W \tag{1.2}$$

Knowledge of the variance-covariance matrix of these variables for a given period of time implies knowledge of the variance or standard deviation of the portfolio over this period.

1.1.1 The Link Between VaR and Modern Portfolio Theory

The above approach to estimating a portfolio risk is rooted historically in the pioneering work of Markowitz (1952) which laid down the cornerstone of modern portfolio theory. Markowitz was the first to show that the risk in a portfolio of securities is equal to the weighted second moments of the multivariate distribution of their returns[3]. In Markowitz's framework risk can be seen as the uncertainty that surrounds the future value of a portfolio; as given by the spread of the probability density function of the portfolio around its expected value.

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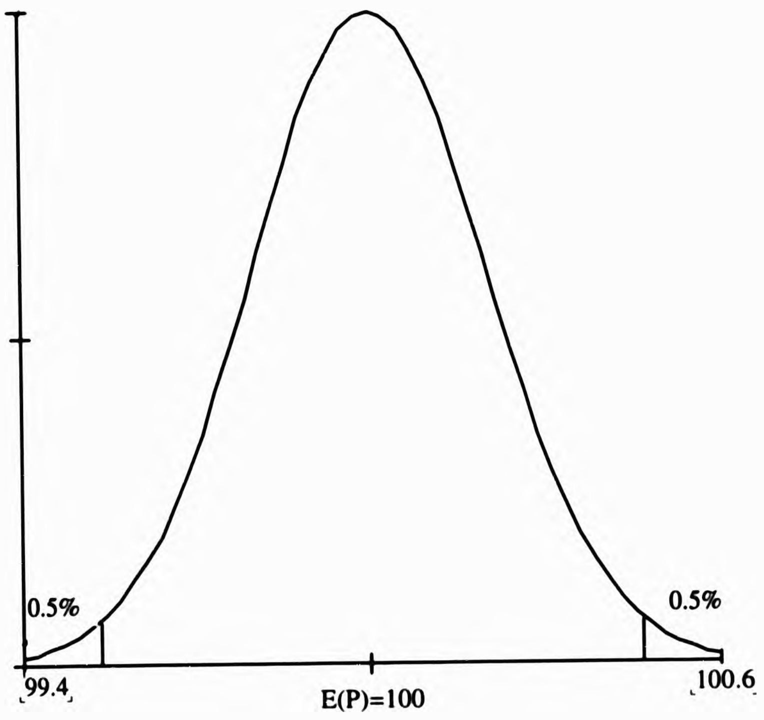
[3] We use the term risk to refer to both variance and standard deviation of returns.

This is shown in figure 1.1 by the area on the far left and right of the expected value of the portfolio. Portfolios with high expected returns are attractive but, assuming market efficiency, are associated with higher risk.

Fig 1.1 p.d.f. of Portfolio Exp. Value



There is a direct link between the way DEaR and VaR measure risk and the one defined in modern portfolio theory by Markowitz (1952, 1956). In the Markowitz portfolio theory, the risk of an asset mix is seen as the variability of actual return around its expected value, which is the centre of the distribution, at the end of the investment period. The area on the far left in figure 1.1 tells us that there is a 0.5% probability that at the end of the investment horizon, the portfolio with standard deviation of

20% faces a loss of 0.46% or more which also represents the VaR of an equal length holding period[4] [5].

Hence, this can be interpreted in a way that allows us to measure the number of days that a similar loss may take place over the entire investment period. An investor who holds a portfolio with a distribution similar to that in figure 1.1 can expect to make a loss equal to or greater than the VaR value once out of two hundred days. That is if the VaR measure is accurate, losses greater than the VaR value should occur on average 0.5% of the time. This is shown in figure 1.2 where a loss greater than 2.33 standard deviations occurs 27 times in the 2609 days interval.

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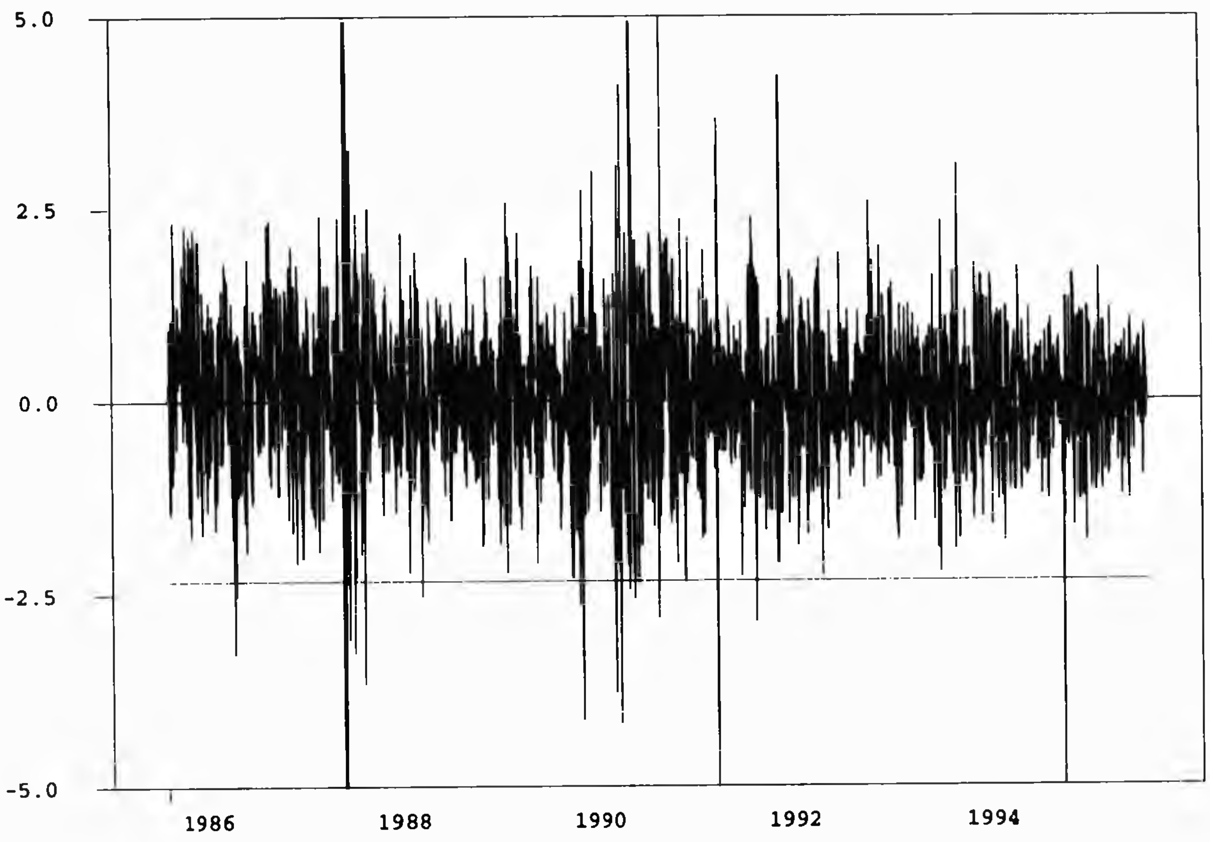
[4] Although upside risk is welcome by any investor, Markowitz treats upside and downside risk in the same manner. For risk management purposes, we are only concerned with potential losses, not gains. Given that on symmetrically distributed returns the DEaR is half the size of the risk in Markowitz definition, we will regard these two forms of risk as equivalent. Furthermore, DEaR and VaR are equally valid for calculating the potential for gain.

[5] Of course this is valid only under the same assumptions on which Markowitz theory is based. These assumptions and their implications will be examined in the next paragraphs of this chapter.

Fig 1.2 Portfolio Daily Profits & Losses



There are additional common properties between VaR and modern portfolio theory. VaR summarises the amount of risk embedded in a portfolio as a single number and, like modern portfolio theory, takes into account the correlation between different types of risk. Since VaR complements itself with the associated likelihood that these losses will materialise, it gives a more objective assessment of the portfolio's risk exposure. Furthermore, the VaR method enables senior management of enterprises to assess the magnitude of the risks involved since the risk shown as the size of potential monetary losses can easily be compared with the portfolio's expected return and with the firm's own capital.

1.2 Distributional Properties of Security Returns and VaR

1.2.1 The i.i.d. Assumptions

To estimate the portfolio risk σ_p, Markowitz suggested using historical (unconditional) variances and covariances. To do so, he made two important assumptions about the distributional properties of securities returns. These are: *security rates of return are independently* and *identically distributed* (i.i.d.). These two properties form the basis of what is known as the "random walk" model of efficient markets[6] [7]. All asset pricing theories, *i.e.* the Capital Asset Pricing Model (CAPM) and the Black-Scholes option pricing formula, are based upon the assumption that markets are efficient. The independence property implies that a series of price changes has no memory: past history cannot be used to increase expected profits. The second property, identically distributed returns, states that the rate of return must conform to some, fixed, probability distribution. The theory does not specify what the shape of the distribution should be. Fama (1965) argued that any distribution which correctly characterises the process that generates the rate of returns is

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[6] The credit for the random walk model is given to Bachelier (1900). He, however, also assumed that price changes are normally distributed. Fama (1965) removed the assumption of normal distributions. Later, Granger and Morgenstein (1970, pp 71-73) defined the "random walk" as a constant expected price change and zero correlation between the price changes for any two different days; hence they removed the identically distributed assumption.

[7] The "random walk" model forms the core of Fama's (1970) definition of efficient markets.

consistent with the theory. The theory however implies that the statistical moments of the distribution should remain constant[8].

When returns are i.i.d. and the moments of the distribution are known, any inferences made about potential portfolio losses will be accurate and unchanging over time. Under these circumstances, the historical variance-covariance approach can be used to estimate portfolio VaR. This is based on the assumption that the changes in the value of the portfolio are on average random and their frequency distribution can be estimated using a Gaussian statistical curve. Normality simplifies VaR calculations because all percentiles are assumed to be known as multiples of the standard deviation. Thus, to calculate the VaR we only need to know one estimate, the standard deviation of the portfolio's change in value over the holding period. Stationarity implies that the probability of occurrence of a specified loss is the same for each day. Independence implies that the size of price movement in one period will not influence the movement of any successive prices. These assumptions simplify the VaR calculation for any holding period and probability. The longer period's VaR can be found by multiplying shorter, *i.e.* daily, horizon standard deviations by the square root of the number of days t in that period.

$$VaR_{T,T+t} = DEaR_T\sqrt{t} \tag{1.3}$$

[8] The series of returns (and in general any time series) with constant statistical moments over different periods is known as stationary. Throughout this thesis under the term "stationarity", will refer to the "weak" form of stationarity, where the means and variances across time are constant. For a discussion see Hamilton (1994), p 45.

However, the convenience of these assumptions must be offset against the voluminous empirical evidence which has found that the tails of the distribution of daily changes in speculative prices exceed those of the Gaussian[9]. The presence of fat tails could be explained by non-stationary distributions. Studies investigating changes in asset means have been inconclusive. Others, as we will see later, found indisputable evidence that a security's risk not only changes over time but also follows an autoregressive process. This latter implies a conditional dependency in the distribution of returns. If that is the case, using a constant volatility method to calculate VaR could be very misleading since the probability of a large loss is not equal across different days. During days with higher volatility we would expect larger than usual losses. Furthermore if there is a tendency for large price changes to be followed by more large changes, known as volatility clustering, the portfolio VaR for that period will be larger than under usual market conditions[10].

Today there is a large body of evidence which suggests that speculative price changes are fat tailed distributed returns with changing conditional moments. This will undermine the ability of VaR to quantify portfolio risk where unconditional estimates of means, variances and covariances

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[9] The presence of fat tails found on the distribution of daily and weekly data is commonly accepted today but for monthly data there is weaker evidence, see Blattberg and Gonedes (1974). In the letter case, when an institution is interested in calculating VaR over a long horizon, the time-varying volatility may not be an important issue.

[10] For some types of investments two or three consecutive adverse price changes may be sufficient to ruin the investor, i.e. contingent claims and investments with leverage. In a leveraged portfolio investors borrow at a fixed rate and invest in a risky asset, usually equity. Leverage increases positive expected return because leverage magnifies volatility. The larger is the leverage factor, the higher the gains will be in case of positive returns. But large adverse returns increase the downside risk and chances of a disaster. Thus the DEaR and VaR are far more important in the management of leveraged investments.

are employed. Excess kurtosis will cause losses greater than VaR to occur more frequently and be more extreme than predicted by the Gaussian distribution. The variability of second moments itself causes an additional source of uncertainty. Under these conditions it will be inappropriate to use historical variances and covariances to estimate VaR.

1.3 Modelling Volatility and Correlation as Time-Varying

1.3.1 Historical vs. Conditional Volatility Models

The problem of fat tailed returns could possibly be alleviated by using a leptokurtotic distribution, although sometimes smoothing the data-set by using the normal curve may improve the prediction if the extreme observations are due to sampling error. When, however, the use of any (constant) known distribution is unsuitable, *i.e.* not all assets returns can be described by the same distribution, the use of simulation methods based on sample values of prices to build the distribution of portfolio returns may be more appropriate to calculate VaR. Monte Carlo simulation uses unconditional variances and covariances of prices to generate a series of sample paths through price span.

Nevertheless, multivariate simulation methods which use historical variances and covariances to replicate asset returns will still provide inefficient portfolio VaR estimates when the second moments of asset returns are time-varying. The time changes of variance and covariances may be captured by using conditional time series models. This class of statistical models makes more effective use of the information set available at time t to estimate the means and variances as time-varying. Multivariate conditional models such as GARCH (Generalised Autoregressive Conditional Heteroskedastic), the Harvey (1991) CAPM and state space techniques have been successfully applied to capture time-varying covariances and hence betas and correlations. The GARCH models of Engle (1982) and Bollerslev (1986) have been designed to remove the systematically changing variance from the data which could account for most of the leptokurtosis observed in the unconditional distribution. Among others Bollerslev *et al.* (1988), Ng (1988) and Bodurtha and Mark (1991), modelled the conditional covariances as a function of past conditional covariances to test the CAPM.

Harvey (1989) uses Hansen's (1982) generalised method of moments (GMM) to test a version of the CAPM that allows for both time-varying expected returns and conditional covariances and found that "..conditional covariances do change through time" (pp315). Harvey's parameterisation, in contrast to multivariate GARCH studies, does not assume a functional form, *e.g.* autoregressive, that the second moments may need to follow.

The above methods have lower forecast error variance and allow for the unconditional distribution of the data to exhibit excess kurtosis and thus better describe the empirical distribution of financial data. They do not, however, come without cost since they are non-linear and computationally intensive. Furthermore, the number of unknown coefficients to be estimated increases with the square of the number of series included in the system. As a result, empirical studies that jointly estimate the conditional momen.s, have to restrict the number of assets to not more than half a dozen at a time[11]. The fact is that in the mean-variance port-

[11] Bollerslev et al. (1988) restricted their multivariate GARCH model to three assets while Harvey (1991) used seven variables to estimate the joint second moments.

folio approach to asset allocation and risk measurement, the number of variances and covariances to be estimated increases with the square of the number of available assets, making the use of conditional models problematic. Moreover, risk managers would be mostly interested in the early identification of shifts in the risk parameters, but the sheer number of portfolio parameters (variances and covariances) to be monitored makes the task of distinguishing parameter shifts from sampling errors fraught with difficulties.

1.3.2 Specifying the Conditional Probability Distribution

As we have seen, two statistical elements are critically important in VaR analysis. The estimates of volatility over the horizon and the probability distribution that describes the portfolio (residual) returns. Time series models are helpful to model conditionally the moments of the distribution of portfolio returns and remove (most) non-linearities from the data. They usually assume that returns are distributed conditionally as normal but its moments are allowed to change over different periods; hence the unconditional distribution may be non-normal and exhibit the expected stylised facts that characterise security returns, mainly excess kurtosis and volatility clusters. The aim of each time series model is to leave any residual returns as i.i.d.

Assuming normality in the conditional distribution is advantageous in the model estimation but may not represent well the distributional properties of the data. In fact, the specification of the probability distribution that the portfolio return innovations are following represents the major challenge in the calculation of VaR. It is known that financial asset price changes are exposed to uncommon events which cause unusually large losses (and gains). Furthermore, it is difficult to find such a conditional time series model which leaves (scaled) residuals with a known (conditional) density that is able to describe uncommon but possible losses. Therefore, assuming (conditional) normality in calculating VaR may lead to the underprediction of tail events.

It is, however, possible to model the density of portfolio returns in a more general form, *e.g.* as a *Student*-t or as a semi-non-parametric distribution, see Gallant *et al.* (1991). A more general specification may be better suited to the financial data since it will allow the conditional return innovations to deviate further from normality. However, specifying the density of the asset's conditional (residual) returns in a more general form is subject to additional problems. These include loss of forecastability, possible overfitting and increased computational difficulties.

1.4 Sizeable Deficiencies in Computing the Variance-Covariance Matrix Conditionally

1.4.1 Partitioning the Variance-Covariance Matrix

The estimation of portfolio variance in (1.2) requires the knowledge of the variance covariance matrix Ω. The number of elements of this matrix, however, increases with the square of the number of assets in the portfolio. When conditional time series models are employed to estimate jointly the statistical moments of even a moderate number of assets the number of unknown parameters exceeds above the point beyond which the model computation is not feasible. Hence, the number of different types of assets in the portfolio restrains the applicability of conditional time series models.

One possibility is first to partition this matrix into $\frac{(N-1)N}{2}$ off-diagonal elements and then to capture the joint dynamics of the second moments for each possible pair-wise combination of investment holdings. The volatility of current investment holdings is then computed as in (1.2). This, although computationally expensive, is a feasible solution since it requires the estimation of $\frac{(N-1)N}{2}$ bivariate systems. The problem this simplification face stem from the way the variance-covariance matrix is partitioned. That is because unless certain preconditions are satisfied there is no guarantee that the resulting variance-covariance matrix comes from a $N{\times}N$ multivariate distribution, *e.g.* the absolute value of the determinant may be greater than one. Hence the portfolio variance estimates are very likely to be biased.

1.4.2 Using the SIM to Build a Conditional Variance-Covariance Matrix

An alternative solution to this problem can be found by applying the Single Index Model (SIM) of Sharpe (1963) to the conditional time series content. Sharpe (1963) proposed a simplified method to solve the problem of optimising a portfolio, which was mainly aimed at cutting down the computation in the variance-covariance matrix Ω when a portfolio was to be diversified across a wide (large) set of assets. Sharpe (1963), based on a footnote of Markowitz's (1959) monograph, proposed the idea that in each market, a single factor (index) accounts for a greater proportion of the variability of security returns than any other factor. One such factor could be the market index itself.

Let Y_i be the return on stock i, and let Y_m, be the return on the market index. The single index model assumes that we can write

$$Y_i = \alpha_i + \beta_i Y_m + \varepsilon_i \qquad \varepsilon\text{-}N(0,\sigma_\varepsilon^2) \qquad (1.4)$$

Formally, the SIM assumes a one factor return generating process. In such a process, the variability of all stock returns can be completely captured by the market index plus firm specific events. The responsiveness of each security to the market is measured by its *beta* coefficient (β) which is the slope in the above linear equation.

The SIM makes the assumption that the idiosyncratic returns, ε_i, are independent across different firms, $E(\varepsilon_i\varepsilon_j)=0$. This leads to a significant simplification of the covariance matrix of returns. The off-diagonal terms of the covariance matrix will take the simple form:

$$Cov(Y_i,Y_j) = \beta_i\beta_j var(Y_m) \qquad (1.5)$$

where Y_i, Y_j are the returns of series and β_i, β_j the beta coefficients of stock i and j respectively, Y_m is series of returns of the common index. The above covariance expression brings a considerable saving in the number of terms need to calculate Ω. The computational advantages of the SIM are expanded in many areas in finance where a variance-covariance matrix is indispensable, among others in the computation of the VaR. As we will see in chapter two, the SIM can be extended to multivariate non-linear time series models to overcome limitations in computing large conditional variance-covariance matrices.

1.5 Time-Varying Volatility and Inter-Market Linkage

By the mid 1980s major stock markets became increasingly deregulated while investors began to make more use of the relaxation of the old barriers to capital movement. To raise equity, companies started to look at different markets, outside their country of origin. The number of multiple listed companies has expanded over the years. Multinational firms, such as Royal Dutch-Shell and Ciba-Geigy are traded on more than a dozen markets[12].

The consequence of the deregulation and relaxation of capital movements was a growing international integration of financial markets. Investors began studying market cycles on a number of markets and were prepared to move their capital when they showed good buying opportunities. Integration or interdependency implies that a change in the prices in one market will affect, to some extent, prices in all other markets. This increasing integration has also been the cause of the transfer of disturbances from one market to others. For example during the 1987 crash there was an "almost" simultaneous sharp drop in equity prices around the world. Similar disturbances are observed each time there is a big drop in prices in the US market.

The extent to which a price movement in one market will influence prices on other markets depends on the degree of their integration. In the Markowitz's portfolio model price changes across different securities are linked through the second moments of their joint distribution. Similarly, the SIM allows security price changes to be linked through their common co-movement with the index. Common measures of interde-

[12] Arbitrage among markets guarantees that the share value of multiple listing companies, after adjustments for exchange rate and transactions costs have been made, is the same across all markets.

pendence are the correlation and beta coefficients against the world index. Historical correlations and betas are easy to compute but restrict the degree of interdependence to be constant over different time periods. But as we have seen there is a large body of empirical evidence that finds conditional variances, covariances and betas of domestic securities do change over time. Among others Hamao *et al.* (1990) Chan *et al.* (1991) and Karolyi (1995) provide evidence that the conditional volatility on international markets changes over time.

Therefore, historical estimates will not reflect the actual interdependence in each period. Any changes in the joint second moments need to be taken into account when searching for market interlinks. We noticed for example that during the crash of 1987 along with other periods of turmoil, such as the Iraqi invasion of Kuwait, equity prices around the world collapsed together. Furthermore, these changes in volatilities may not be contemporaneous. Studying eventual leads and lags in the way the betas of national markets change may open the way to understanding how volatility is transmitted from one market to others. This is the issue to be investigated in chapter five.

1.6 The Empirical Distribution of Speculative Prices Changes

As we have seen, the distributional properties of the changes in the prices of financial assets are critical in estimating VaR. Normality simplifies calculations but the VaR analysis requires precision, not simplicity. Precision is waived if the data are not normally distributed. The random walk theory has stimulated a number of researchers, (Kendall (1953), Osborne (1959), Mandelbrot (1963), Fama (1965), Officer (1972), Praetz (1972), Blattberg and Gonedes (1974)), to analyse the distributional properties of speculative prices by investigating the randomness and the form of the distribution of successive changes in speculative prices[13].

1.6.1 Independence

The investigation of independence has concentrated on testing for serial correlation in successive price changes. The general conclusion is that successive price changes are autocorrelated but are too weak to be of economic significance. This leads most studies to accept the randomness hypothesis. Fama (1965) examined thirty stocks and found small serial correlation coefficients with large standard errors. In only a few cases did the coefficient exceed its standard error by a factor of two or more but their significance may be overstated because the distribution of the data he examined was non-Gaussian. He concluded that "..dependency of such a small order of magnitude is, from a practical point of view, probably unimportant for both the statistician and the investor" (p. 70).

Given that the distribution of returns may be non-normal, lack of autocorrelation does not imply serial independence. A number of studies found that returns are generated by non-linear processes which allow successive observations to be linked through their second moments. This phenomenon was initially noticed by Mandelbrot (1963), who observed that large price changes tend to be followed by large price changes in either direction. Nevertheless, this observation was not explored until recent developments in econometrics made powerful techniques available to model these dependencies in security returns. For instance, the

[13] The form of the early studies was mostly technical and the first complete economic rationale for the random walk model was given by Samuelson (1965).

GARCH family of Engle (1982) and Bollerslev (1986) and the stochastic volatility models of Mellino and Turnbull (1990) and Harvey *et al.* (1994).

1.6.2 The Form of the Distribution.

In finance it is natural to assume normality in conditional predictions and unconditional prediction for risk analysis. The normal (or Gaussian) distribution has several advantages. All moments of positive order exist. It is mathematically tractable where results can be obtained in a closed form. It is completely characterised by its first two moments; thus, establishing the link with mean-variance analysis, Markowitz (1952, 1956). Indeed it was Markowitz who assumed that the rate of returns follows a multivariate normal distribution. He did this because it was convenient at that time for computing the optimal portfolio[14]. Since the normal distribution remains stable under addition any portfolio invested in stationary and jointly normally distributed assets will also be normally distributed with stable moments[15]. Then two parameters, the mean and variance, are sufficient to describe it.

The Gaussian distribution, however, has several disadvantages which are especially relevant in VaR analysis. As the tails of this distribution decay exponentially towards the axis, extreme realisations are very unlikely. This seems to argue against the empirical evidence for the distribution of asset returns. Indeed, when researchers began to examine the form of the (unconditional) distribution of speculative price changes they found more kurtosis (fatter tails) than was predicted by the Gaussian distribu-

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[14] The optimal asset mix problem can easily be defined and solved if returns are multivariately normal.

[15] Under the central limit theorem if X1 and X2 are normally distributed, so is X=aX1 + bX2.

tion[16]. Leptokurtotic distributed returns were first reported by Kendal (1953) who analysed weekly price changes of British stocks. Moore (1962) plotted on normal probability paper the weekly log returns of eight NYSE common stocks and found that there were many values at the extremes of the histograms. However, these authors elected to drop the outliers from their analysis to allow returns to follow a Gaussian distribution. The credit for being the first to observe that the rates of change in asset prices are not well described by the normal distribution goes to Mandelbrot (1963). Mandelbrot noticed that the outliers observed in the empirical distribution of stock and commodity returns are numerous and tests carried out on the "trimmed" data lost power. Mandelbrot suggested that other probabilistic distributions can be used that can more properly represent both the main body of the data and the observed fat tails. He proposed that returns follow a stable symmetric Paretian distribution with infinite variance.

Fama (1965) found that the extreme tails of the distribution in all thirty stocks that made up the Dow Jones Industrial Average contained more observations than would be expected under a Gaussian distribution. In order to explain this departure from normality Fama tested two variations of the Gaussian model. First he assumed that the distribution of daily within the week returns had a lower variance than that followed by weekend and bank holiday returns, but both distributions are Gaussian. Nonetheless in his empirical tests Fama failed to support this hypothesis. The alternative hypothesis he examined is that the distribution of returns

[16] Excess kurtosis has been observed in daily (Mandelbrot (1963), Fama (1965)) and weekly (Kendal (1953)) returns. However, there is evidence that the distribution of monthly data conforms well to the normality hypothesis, Blattberg and Gonedes (1974). Fama (1976) argued that the normal distribution provides a good approximation for monthly returns.

is normal but its moments change across time. Shifts in the variances and/or means can explain the excess kurtosis in the empirical distribution of the security returns. But he restricted the empirical analysis to testing changes in means only. He rejected this hypothesis on all five stocks that he tested[17]. Fama (1965) came to the conclusion that "..the Mandelbrot hypothesis fits the data better than the Gaussian hypothesis", (p 90).

The Paretian distribution, is described by four parameters. One of them, the characteristic exponent, known also as the "peakdness" parameter, determines the total probability contained in the extreme tails. This parameter, which measures the tail of the distribution, is bounded by the interval 0.0 and 2.0 and the lower the value is, the fatter are the tails of the distribution. The normal distribution is a special case of the stable symmetric Paretian with peakedness parameter of 2.0. This type of distribution is stable under addition: when observations with the same characteristic exponent are summed the characteristic exponent values of this distribution does not change. A problem however with this type of distribution is that the variance and other higher order moments are not defined except for the special case of the normal. However, Fama and Roll (1968) show through the use of simulation that even with an characteristic exponent of less than 2.0, the parameter for the central tendency can be well approximated by the sample mean while the dispersion can also be redefined and can exist even if the characteristic exponent is less than 2.0. Fama (1971) shows that if all asset returns follow a stable and symmetric Paretian distribution, the traditional CAPM holds.

[17] The analysis has been restricted to only the five stocks which "..seemed to show changes in trend that persisted for rather long periods of time..", p. 58.

Other studies that have examined the distribution of stock returns, *e.g.* Officer (1972), Blattberg and Gonedes (1974) have found that the characteristic exponent rises as individual daily stock returns are added to the portfolio. This indicates that daily equity returns do not follow a stable distribution including a stable normal. Praetz (1972) and Blattberg and Gonedes (1974) suggested that a *Student's -t* distribution is more appropriate in describing the daily returns on both stock price indices and individual stock prices. The *Student -t* distribution is described by three parameters, central tendency, dispersion and degrees of freedom. As long as the degrees of freedom are less than infinity the *Student -t* distribution has fatter tails than the normal and so is consistent with the data[18]. Furthermore, the *Student -t* distribution fits well for daily or weekly data but, as the interval length over which returns are measured increases, the observed distribution converges to a normal. This has implications at a theoretical level since investors who have short horizons should use CAPM based on the *Student -t* distribution. To date no such a model exists, (Hagerman, 1978, p 1215).

Others suggested that the speculative price changes follow a mixture of normal distributions. Thus, they are described by a combination of normal distributions possessing different variances and possibly different means. Clark (1973) examined a normal distribution which allowed the variance to follow a lognormal distribution and Kon (1984) examined a discrete mixture of normal distributions. Hsu *et al.* (1974) considered a normal process with random jumps in the variance that occur at discrete points in time. Bookstaber and McDonald (1987) proposed a more general distribution, the GB2, to describe security returns. This distribution

[18] The Student -t is not a stable distribution because, as observations are added, the degrees of freedom (d.f.) change and so does the distribution. However, d.f. do not change in the modelling of data.

is extremely flexible and includes both the lognormal and the *Student -t* distributions but its generality comes at the cost of four parameters. Friedman and Vandersteel (1982) presented evidence more consistent with finite variance data that are non-stationary.

1.6.3 Non-Stationarity

Fama (1965) investigated the possibility that serial dependence in price changes may account for the observed fat tails in the empirical distributions of stock returns. Nevertheless, the serial correlation and runs tests he conducted ruled out dependency and suggested that the cause for the fat tailed empirical distributions must be found somewhere else. One such possibility, Fama argued, is non-stationarity. Changes in means or variances of returns across time will cause excess kurtosis in returns distribution. Fama himself failed to provide any evidence to support this hypothesis but he only investigated changes in means. Other studies have investigated the possibility that the second moments are time-varying as well. Mandelbrot (1963) was perhaps the first to recognise that asset volatility is not only constant but also autoregressive. Joyce and Vogel (1970) have noted that variance estimation is sensitive to the period over which the data are selected as well as to their periodicity. Bones *et al.* (1974) investigated non-stationarity of both means and variances in weekly price changes of 33 electric utility companies. Their evidence is inconclusive for the means but the evidence with respect to the variances supports the time-varying hypothesis.

Excess kurtosis in the empirical distribution of financial returns can be caused by the conditional dependency in the second moments. If returns have some kind of conditional dependency, their unconditional distribution will always have fatter tails than the conditional one. With recent advances in econometrics, more powerful tests against heteroskedasticity have been developed and a number of models have been proposed to capture the dynamics governing the second moments of financial asset returns. Probably, the most popular of these techniques belongs to the family of generalised autoregressive conditional heteroskedasticity (GARCH) models of Engle (1982) and Bollerslev (1986). The GARCH methodology estimates current period's, variances as deterministic functions of the previous period's squared innovations along with the (previous period's) conditional variances. A GARCH model is intended to capture the clusters in volatility of the financial data rather than to model variances in an economic sense. Among others, Engle and Bollerslev (1986), Bollerslev (1986) and Engle *et al.* (1987)[19] employed GARCH models to capture the changes in the variances of security returns. Pagan and Schwert (1990) used parametric (GARCH) and non parametric (Kernel and Fourier) methods to capture the volatility changes for the US stock data from 1834-1925. Further supportive evidence for time-varying second moments is reported in the numerous studies which examined the informational content and predictive power of volatility implied by financial options[20].

Other authors have investigated the causes of the changes in the volatility of security returns. Beaver (1968) and Merton (1976) argue that changes in the variance are caused by the arrival of new information. Diebold and Nerlove (1989) who found autoregressive volatility (clusters) in weekly spot foreign exchange data from 1973 until 1985, argued that

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[19] A good review of studies which applied GARCH methods to model the changes in financial time series variances can be found in Bollerslev et al. (1992).

[20] See for example Latane' and Rendleman (1976), Chiras and Manaster (1978), Day and Lewis (1992) and Jorion (1995).

these volatility clusters are due to a serially correlated news arrival process which sometimes is of dubious relevance or significance. Christie (1982) shows that one type of news that influences the level of equity volatility is the change in the interest rate. However, his theoretical explanation could not determine ex-ante if the two variables are positively or negatively correlated. Christie then carried out an empirical investigation and found firm evidence that the riskless interest rate has a strong positive effect on volatility. This is consistent with Fama and Schwert (1977) who reported earlier that on average, the value of the firm is inversely related to the expected (interest rate) and unexpected inflation. Christie also quoted a number of other variables, such as dividends, that can have an impact on volatility.

1.7 Conclusions

This chapter has highlighted the inappropriateness of the historical variances and covariances in the portfolio VaR analysis. VaR employing historical portfolio risk estimates are based on the implicit assumption that returns are normally distributed with constant variances and covariances across time. However, the stylised facts on speculative prices changes point to the contrary. In the subsequent chapters, we will investigate the appropriateness of conditional volatility models in the VaR analysis. In chapter three we will empirically test the portent of using bivariate GARCH models, in conjunction with the SIM of Sharpe (1963), in the estimation of portfolio VaR. This will be followed by the introduction of a simplified approach through the use of historical returns. We will argue that the latter approach is superior to the traditional ones because it utilises more efficiently all available information regarding the dynamics that govern investment holdings. Next, we will examine the methodological issues involved in this thesis.

Appendix 1A

A1.1 The Implications of Non i.i.d. Returns.

A1.1.1 General Implications

In the mean variance framework and in pricing models like CAPM and contingent assets the distribution of returns of the underlying asset is assumed to be normal. But a number of studies, quoted earlier, provided strong evidence that the tails in the empirical distributions of security returns exceed those of the Gaussian. The investor who holds such a fat tailed asset is exposed to a greater risk than implied by a Gaussian distribution. In statistical analysis the estimates of variances and covariances become inefficient and standard t and F tests of significance are unreliable. Diebold (1988) shows that if the residuals are heteroskedastic the Ljung-Box test statistic, a commonly used test for serial correlation, tends to yield biased results because standard errors are likely to be underestimated. In some cases these tests are used to validate the modelling procedures applied in market efficiency testing[1].

[1] The fact that returns are not i.i.d., and therefore potentially predictable, does not necessarily contradict market efficiency.

The dependency of returns will affect, among, other things, the portfolio mix choice and the price of contingent claims. When returns are serially correlated, variances estimated from longer-interval returns may not be proportional to variances estimated from shorter-interval returns[2]. When returns are positively (negatively) serially correlated the variance should grow at an increasing (decreasing) rate as the return interval increases. Hence, variances that increase disproportionately with time will affect portfolio selection decisions. For example when portfolio returns are positively serially correlated (portfolio variance increases at an increasing rate with time) then more conservative portfolios will be chosen for longer investment horizons. By contrast, when portfolio returns are negatively correlated, more aggressive portfolios will be chosen for longer investment horizons as their variance will tend to grow at a decreasing rate over time.

For investments like options, the future volatility of the underlying asset is of primary importance in pricing, e.g. Black-Scholes (1973), and a wrong assessment can have serious consequences for the investor. If for example returns are serially dependent and we estimate annual return variances, by extrapolating weekly return variances we will overestimate the price of the long term option. Investors possessing a more precise estimate of an asset's current or future risk level may use that information in their trading tactics. The market sooner or later will re-evaluate the security's risk level and will adjust its price leaving some investors with excess profits.

The latter states that forecast errors of returns are not predictable; see Hsieh (1991).

[2] Under the central-limit theorem the variances of independent and normally distributed returns will be proportional to the respective time interval. If σ^2 is the variance of monthly returns the variance of annual returns will be $12\sigma^2$.

A1.1.2 Errors in *Beta* Estimation

Traditionally the betas in (1.4) are estimated using ordinary least squares (OLS). The natural context for OLS techniques and their application is that of stationary Gaussian time series since it assumes that the error terms $\varepsilon_{i,t}$ are independently distributed with zero mean and constant variance, $\varepsilon_{i,t} \sim IN(0,\sigma^2)$. When OLS is used for heteroskedastic series, *i.e.* stock returns, it produces erroneous risk estimates and severe problems of interpretation arise. The time-varying hypothesis in second moments of security returns has perhaps achieved more attention in measuring systematic risk than anything else in portfolio analysis. As a result a substantial volume of literature in finance has attempted to assess the time invariance of beta in speculative prices, mainly common stocks. Successive studies have investigated the process generating betas and tried to answer such questions as: are systematic risk changes stochastic? Are beta changes serially independent or serially correlated? If they are serially correlated, is the process stationary? And how can these changes be measured? What are the implications for the mean of security returns of shifts in the systematic or unsystematic nature of their risk? Rosenberg (1985) argues that historical betas have little predictive power and identifies two main reasons to explain why historical betas fail to be the "true" (conditional) betas. First, estimates from OLS over a period of time are constant for that period. But beta is a measure of the relationship between a stock's return and the market return over a time interval which can be as little as a week or a day. And because of the changing nature of the company, the changing nature of the market and the changing nature of the risks that exist in the market there is reason to believe that beta changes over time. Second, the residual returns of the stock cause the estimated regression coefficient to differ from the underlying value by an estimation error. The historical beta differs from the average true beta by the amount of this estimation error.

Initial studies conducted by Blume (1968), Fisher (1970) and Gonedes (1973) suggest that security betas with the market are not stable over long period of time[3]. Among others Blume (1971), Fabozzi and Francis (1978), Sunder (1980), Alexander and Benson (1982), Ohlson and Rosenberg (1982) and Bos and Newbold (1984), provide evidence that security betas not only are time-varying but also can be better described by some type of stochastic model. In addition, Bos and Newbold (1984) found that the variation in beta is stochastic but they failed to reject the hypothesis that those changes are serially independent. In contrast, Ohlson and Rosenberg (1982) provide evidence that systematic risk exhibits both a (stationary) first order autoregressive component and a serially independent random element[4].

A1.1.3 Implications on CAPM

The distributional assumptions about security returns also become a necessary condition for commonly used econometric techniques employed in tests of the CAPM[5] and studies of mean variance efficiency (MVE)[6]. Tests applied in these studies often use the variances and covariances of

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[3] Nevertheless, when securities are aggregated the portfolios formed tend to have stable betas with the market, e.g. Blume (1971) and Levy (1971).

[4] Thus, security betas follow a stochastic but stationary process.

[5] Capital Asset Pricing Model, see Sharpe (1964), Lintner (1965) and Mossin (1966).

[6] Followed Roll's (1977) study, that the CAPM is essentially unstable because the market portfolio is unobservable, a number of studies have focused on tests of the mean-variance efficiency (MVE) of the underlying portfolio under consideration (e.g. stock market index).

historical returns as measures of risk. It is therefore important for these studies to know what distribution should be used when testing these hypotheses. Most of these tests are based on the i.i.d. multivariate normality assumptions. Since they are depending on the distribution of the returns, these variances and covariances may not be a trustworthy or sufficient estimator to use and alternative measures of dispersion may be more appropriate.

When the CAPM is empirically tested it is usually written as follows:

$$(Y_t - R_f) = \alpha + \beta(R_{pt} - R_f) + \varepsilon_t \tag{A.1.1}$$

where Y_t is the (Nx1) vector of returns of the asset i, i=1,..N in period t; R_{pt} is the return of the market portfolio whose efficiency is being tested, in the same period; R_f is the riskless rate of lending and borrowing; $\alpha = (\alpha_1,\ldots,\alpha_N)'$ is a vector of regression intercepts; $\beta = (\beta_1,\ldots,\beta_N)'$ is a vector of regression slopes and $\varepsilon_t = (\varepsilon_{1t},\ldots,\varepsilon_{Nt})'$ is a vector of error terms assumed to be i.i.d. and normally distributed[7] with zero mean and covariance matrix Ω. The Sharpe-Lintner CAPM states that the following linear relationship should exist:

$$E(Y_t - R_f) = \beta E(R_{pt} - R_f)$$

The above relationship forms a testable hypothesis to verify the CAPM. If the CAPM holds then the null hypothesis below should hold:

$$H_0 : \alpha = 0$$

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[7] As we can see later the normality assumption can be relaxed.

The test of H_0 relies on the assumption that the joint distribution of the rate of return of the risky assets and the market proxy is bivariate normal and stationary. Thus, the test results are valid as long these assumptions are true. The empirical evidence quoted earlier does not support the normality assumption.

Apart from all the concerns about normality raised by previous studies, most tests of asset pricing models have assumed normality in the residuals and made similar distributional assumptions about the coefficients in the linear model in (1.4) (Fama and MacBeth (1973), Shanken (1985)). Gibbons *et al.* (1989) proposed a statistic to test the joint hypothesis $(\alpha_1,..,\alpha_N)=0$. Nevertheless this test is still based on the assumptions that the error terms in (1.3) are multivariate normal. Affleck-Graves and McDonald (1989) used simulation to examine the effect that non-normalities have on the Gibbons *et al.* (1989) test. They found that in the presence of severe non-normalities the "..size and power of the test can be seriously misstated" (p. 889).

The importance that the stability systematic risk over time of has for this type of testing was recognised by Blume (1971). Thereafter, the evolution of beta coefficients over time has been the subject of investigation in a large number of studies since the early 1970s (Blume (1971), Fisher (1970), Black *et al.* (1972), Vasicek (1973), Fabozzi and Francis (1978) and Alexander and Benson (1982)). All these studies provide evidence that both individual stocks and portfolio betas with the market proxy do change over time. Further evidence for time-varying joint second moments on security returns is provided in a number of studies that employed conditional multivariate time series to test pricing models and capture the time variation in variances and covariances, *i.e.* Bollerslev *et al.* (1988) [8].

A1.1.4 Non-Stationary Returns and the Risk Premium

The relationship between risk and return in an investment is a major concern in a wide range of academic and business applications. The CAPM expresses this theoretical relationship as:

$$E(R_p) = R_f + k\ Cov(R_p,R_m) \qquad (A.1.2)$$

where R_f is the riskless rate of interest and $Cov(R_p,R_m)$ is a measure of the risk of the portfolio. The second term on the right hand of the equation, known as the risk premium, quantifies the reward that the investors are expected to be paid for preferring to hold that risky portfolio over the risk free asset. The term $\frac{E(R_m)-R_f}{\sigma^2_m}$ alone is known as the market price of risk and is the same for all efficient portfolios. Thus, in equilibrium the expected return on the portfolio, $E(Rp)$, is equal to the riskless rate of interest plus a risk premium measured as the product of the scaling factor k and its covariance with the market. In the unconditional CAPM, the expected return, its covariance with the market and the scaling factor k are all assumed to be constant over time. Investors can only increase their portfolio expected return by holding a portfolio with higher volatility. Since the variance of returns for individual series is unlikely to be invariant over time, so is the joint second moment. Like the variances, covariances and so correlations betas, may change as well.

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[8] A review of the studies which investigated the non-constant hypothesis of security betas is in the next chapter.

Shifts over time in a portfolio's covariance with the market implies that the compensation required by risk averse investors for holding that portfolio must fluctuate as well. In addition, a misleading evaluation of an asset's covariance (beta) may lead to a significant miscalculation of its equilibrium price.

Heteroskedasticity, other than making future investments' risk exposure uncertain, undermines the validity of equilibrium pricing models, such as the CAPM. Several studies have pointed out that the empirical failure of the CAPM is due to a significant degree of variability over time of the variance-covariance matrix. Among them, Giovannini and Jorion (1987), have argued that the time variation of the conditional second moments might have important implications for the empirical performance of various asset pricing models. As a result, a number of studies have formulated the CAPM within the family of ARCH models and found that there is a substantial improvement in the performance of the CAPM. Engel and Rodrigues (1989) have estimated a six country CAPM in which the conditional covariance both follows an ARCH process and is related to macroeconomic variables. Frankel (1988) argued that the alternative ARCH models used to explain the time-varying path of the risk premium and beta seem to disclose a set of different beta magnitudes and paths.

Bollerslev *et al.* (1988) have applied a multivariate GARCH to the excess return of three portfolios (stocks, bonds, and bills) and they computed the time-varying or conditional betas for each of them. They found some evidence that the risk premia are better represented by covariances with the (implied) market than by own variances. Among others Baillie and Mayers (1991) and Ng (1991) have used multivariate GARCH models for estimating a time-varying covariance matrix among a set of financial time series.

The assumption of normality for the distribution of asset returns is, in general, not necessary from a theoretical perspective to derive the mean variance efficiency models. Normality is rather adopted for statistical convenience to derive the finite sample properties of statistical tests on asset pricing models. Owen and Rabinivitch (1983) argued the CAPM and multibeta models remain valid under elliptically distributed returns[9]. Zhou (1993) showed that mean-variance efficiency, rejected under normality, can still be valid under alternative elliptical distributional assumptions.

Today, a number of researchers have used more flexible statistical methodologies that require much weaker distributional assumptions to derive mean-variance efficiency models. McKinlay and Richardson (1991) used a generalised method of moments (GMM) approach to test portfolio efficiency. The GMM method is robust against departures from normality and allows the disturbance term to be serially dependent and conditionally heteroskedastic. In a similar experiment Chou (1996) applied a bootstrapping simulation to test mean-variance efficiency. Bootstrapping simulation does not pre-impose the form of the distribution from which draw random numbers but this is imposed by the data themselves.

A1.1.5 Optimal Portfolio Choice

The question of the time invariability of the statistical moments in security returns and the difficulty of measuring the necessary inputs to the portfolio optimisation problem, namely expected returns, variances and covariances has restricted the empirical application of the mean-variance

[9] The elliptical class of distributions embodies the multivariate normal, multivariate Student's -t and mixture of multivariate normals.

framework to portfolio selection, at least in the form proposed by Markowitz (1956). Empirical studies which examined the effect of error in these parameters have raised questions about the trustworthiness of the ex-post mean-variance analysis.

Frankfurter *et al.* (1971) used multivariate Monte Carlo simulation to study how errors in estimating the required parameters can affect the mean variance efficiency of a three asset portfolio. They found strong evidence that portfolios selected according to the mean-variance criterion are unlikely to be more efficient than portfolios selected at random. Their experiment allowed only for sampling errors for the multivariate normal process which was assumed to be time invariant. They argued that "this error can only be magnified when more realistic conditions, such as estimation on the basis of judgement and time dependency are taken into account" (p 1262). Jobson and Korkie (1980), on comparing the optimal weights to the distribution of sample values obtained in simulations, found that the portfolios selected using ex-post mean variance analysis performed very poorly. Their general conclusion is that small variations in sample means often lead to large portfolio re-adjustments[10].

A number of studies have examined the ability of various techniques to forecast the correlation matrix among a set of security returns and the impact that such forecast errors have on the portfolio mix. Eun and Resnick (1984) have evaluated the performance of twelve alternative forecasting models (a *full historical,* three *mean models* and eight *index models*) that may be used to estimate the correlation structure of international share prices. Their empirical test established, unexpectedly, that the *full historical model* was the second best performing model. How-

[10] The impact that errors in means have on the portfolio weights will depend very much on the investor's level of risk tolerance.

ever, in another empirical study, Wainscott (1990) examined the correlation coefficients between US stocks and bonds and he found the historical correlations to be an unsatisfactory predictor of future correlations. Although the portfolios examined in this study were diversified among only three classes of assets, cash, US government bonds and stocks, the author made the concluding remark that changes in the correlation between stocks and bonds significantly affects the asset mix. He also noticed that the differences in the portfolio's mix will increase as more asset classes and more uncertainty about the future covariance matrix are added to the inputs of the portfolio optimiser. The above remarks can be interpreted as follows. The optimal portfolio for a target level of return, as produced by any exact optimiser is unlikely to be unique. As Michaud (1989) noted, because means, variances and covariance are estimated with error, then for every true point on the efficient frontier there is a neighbourhood that includes an infinite number of statistically equivalent portfolios. How large this neighbourhood is depends upon factors such as the target return, input number of assets for the quadratic algorithm, their risk and return trade-off, and the corresponding confidence interval of the forecast.

Thus, the implications of errors in means, variances and covariances for the optimal portfolio may be severe. But even if these parameters are known with certainty at a particular time, changes in the means, variances or covariances of any asset eligible to be included in a portfolio will probably have an (instant) impact on its risk return trade-off and may move it away from the efficient frontier. That is because variances and covariances computed from historical prices will no longer be the true risk estimates, and the portfolio calculated using the unconditional mean-variance optimisation criteria will become sub-optimal. And as we have seen earlier there is strong evidence that the joint second moments of speculative price changes are changing over different periods.

Chapter 2

Data and Methodology

As we have seen in the previous chapter it is important, when estimating a portfolio VaR, to model correctly the asset's volatility and correlations. Given the empirical evidence about fat tails and heteroskedastic returns researchers started to look to different methods of improving risk estimates. In the early studies emphasis was given to improving beta estimates. Beta modelling is at the core of our analysis because we will employ the SIM algorithm to update, daily, a large correlation matrix.

The first section of this chapter will begin by investigating the statistical methods and other techniques (*i.e.* adjustment, fundamental) proposed from early studies for improving beta estimation. Consequently we will outline modern time series methods used to capture the changes in security volatilities and betas. These include beta adjustment OLS techniques and exponential smoothing. Finally we will examine non-linear time series, *i.e.* GARCH and stochastic variance, which model the joint second moments of a set of asset returns as autoregressive. These methods are unique because they are designed to capture any volatility clusters. As we have seen in the previous chapters, daily changes of almost all speculative prices are known to contain clusters in their volatility. After considering what different models are aiming to achieve but also their computational feasibility, we concluded that (in the next stage of our analysis) the most appropriate model to use for estimating a portfolio's DEaR is the bivariate GARCH. Finally, the data-set to be employed in this thesis is described. This will consist of daily closing prices for thirteen national markets over a ten year period. Its use in the empirical chapters will be discussed thereafter.

2.1 Modelling the Correlation Matrix. Unconditional Models.

Researchers over the years have attempted different, mainly statistical, methods to handle the problems associated with changing volatilities and correlations. Given the difficulty in handling jointly the second moments of a large set of assets, these studies have focused on modelling betas which then can be used to build a correlation matrix. In this section we will describe early models designed mainly to adjust historical betas but also more recent techniques based on non-linear time series.

2.1.1 Adjustment Methods

To deal with time variability in security beta, early studies have suggested a number of adjustment techniques. These are mainly based on the principle that the historical beta contains part of the true beta and a *random* error term. The adjustment techniques, *e.g.* Blume (1971) and Vasicek (1973), just weight a set of security betas toward a central tendency value. Blume observed that the forecast betas tend to be closer to one (1.0) than the estimate obtained from historical data. That is because the true betas of any stock tend to have a value of one due to sampling error, the historical estimate deviates from that central value.

Blume (1971) was the first who proposed to modify past betas to capture this tendency. Blume estimated historical betas of a set of stocks over two consecutive non-overlapping periods, 1948-1954 and 1955-1961. He then used a cross sectional linear regression. This required regressing the betas of the second period against a constant and the beta of the first period. Denoting $\beta_{i,a}$ as the adjusted (forecast) values of beta and $\beta_{i,h}$ the current period betas he estimated the following equation form:

$$\beta_{i,a} = 0.343 + 0.677\ \beta_{i,h} \qquad \text{with } i = 1,\ldots,N \qquad (2.1)$$

which measures the average tendency of the forecast betas to be one rather than their historical estimates. Hence the Blume adjustment gives the adjusted beta, $\beta_{i,a}$, a 34.3% weight to the market beta of one and a 67.7% weight to the past period's (historical) estimate of beta, $\beta_{i,h}$, of the same stock. Hence, the relationship described in (2.1) implies that the forecasting beta is 0.343 + 0.677 time the past period's beta. If the past period's beta for a particular stock was 1.5 the forecast would be 0.343 + 0.677(1.5)=1.36. Similarly, historical betas less than one are made larger, but will still be less than one, hence the tendency of forecast betas to be equal to one. However, this technique has the undesirable property of forecasting with a trend[1].

Vasicek (1973) observed that the deviations from the average beta of a group of stocks are proportional to the sampling error (standard error of the estimate). Large standard errors imply greater chances of big deviations from the true beta and so the adjustment should be greater. He

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[1] If the average betas between the two periods has increased (decreased) it assumes that the average beta will continue to increase (decrease) over the next periods.

therefore suggested a Bayesian approach to the adjustment of betas. Information obtained from a weighted average of betas across a sample of stocks for a period t is used to adjust historical (sample) betas in keeping with a minimum expected loss criterion.

Let us denote $\bar{\beta}_t$ the mean of the distribution of the historical estimates of beta over the same sample of stocks for the period t, and $\sigma^2_{\bar{\beta},t}$ their variance. Further, denote $\sigma^2_{\beta i,t}$ as the variance of the historical estimate of beta for security i, and β_i measured over the same period. The adjusted beta is given by:

$$\beta_{i,a} = \frac{\sigma^2_{\beta i,t}}{\sigma^2_{\bar{\beta},t} + \sigma^2_{\beta i,t}} \bar{\beta}_t + \frac{\sigma^2_{\bar{\beta},t}}{\sigma^2_{\bar{\beta},t} + \sigma^2_{\beta i,t}} \beta_{i,h} \qquad (2.2)$$

This technique, like that of Blume, tends to shrink betas toward one and hence may be the source of forecast bias. However, unlike the Blume technique, it does not force beta forecasts toward a trend. But the Vasicek technique adjusts those betas with higher standard errors further toward the mean than it adjusts betas with low standard error. Hence, high beta stocks are adjusted more toward a mean than low beta stocks.

The ability of these techniques to forecast betas has been investigated by Klemkosky and Martin (1975). They examined three adjustment techniques, the Blume, the Vasicek and the Merril Lynch, Pierce, Fenner, & Smith Inc. (MLPFS). This last method is very similar to the Blume method. Klemkosky and Martin verified that both techniques lead to more accurate forecasts of future betas than did the unadjusted betas. They noted that the forecasting accuracy of betas to predict subsequent time period (betas) grew progressively worse as beta levels departed significantly from average. In addition, they used a decomposition technique to uncover the origins of the beta forecast error. They partitioned the mean square forecast error[2] (MSE) of beta predictability into three components, bias, inefficiency and random error. Klemkosky and Martin argued that, although the differences among the three techniques were small, Vasicek's technique had a slightly higher performance since it scored the lowest mean squared error in two out of three time periods.

2.1.2 Fundamental Adjustment

A different, causal or economic approach, assumes that the variation of a stock's beta can be explained by economic related factors. Such a model has been proposed by Beaver *et al.* (1970). The variables they used are:

- *Dividend pay-out* (dividends/earnings)
- *Asset growth* (annual change)
- *Liquidity* (current assets/current liabilities)
- *Leverage*
- *Asset size* (total assets of a company)
- *Earning volatility* (standard deviation of earnings/price ratio)
- *Accounting beta* (the slope of the regression of the firm's earnings against the average earnings of the market)

Other studies have followed this kind of analysis and a different number of factors was proposed by each study. For example Rosenberg and Marathe (1975) suggested that 101 factors are necessary to explain betas' variability. However, they restricted the number of factors affecting each

[2] They define the MSE as the average squared distance between the estimated beta coefficient for the period t and the predicted coefficient which equals the last period's beta.

stock's beta by imposing a set of industry dummy variables in the regression equation. The ability of fundamental betas to predict future betas has been examined by, among others, Breen and Lerner (1973), Gonedes (1973), Melicher (1974). The criticisms are mixed with some studies reporting significant improvement in forecasting capability of models when using fundamental variables while others do not.

2.2 Modelling the Correlation Matrix. Conditional Models.

Given all the evidence against the normal distributional hypothesis, valid statistical corollaries require statistical techniques robust to non-normalities or corrections for non-normalities, (see Korajczyk (1985) and Krasker (1980)). The methods described above only seek to adjust sampling errors. A different approach is to use statistical procedures to measure continuously the variation of betas. Usually these models employ only past returns. Thus, they are more restrictive than the fundamental ones but they require less data. On the other hand, they may become more complex and present greater computational difficulties.

2.2.1 Stochastic Time Series Models

One type of conditional model uses stochastic analysis to describe the construction of volatility and betas. The simplest case is the *constant coefficient* model where the i*th* security's beta, $\beta_{i,t}$, is equal to the previous period's beta :

$$\beta_{i,t} = \beta_{i,t-1} = \beta_i^* \qquad \text{for all } t. \qquad (2.3)$$

This estimate of beta is the most common one employed in finance and is the process assumed by OLS estimation. This is often used as the null hypothesis for most tests of beta variability, *e.g.* Fabozzi and Francis (1978). A related parameterisation is to allow $\beta_{i,t}$ to deviate from its underlying mean by a random error term. This is given by

$$\beta_{i,t} = \beta_i^* + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim N(0,\sigma^2) \tag{2.4}$$

In the above model specification, referred as *dispersed coefficient* by Schaefer *et al.* (1975), the transient beta returns quickly to its underlying mean value. If, however, the beta follows a *random walk* then its path can be expressed as

$$\beta_{i,t} = \beta_{i,t-1} + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim N(0,\sigma^2) \tag{2.5}$$

Hence, although β_t is not-stationary, it is serially correlated. A more comprehensive model is the *return to normality* model, in Rosenberg (1973), given by

$$\beta_{i,t} = \alpha_i\beta_{i,t-1} + (1-\alpha_i)\beta_i^* + \varepsilon_{i,t} \qquad \varepsilon_{i,t} \sim N(0,\sigma^2) \tag{2.6}$$

In the above process there is a tendency for beta to regress towards a mean level, which is measured by $1-\alpha_i$. If the value of α_i is one, there is no tendency for beta to return to an underlying mean and the model reduces to the random walk equation. If α_i is set to zero, then it reduces to a dispersed coefficient one. The Vasicek adjustment technique is a restricted case of (2.6) since it sets the β_i^* to one and makes no restriction on the adjustment rate toward the underlying mean. This model attracted a lot of interest in the financial literature over the years, *e.g.* Bos and Newbold (1984) and Collins *et al.* (1987).

Other time series models include time-varying regression parameters which are based on the principle that the systematic risk variation is due to a stationary part or trend and a random or noise component[3]. A general parameterisation of such a model can be written as:

$$Y_{i,t} = \beta_{i,t} X_t + B_i V_{i,t} + v_{i,t} \tag{2.7.a}$$

$$\beta_{i,t} = \gamma_i + \delta \xi_t + u_{i,t} \tag{2.7.b}$$

where γ_i and δ are unknown constants, ξ_t is a deterministic function of time and $v_{i,t}$ and $u_{i,t}$ are zero mean and constant variance, serially independent random disturbances. The vectors X_t (variables common to all stocks) and $V_{i,t}$ form the set of explanatory variables that affect the mean. Accordingly, the randomly time-varying beta, $\beta_{i,t}$ is distributed with mean

$$E(\beta_{i,t}) = \gamma_i + \delta \xi_t$$

and variance

$$Var(\beta_{i,t}) = E(u_{i,t}^2) = \sigma_i^2$$

[3] For example, see the variable mean response (VMR) random coefficients models described in Lin et al. (1992).

Unbiased and consistent estimates of the parameters can be obtained by feasible generalised least squares (FGLS) estimation. Alternatively Kalman filter techniques may be employed to calculate the models in (2.6) and (2.7).

Other time series methods for estimating betas include Bayesian models suggested by Bawa *et al.* (1979).

2.3 Conditional Heteroskedastic Models.

The success of each model in producing estimators and predictors of stocks' volatility and beta is determined by its ability to take full advantage of the information that is present in the available data. Among the properties characterising financial time series data is clustering in volatility. None of the models described earlier makes use of this information when estimating the changes in variances and covariances. Allowing for changes in variances is a more general specification than allowing for changes only in means and can also fit well with the speculative price changes which, as is known, show volatility clusters. As we have seen, changes in variances can also account for the fat tails in the empirical distribution of security returns.

One such family of models that allows the variance to change as new information becomes available can be found in the GARCH methodology based on the work of Engle (1982) and Bollerslev (1986). Suppose that a security's returns Y_t are modelled as:

$$Y_t = \Phi_t^T \delta + \varepsilon_t \qquad (2.8.a)$$

where Φ_t is a vector of variables with impact on the conditional mean of Y_t and δ is the corresponding vector of parameters. Conditional on I_{t-1}, the information available up to time (t-1), the expected value of ε_t is zero, *i.e.* $E_{t-1}(\varepsilon_t) = 0$, and the corresponding conditional variance is:

$$V_{t-1}(\varepsilon_t) = h_t$$

Engle (1982) modelled the variance of the errors, $E_{t-1}(\varepsilon_t^2)$, as an autoregressive process, and referred to this as the *Autoregressive Conditional Heteroskedastic (ARCH)* model. Hence, the first order ARCH model is

$$\varepsilon_t \mid I_{t-1} \sim N(0,h_t)$$

where:

$$h_t = \omega + a\,\varepsilon^2_{t-1} \tag{2.8.b}$$

where $\omega > 0$, $a \geq 0$. The variable h_t denotes the conditional variance of ε_t, and if $a>0$, h_t is time-varying. Thus, the conditional distribution of Y_t is normal but its conditional variance is a linear function of past squared errors. OLS is a special case since it restricts a to be zero and treats the conditional variance as a constant. Thus, linear models rule out the presence of heteroskedasticity in residual returns.

The ARCH model in (2.8.b), has one particularly interesting property. It allows the errors ε_t to be serially uncorrelated but not necessarily independent, since they can be related through their second moments, when a is positive. Thus changes in the volatility of the series may be predictable. Predictability of variance arises from the non-linear dependence in the returns themselves, rather than exogenous change in other structural factors, *e.g.* fundamental models. An additional appealing property of ARCH models is that they allow the series to have excess kurtosis. As Engle (1982) shows, the ARCH(1) model requires that 3a<1 for the unconditional fourth moment to exist. The fourth moment is given by

$$\frac{E(\varepsilon_t^4)}{\sigma_\varepsilon^4}=\frac{3(1-a^2)}{(1-3a^2)} \tag{2.9}$$

which, for a<1 , is greater than 3, the corresponding coefficient of the normal distribution. Therefore, ARCH models allow the unconditional distribution of Y_t to exhibit fat tails (excess kurtosis) without violating the conditional normality assumption[4], and therefore to be symmetric. As we have seen earlier a number of empirical studies have found that the distribution of price changes, or their logarithm, in a variety of financial series tends to be symmetric, but with fatter tails than those of the normal distribution, *e.g.* Mandelbrot (1963), Fama (1965). In general, in the ARCH methodology the conditional variance of Y_t is expressed as a (non-linear) function of past information. It validates earlier concerns about heteroskedastic stock returns and meets the need for modelling the volatility as conditional on past information.

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[4] The estimates of an ARCH model are still consistent, even if the assumption of conditional normality for the distribution of ε_t is violated. Engle and Gonzalez-Rivera (1991) argued that the assumption that the conditional density is normal usually does not affect the parameter estimates of an ARCH model even if it is false.

A generalisation to model (2.8) by Bollerslev (1986) includes past values of h_t in the definition of conditional variance. This is known as a generalised ARCH or GARCH model. Engle and Bollerslev (1986) stated that a low order GARCH process may have properties similar to high order ARCH but with significantly fewer parameters to estimate. The GARCH model, in its simplest form, is given by:

$$h_t = \omega + a\varepsilon^2_{t-1} + \beta h_{t-1} \tag{2.10}$$

so the current variance depends upon yesterday's surprise, ε_{t-1}, and variance, h_{t-1}.

2.3.1 Exponential Smoothing (ES) or GARCH

Today perhaps the most popular volatility method is the exponential smoothing (ES) proposed by JP Morgan (see Riskmetrics (1995)). Given a series of returns, Y, the ES model[5] for conditional variance is given as:

$$h_t = \lambda h_{t-1} + (1-\lambda)Y^2_{t-1} \qquad 0 < \lambda < 1 \tag{2.11}$$

where h_t is today's variance. From (2.11) it emerges that the current level of volatility, h_t, is function of yesterday's volatility and the square of yesterday's returns, Y_{t-1}. The above model has the advantage that it uses only one parameter, λ. Optimal values for the λ can be obtained by

[5] Riskmetrics (1995), however, uses today's returns to estimate the today's volatility. In this study we will use the specification in (2.11) since it uses the same set of information as the GARCH model.

minimising the sum of the normalised squared errors. This is equivalent to the maximum likelihood estimation used in GARCH models. However, it assumes that the conditional mean of Y is zero for each period t. When the conditional mean is different from zero the ES in (2.11) overestimates the variance. The ES can be written as a two equation system by introducing a conditional mean equation like the one of (2.8.a) but this will require maximum likelihood techniques, similar to those used in GARCH, to find optimal values for λ, and any other parameters in the mean equation.

The ES and the GARCH models have many similarities, *i.e.* today's volatility is estimated conditionally upon the information set available at each period, t. Both the GARCH(1,1) model in (2.10) and the (ES) model in (2.11) use last period's returns to determine current levels of volatility. They imply that today's volatility is known immediately after yesterday's market closure[6]. Since the latest available information Y_{t-1}, is weighted in a more effective way, it can be shown that both models will provide more accurate estimators of volatility than unconditional models, i.e. historical volatility.

However, several differences exist in the operational characteristics of the two models. The GARCH model, for example, uses two independent coefficients, α and β, to measure the impact of last period's errors and volatility in determining current volatility. On the other hand the ES model uses only one coefficient, λ, and forces last period's innovation and volatility to have a unit effect on current period's volatility. Thus, a

[6] Note that there is no stochastic term in either (conditional variance) equation.

large shock will have longer lasting impact on volatility in the model (2.11) than in (2.10).

The terms α and β in GARCH do not need to sum to unity and one parameter is not a complement of the other. Their estimation is achieved by maximising the likelihood function. This is a very important point since the values of α and β are critical in determining the current levels of volatility. Incorrect selection of the parameter values will adversely affect the estimation of volatility. The assumption that α and β sum to unity (see model 2.11) is, however, very strong and presents a hypothesis that can be tested rather than a condition to be imposed.

Furthermore, the GARCH model has an additional parameter, ω, that acts as a floor and prevents volatility dropping below that level. In the extreme case when α and β equal zero, volatility is constant and equal to ω. The value of ω is estimated together with α and β using maximum likelihood estimation and the hypothesis $\omega = 0$ can be tested easily. The absence of the ω parameter in the ES model allows volatility, after a few quiet trading days, to drop to very low levels, see Giannopoulos and Eales (1996).

Another appealing feature of GARCH modelling is that it allows a flexible parameterisation in the variance and mean equations. Among others, Black (1976) observed that volatility tends to be higher when prices are falling than when prices are rising. Thus, there is an asymmetry in volatility also known as the leverage effect. The GARCH(1,1) model in (2.10) can be easily modified to allow for asymmetries in volatility. One such model specification is the exponential GARCH (EGARCH) of Nelson (1988). In the EGARCH(1,1) model the variance of the residual error term for the is given by

$$\ln(h_t) = \omega + \beta \ln(h_{t-1}) + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \phi\left(\left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) \tag{2.12}$$

This type of modification enables negative returns to have a greater impact on the current estimate of volatility than positive returns. When the estimated value for γ is negative, past errors have greater impact on current variance than the analogous positive errors. Hence, h_t in equation (2.12), is expressed as a function of both the magnitude and sign of lagged errors[7].

In general, we can say that both ES and GARCH, use past information in a more efficient way to compute current variances[8]. The GARCH methodology seems to be superior but the ES is computationally easier[9].

2.3.2 Other Conditional Heteroskedastic Models. Stochastic Volatility

An alternative approach in modelling the portfolio's volatility as time-varying and conditionally heteroskedastic is provided by the Stochastic Volatility (SV) family of models. Given the portfolio's return in (2.8.a) the variance, σ^2_t, can be written as an unobserved variable which is presumed to follow a known stochastic process. Hence the SV can be written as

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[7] There are a number of other asymmetric specifications. For a survey of these models see Bera and Higgins (1993). In chapter four the Asymmetric GARCH (AGARCH) of Engle (1990) will be employed to estimate a hypothetical portfolio's volatility.

[8] Hereafter we will treat ES as a special case of GARCH, integrated GARCH, and will not quote it as a model itself.

[9] For an empirical comparison of the two models see Giannopoulos and Eales (1996).

$$Y_t = \Phi_t^T \delta + \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim N(0, h_t) \qquad (2.8.a)$$

$$\ln(h_t) = \omega + \beta \ln(h_{t-1}) + \eta_t \qquad \eta_t \sim N(0, \sigma^2_\eta) \qquad (2.13)$$

Hence the stochastic volatility, h_t, follows a first-order autoregressive [AR(1)] process in logarithms. The two error terms, ε_t and η_t, may or may not be correlated. As Harvey (1993) argues, the major disadvantage of the SV models is in writing the likelihood function. He suggests writing the model in a state space form and using the Kalman filter to estimate it. The model is not conditionally normal but Harvey suggests treating the model as if it was normal and maximising the resulting quasi-likelihood function. SV models can also be estimated using Hansen's (1982) generalised method of moments (GMM), (see Melino and Turbull (1990)). Danielson (1994) estimated a SV model using simulated maximum likelihood methods.

The SV and GARCH models have many similarities and can explain some stylised facts that characterise the changes in speculative prices. For example both allow the unconditional distribution of the data to exhibit fat tails. Further, in both models squared past innovations follow a positive autoregressive process. The obvious difference between the two models is that in the SV model, changes in volatility are explained exclusively by changes in the information incorporated into prices. On the other hand, the volatility in the GARCH models is a function of past prices. Hence, the SV allows the processes of mean and variance of returns to be independent of each other. Moreover, SV models do not assume that the volatility in period t is known with certainty as at the previous period. Taylor (1994) employed both GARCH and SV type of models to estimate the volatility of daily Deutsche Mark to US dollar exchange rates. He argues that "..exponential ARCH(1,0) describes the DM/$ returns best within the ARCH family and hence the ARV[10] model is a credible choice when the distribution of the average variance is required", (p. 201). He recommends, however, to use ARCH models to obtain the estimate for the average variance[11].

2.4 Multivariate Conditional Heteroskedastic Models

All models described above are univariate. However, risk analysis of speculative prices examines both an asset's return volatility and its comovement with other securities in the market. Hence, these models could find more prominent use in empirical finance and in VaR analysis if they could describe risk in a multivariate context.

In a general multivariate GARCH(1,1) model, the conditional variance-covariance matrix H_t may be described as[12]:

$$\mathrm{vech}(H_t) = \Omega + A\,\mathrm{vech}(\varepsilon_{t-1}\varepsilon_{t-1}^T) + B\,\mathrm{vech}(H_{t-1}) \qquad (2.14)$$

where Ω is a $[N(N+1)/2]\mathrm{x}1$ dimensional vector and A and B are $[N(N+1)/2]\mathrm{x}[N(N+1)/2]$ dimensional matrices. However in this model there

[10] ARV stands for Autoregressive Random Variance, as Taylor (1994) refers to the stochastic volatility model.

[11] In order to use ARCH estimates as initial values for the variance in the stochastic volatility simulation.

[12] vech(*) is the operator which stacks lower triangular elements of the matrix (*) into a vector.

are $(N(N+1)+N^2(N+1)^2)/2$ parameters to estimate which makes its use prohibitive in large portfolios.

When a conditional variance-covariance matrix needs to be computed for a larger number of assets, then the above computational exercise may be simplified by re-specifying the full NxN variance-covariance matrix to N, 2x2 blocks. Each block is composed of the market return series and one of the N assets. Thereafter, a bivariate GARCH model can be used to compute the 2x2 conditional variance-covariance matrix between asset i and the market.

However, even in the two variable case there are 21 parameters to be estimated in the variance-covariance equation (2.14) alone. In practice, it is therefore usually necessary to restrict the model. Engle *et al.* (1984) restrict each element of the H_t to depend only on its own lagged squared errors or cross products. Thus, a diagonal bivariate GARCH(1,1) is written as:

$$Y_{1,t} = \Phi_{1,t}^{T}\,\delta_1 + \varepsilon_{1,t} \qquad (2.15.a)$$

$$Y_{2,t} = \Phi_{2,t}^{T}\,\delta_2 + \varepsilon_{2,t} \qquad (2.15.b)$$

$$\text{with} \quad \begin{vmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{vmatrix} \sim N(0, H_t)$$

where $Y_{1,t}$ represents the return on a single asset over the period (t-1,t) and $Y_{2,t}$ represents the return on the market index over the same period. Conditional on information available up to time (t-1), the vector ε_t follows a bivariate normal distribution with zero mean vector and conditional variance-covariance matrix $\mathbf{H}_t$. When it is restricted to its diagonal specification, $\mathbf{H}_t$ can be decomposed as

$$h_{1,t} = \omega_1 + a_1 \varepsilon^2_{1,t-1} + b_1 h_{1,t-1} \quad (2.16.a)$$

$$h_{12,t} = \omega_{12} + a_{12} \varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{12} h_{12,t-1} \quad (2.16.b)$$

$$h_{2,t} = \omega_2 + a_2 \varepsilon^2_{2,t-1} + b_2 h_{2,t-1} \quad (2.16.c)$$

Here $h_{1,t}$ and $h_{2,t}$ can be seen as the conditional variances of a locally diversified portfolio and the World market index respectively. These are expressed as past realisations of their own squared disturbances. The covariance of the two return series, $h_{12,t}$, is a function of the cross product between past disturbances in the two markets. The ratio $\frac{h_{12t}}{h_{2t}}$ forms the local stock's time-varying beta. Thus, the conditional beta for each local portfolio with the market is estimated individually from the rest of the series.

The SV model can also be extended to include multiple assets, see Harvey *et al.* (1994). Like in the univariate cases, estimation of the multivariate SV model is not straightforward since volatility is a dynamic latent variable and conventional estimation methods are not applicable. Simulated maximum likelihood techniques can be applied but at a high cost since multivariate SV models have a large number of parameters to estimate.

An easy and relatively cheaper to implement solution is provided by the multivariate specification of ES. Based upon the univariate formulation in (2.11) the covariance between Y_1 and Y_2 in the ES can be written as

$$COV_{12,t} = \lambda\, COV_{12,t-1} + (1-\lambda)Y_{1,t-1}Y_{2,t-1} \qquad 0 < \lambda < 1 \tag{2.17}$$

However, multivariate ES are exposed to the same limitations as that of univariate ES. When precision is important its use should be avoided.

2.5 Selecting the Right Model

The success of each model in producing estimators and predictors of stocks' betas is determined by its ability to take full advantage of the information that is present in the available data. Among the properties characterising financial time series data is clustering in volatility. None of the traditional OLS based models mentioned earlier (such as SIM) makes use of this property. However, conditional heteroskedastic models, like GARCH and SV take full advantage of current and past information when estimating variances and covariances. But the implementation of these model are restricted to 2 or three assets at a time. The empirical success of the GARCH family has been well documented in the financial literature. Furthermore, the likelihood of multivariate GARCH is defined and this makes their computation substantially easier to that of multi-variate SV.

We will, therefore, adopt a bivariate GARCH model to estimate the correlation matrix of a internationally diversified portfolio. Given that our portfolio consists of N assets, we will adopt the SIM of Sharpe (1963) instead of estimating jointly all possible pair wise coefficients. Thus we estimate conditionally N betas using N similar bivariate systems of equations. The above model parameterisation has the advantage that it can be repeatedly computed for any number of securities and for their conditional beta to be estimated. Hall *et al.* (1988) used a similar model to compute conditional betas for different sectors of the FT index. Hall and Miles (1992) used a bivariate ARCH in mean model to estimate the conditional risk of local bond portfolios against a world bond portfolio. They noticed that because this procedure does not estimate jointly the conditional variance-covariance matrix $\mathbf{H}_t$, it may involve some loss of efficiency which is compensated by simplifying the computation process.

2.6 Data

In this thesis we selected closing daily price indices from thirteen national stock markets[13] over a period of 10 years, from the first trading day of 1986 (2 January) until the last trading day of 1995 (29 December). Hence 2608 consecutive price changes have been collected. The thirteen markets have been selected in a way that matches the regional and individual market capitalisation of the world index. Our data sample represents the 93.3% of the Morgan Stanley International world index capitalisation[14]. The *Morgan Stanley Capital International (MSCI) World Index* has been chosen as a proxy for the world portfolio. The equity indices that have been chosen are reported in table 3.1.

[13] The terms "local market", "national market", "domestic market", "local portfolio", "national portfolio" refers to the national indices and will be used interchangeably through this study.

[14] Due to investment restrictions for foreign investors in the emerging markets and other market misconceptions along with data non-availability, our study is restricted to developed markets only.

To quantify a portfolio's VaR, all investment returns need to be expressed in a common currency. It is a common practice among practitioners to use the US dollar as a numerative currency. Hence our price series are multiplied by the daily spot dollar rates quoted by Barclays Bank International. Daily returns are computed for all indexes as

$$R_t = \ln(\text{closing price index at } t) - \ln(\text{closing price index at } t\text{-}1)$$

where ln denotes the natural logarithm. When the data are translated in US dollars the closing prices are multiplied by their equivalent fx quotation. The dollar value of the world index is published daily by MSCI.

Table 3.1 The data series

EUROPE	
Denmark	*Copenhagen SE General, Total Price Index*
France	*MSCI, Price Index*
Germany	*Frankfurt Commerzbank, Total Price Index*
Italy	*Banca Commerciale Italiana, Price Index*
Netherlands	*CBS Total Return General, Price Index*
Spain	*Madrid Stock Exchange, Total Price Index*
Sweden	*MSCI, Price Index*
Switzerland	*Credit Swiss General, Price Index*
UK	*FT-SE 100-share, Price Index*
ASIA	
Hong Kong	*Hang Seng Bank, Price Index*
Japan	*Nikkei Stock Average (225), Price Index*
Singapore	*Straits T Industrial, Price Index*
AMERICA	
USA	*Dow Jones Industrial, Price Index*
MARKET	
World	*MSCI World, Price Index*[15].

[15] As with local indices, the Morgan Stanley provides a form of the world market index with the aggregated stocks not translated into a common currency. Thus, the one period world market return is just the weighted return of the aggregate local portfolios, net of any currency fluctuations.

In chapter three we are going to use each national market's returns to estimate its conditional variance and covariance with the world index. The estimation is performed on both local currency and US dollar returns. The variances and betas of the US dollar nominated returns are consequently used to estimate a hypothetical portfolio's DEaR over the 10 year period. Furthermore, the US dollar returns are used as the historical returns of a hypothetical portfolio, needed to conduct "stress" analysis on our DEaR estimate. These portfolio historical returns are also employed during the analysis in chapter four of a simplified approach of the conditional estimation of VaR. In chapter five, we will need to employ risk estimates net of any currency fluctuation (currency risk is fully hedged) to study volatility transmission mechanisms from one national market to the other.

2.7 Conclusions

In summary, the problem, of using historical estimates of assets' means, variances and covariances in VaR analysis, is well known to market practitioners. As a result a number of methods have been proposed to overcome the instability problem and to estimate in the best possible way current variances and covariances. Perhaps today the most popular method is the exponential smoothing (ES) proposed by JP Morgan, RiskMetrics (1995). A more sophisticated approach can be found in the GARCH methodology based on the work of Engle (1982) and Bollerslev (1986). Both approaches use past information in a more efficient way to compute current variances. The GARCH approach allows for very flexible specification in the mean and variance equations, it is based on fewer assumptions and is estimated using maximum likelihood, which asymptotically guarantees efficiency and consistency of the parameter estimates. On the other hand, the one parameter ES is very restrictive, it is a particular specification of GARCH, but is easy to compute. In the next chapter, we estimate VaR and DEaR using the bivariate GARCH in conjunction with the Single Index Model of Sharpe (1963) where we evaluate its ability to measure portfolio risk and compare it with an exponential smoothing benchmark.

Chapter 3

Building a Time-Varying Variance-Covariance Matrix

The estimation of VaR and DEaR requires an estimation of portfolio volatility. Unfortunately, the historical volatility on a bank portfolio is an ill-suited measure of its current volatility because investment weights may change rapidly and even individual security's volatility may shift over time[1 2]. Moreover, the composition of the volatilities of individual components in portfolio volatility requires the knowledge of the correlation matrix of returns of the different components. This correlation matrix is also possibly subject to shifts over time. Multivariate GARCH has been successfully employed to capture changes in variances and correla-

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[1] This chapter is based on the work of Giannopoulos (1995). However, here daily data measured on both local and US dollar terms are used. Further, some new paragraphs have been added (i.e. on VaR and stress analysis) to reflect the scope of this chapter, which differs from the above publication.

[2] I would like to thank, without implicating, the participants at the 1994 Annual Royal Economic Society Conference, held in Exeter, the participants at the 1994 Southern Finance Association Meeting, Charleston, S.C. and the referees of the European Journal of Finance for helpful comments.

tions. However, this is computationally intensive and its use is restricted to few assets at a time. In this chapter we will show how the GARCH methodology can be implemented in the estimation of large correlation matrices. We then evaluate the ability of such a matrix to measure portfolio risk and compare the results with a benchmark, the exponential smoothing technique. We find that both the systematic and the specific risk of national equity markets change over time where the bivariate GARCH captures a large part of this variation. However portfolio variance estimated upon these conditional betas is found to be a biased estimator with little explanatory power for portfolio volatility.

3.1 Preliminary Analysis

3.1.1 Univariate Statistics

Descriptive statistics on the return series are shown in table 3.2. Part 1 contains the statistics when returns are expressed in local currency. The distribution of all local portfolios is characterised by negative skewness and kurtosis greater than that predicted by the Gaussian distribution. The Jarque-Bera (1980) test rejected the null hypothesis of normality across all series, providing further evidence that non-linearities are present in the data. When returns are translated into US dollars, table 3.2 part 2, the volatility for all series increases, further reflecting the extra source of uncertainty which is due to the foreign exchange fluctuations, while departures from normality remain large and significant across all national equity portfolios. The above indicates further that linear estimates, *i.e.* OLS, of volatility are ill-suited measures of risk.

Table 3.2 Univariate statistics of local market return

Part 1. Returns in local currency

	Mean (p.a.)	St Dev. (p.a.)	Skewness	Kurtosis	normality	p value
DENMARK	0.041	0.112	-1.087	13.290	512.700	0.000
GERMANY	0.048	0.197	-0.928	12.935	373.830	0.000
FRANCE	0.075	0.181	-0.719	8.177	224.000	0.000
HONGKONG	0.169	0.262	-6.446	146.799	17957.620	0.000
ITALY	0.025	0.201	-0.964	10.885	403.540	0.000
JAPAN	0.040	0.215	-0.244	13.661	25.890	0.000
NETHRLD	0.110	0.147	-0.948	25.317	387.850	0.000
SINGAPORE	0.120	0.213	-4.806	116.728	9993.820	0.000
SPAIN	0.112	0.177	-0.434	7.586	81.310	0.000
SWEDEN	0.137	0.197	-0.161	6.823	11.180	0.004
SWITZERL	0.033	0.157	-2.259	26.278	2216.010	0.000
UK	0.093	0.150	-1.737	26.184	1304.920	0.000
US	0.116	0.169	-5.557	135.458	13337.090	0.000

Part 2. Returns in US dollars

	Mean (p.a.)	St Dev. (p.a.)	Skewness	Kurtosis	normality	p value
DENMARK	0.080	0.161	-0.147	11.378	9.420	0.009
GERMANY	0.100	0.214	-0.680	9.659	200.190	0.000
FRANCE	0.117	0.196	-0.502	6.181	109.000	0.000
HONGKONG	0.170	0.263	-6.427	145.928	17851.460	0.000
ITALY	0.030	0.233	-0.847	9.028	311.840	0.000
JAPAN	0.104	0.242	-0.359	10.937	55.920	0.000
NETHRLD	0.139	0.177	-5.755	134.582	14281.910	0.000
SINGAPORE	0.158	0.220	-4.139	109.094	7394.860	0.000
SPAIN	0.130	0.206	-0.403	6.972	70.310	0.000
SWEDEN	0.150	0.224	-1.092	19.904	515.080	0.000
SWITZERL	0.089	0.193	-2.129	24.185	1964.050	0.000
UK	0.086	0.191	-0.735	10.257	234.060	0.000

Note: The test for normality is the Jarque-Bera test, $N((\sigma^3)^2/6 + (\sigma^4-3)^2/24)$. The last column is the significance level.

The serial correlation coefficients for the returns, together with the Ljung-Box Portmanteau statistics (Q statistic) are shown in table 3.3. For almost all markets there is strong evidence or the presence of first order serial correlation in both cases when returns are expressed in local and common currency. For some markets the serial correlation persists for even longer. The Ljung-Box statistic of order six reveals a significant higher order serial correlation for all local portfolios. However, the serial correlation on daily market indices, and especially on broad market indices, is most likely caused by thin trading. Serial correlation is still present when returns are measured in US dollars but tends to be smaller.

Table 3.3 Autocorrelations of returns

Part 1.returns local currency

Series	r1	r2	r3	r4	r5	r6	Q(6)	-p-
DENMARK	0.206	0.020	0.028	0.023	0.011	-0.004	116.200	0.000
GERMANY	0.011	-0.036	-0.011	0.013	0.027	-0.045	11.450	0.075
FRANCE	0.086	0.023	-0.025	0.011	-0.016	0.003	23.250	0.001
HONGKONG	0.013	-0.007	0.080	0.012	0.044	-0.027	24.300	0.000
ITALY	0.147	-0.025	0.036	0.036	-0.005	-0.034	67.680	0.000
JAPAN	0.021	-0.073	0.005	0.037	-0.027	-0.024	22.200	0.001
NETHRLD	-0.025	-0.028	-0.024	0.044	0.039	-0.014	14.550	0.024
SINGAPORE	0.143	-0.082	-0.010	0.060	0.027	0.046	88.180	0.000
SPAIN	0.220	0.055	0.016	0.042	0.009	-0.010	140.140	0.000
SWEDEN	0.144	-0.004	-0.006	0.059	0.037	-0.045	72.640	0.000
SWITZERL	0.034	0.029	-0.021	0.041	0.071	-0.013	24.600	0.000
UK	0.069	0.004	0.017	0.074	0.010	0.001	27.930	0.000
US	0.014	-0.075	-0.015	-0.041	0.060	-0.013	29.830	0.000

Part 2. returns in US dollars

Series	r1	r2	r3	r4	r5	r6	Q(6)	-p
DENMARK	0.104	-0.010	0.015	-0.001	0.007	-0.014	33.300	0.000
GERMANY	-0.024	-0.025	-0.027	0.030	0.009	-0.036	13.270	0.039
FRANCE	0.067	0.006	-0.018	0.034	0.013	-0.011	20.040	0.003
HONGKONG	0.011	0.002	0.068	0.011	0.042	-0.036	25.160	0.000
ITALY	0.118	-0.025	0.020	0.027	-0.014	-0.039	54.980	0.000
JAPAN	0.022	-0.031	0.001	0.027	-0.031	-0.008	10.260	0.114
NETHRLD	-0.043	-0.024	-0.028	0.038	0.041	0.001	19.950	0.003
SINGAPORE	0.115	-0.066	0.000	0.058	0.029	0.028	71.490	0.000
SPAIN	0.130	0.035	0.015	0.056	0.008	-0.016	69.180	0.000
SWEDEN	0.098	-0.020	-0.022	0.061	0.031	-0.043	53.990	0.000
SWITZERL	-0.008	0.016	-0.037	0.044	0.040	-0.004	16.760	0.010
UK	0.056	0.027	0.028	0.049	0.007	0.008	22.660	0.001

In table 3.4, the correlations between squared contemporaneous and squared lagged returns are reported. The motive behind the analysis of correlation of squared returns is the same as the motive behind ARCH. The findings reveal that volatility clustering is present in all national portfolios of our data sample.

Table 3.4 Autocorrelations of squared returns

Part 1. returns in local currency

Series**2	r1	r2	r3	r4	r5	r6	Q(6)	-p
DENMARK	0.243	0.052	0.035	0.036	0.037	0.038	175.380	0.000
GERMANY	0.174	0.188	0.153	0.094	0.132	0.080	317.380	0.000
FRANCE	0.131	0.337	0.146	0.095	0.234	0.070	576.930	0.000
HONGKONG	0.034	0.005	0.033	0.002	0.086	0.004	25.650	0.000
ITALY	0.175	0.076	0.171	0.103	0.075	0.102	241.340	0.000
JAPAN	0.223	0.081	0.109	0.116	0.080	0.030	232.650	0.000
NETHRLD	0.442	0.478	0.302	0.199	0.280	0.164	1723.710	0.000
SINGAPORE	0.268	0.244	0.249	0.100	0.019	0.062	541.540	0.000
SPAIN	0.261	0.291	0.158	0.176	0.139	0.100	621.500	0.000
SWEDEN	0.289	0.138	0.247	0.210	0.148	0.129	644.710	0.000
SWITZERL	0.120	0.232	0.186	0.103	0.394	0.079	718.000	0.000
UK	0.603	0.274	0.140	0.151	0.109	0.054	1297.220	0.000
US	0.089	0.145	0.065	0.013	0.104	0.024	117.230	0.000

Part 2. returns in US dollars

Series**2	r1	r2	r3	r4	r5	r6	Q(6)	-p
DENMARK	0.086	0.045	0.037	0.012	0.008	0.024	33.460	0.000
GERMANY	0.154	0.181	0.149	0.086	0.094	0.053	308.840	0.000
FRANCE	0.147	0.282	0.137	0.069	0.174	0.076	509.300	0.000
HONGKONG	0.035	0.006	0.033	0.003	0.086	0.005	30.910	0.000
ITALY	0.122	0.074	0.182	0.085	0.036	0.075	213.610	0.000
JAPAN	0.201	0.055	0.084	0.110	0.079	0.034	221.760	0.000
NETHRLD	0.019	0.014	0.010	0.005	0.012	0.013	3.130	0.792
SINGAPORE	0.249	0.242	0.234	0.088	0.016	0.059	592.380	0.000
SPAIN	0.158	0.185	0.075	0.085	0.089	0.103	286.690	0.000
SWEDEN	0.064	0.031	0.049	0.038	0.023	0.025	31.920	0.000
SWITZERL	0.059	0.090	0.069	0.041	0.126	0.029	110.130	0.000
UK	0.476	0.251	0.173	0.182	0.152	0.068	1202.920	0.000

Cross correlations between world market returns and each domestic portfolio are shown in table 3.5 [3]. Japanese and US stocks have the highest contemporaneous correlation with the world factor (0.70) and (0.68) respectively, followed by the UK and the Netherlands at (0.63) and (0.61). Low correlation coefficient estimates are critical in risk diversification.

[3] Columns -r1,..-r6 refer to the number of days that returns in local markets lag the world index. Thus the entry -r3, for example, is the cross-correlation coefficient between the world and a domestic portfolio 3 days later. The column r0 contains the contemporaneous correlations and the columns r1 through r6 refer to the number of days the domestic portfolio leads the world.

The square of the correlation coefficient, known as the R^2, tells us the proportion of a market's variance explained by changes in the world index. Hence the US stocks have a common volatility with the world index of 46% while Italian stocks have a common volatility of only 11%.

Table 3.5 Cross correlations of domestic portfolio returns with the world index

Part 1: returns in local currency

Series	-r6	-r5	-r4	-r3	-r2	-r1	r0	r1	r2	r3	r4	r5	r6
DENMARK	0.00	0.02	0.03	0.01	0.01	0.22	0.35	-0.01	-0.05	0.00	0.08	0.00	0.00
GERMANY	-0.03	-0.01	0.04	-0.01	-0.05	0.10	0.52	0.11	-0.05	-0.05	0.05	0.03	-0.02
FRANCE	0.04	-0.02	0.03	-0.02	-0.01	0.10	0.51	0.13	-0.03	0.01	0.04	0.03	-0.01
HONGKONG	-0.04	0.14	0.11	-0.13	0.02	0.19	0.38	0.00	-0.03	-0.01	-0.02	-0.01	0.00
ITALY	-0.02	0.00	0.02	0.03	-0.03	0.19	0.33	0.04	-0.03	0.00	0.04	-0.02	-0.03
JAPAN	-0.04	0.00	0.06	0.00	-0.05	0.22	0.70	-0.01	-0.07	-0.02	0.06	-0.05	-0.02
NETHRLD	0.01	0.00	0.05	-0.01	-0.05	0.10	0.61	0.11	-0.04	-0.06	0.05	0.03	0.01
SINGAPO	0.03	-0.01	0.06	-0.03	-0.01	0.36	0.36	-0.03	-0.09	0.03	0.08	-0.01	0.03
SPAIN	-0.01	0.04	0.06	0.05	0.05	0.27	0.38	0.05	-0.01	0.00	0.04	-0.03	0.01
SWEDEN	-0.04	0.01	0.04	0.00	-0.03	0.20	0.44	0.08	-0.02	-0.04	0.06	0.02	-0.01
SWITZRL	0.01	0.06	0.09	-0.02	0.01	0.16	0.56	0.11	-0.05	-0.04	0.04	0.04	-0.03
UK	0.02	-0.03	0.08	0.05	-0.04	0.12	0.63	0.12	-0.02	-0.02	0.07	0.03	-0.01
US	-0.02	0.00	0.02	-0.04	-0.05	-0.05	0.68	0.36	-0.06	-0.03	0.00	0.08	-0.04

Part 2: returns in US dollars

Series	-r6	-r5	-r4	-r3	-r2	-r1	r0	r1	r2	r3	r4	r5	r6
DENMARK	0.01	0.01	0.03	0.01	-0.01	0.11	0.31	0.02	-0.05	0.01	0.03	0.01	0.00
GERMANY	-0.03	-0.02	0.04	-0.02	-0.04	0.06	0.45	0.10	-0.03	-0.05	0.06	0.02	-0.01
FRANCE	0.03	-0.02	0.06	-0.02	-0.01	0.07	0.45	0.12	-0.03	0.01	0.03	0.03	0.00
HONGKONG	-0.02	0.12	0.11	-0.10	0.01	0.12	0.30	-0.01	-0.02	-0.01	-0.01	-0.02	0.00
ITALY	0.00	-0.01	0.03	0.03	0.00	0.11	0.32	0.07	-0.02	0.01	0.05	-0.03	-0.03
JAPAN	-0.04	0.02	0.04	0.02	-0.01	0.17	0.65	0.01	-0.04	-0.02	0.05	-0.02	-0.01
NETHRLD	-0.01	0.00	0.07	-0.01	-0.01	0.07	0.44	0.08	-0.04	-0.04	0.04	0.03	-0.01
SINGAPO	0.03	0.02	0.05	-0.01	-0.02	0.26	0.30	-0.01	-0.06	0.03	0.08	0.00	0.01
SPAIN	-0.02	0.04	0.06	0.05	0.03	0.19	0.34	0.06	-0.02	0.01	0.04	-0.02	0.01
SWEDEN	-0.02	0.01	0.05	0.00	-0.04	0.14	0.36	0.07	-0.02	-0.02	0.05	0.01	-0.01
SWITZRL	-0.01	0.02	0.08	-0.03	0.00	0.08	0.43	0.09	-0.03	-0.01	0.04	0.03	-0.02
UK	0.03	-0.02	0.04	0.02	-0.02	0.06	0.25	0.03	-0.01	-0.02	0.05	0.02	-0.02
US	-0.02	0.01	0.03	-0.02	-0.04	-0.05	0.54	0.28	-0.06	-0.01	0.00	0.07	-0.04

Note: -rN indicates the number of days the world market return leads the local portfolio and +rN is the number of days the local portfolio return leads the world market.

For risk monitoring purposes, *i.e.* to estimate the portfolio DEaR, it is necessary to employ unbiased and efficient daily correlation estimates. Historical correlations, however, restrict the common volatility to be equal at each day over the sample. This is inconsistent with empirical evidence reported earlier.

Table 3.6 Cross correlations of domestic portfolio squared returns with the world market

Part 1 : returns in local currency

Series	-r6	-r5	-r4	-r3	-r2	-r1	r0	r1	r2	r3	r4	r5	r6
DENMARK	0.04	0.03	0.02	0.04	0.19	0.58	0.41	0.29	0.06	0.08	0.18	0.03	0.00
GERMANY	0.12	0.25	0.13	0.24	0.21	0.16	0.46	0.16	0.23	0.07	0.06	0.10	0.05
FRANCE	0.20	0.38	0.14	0.22	0.22	0.10	0.56	0.21	0.31	0.10	0.09	0.16	0.04
HONGKONG	0.05	0.74	0.34	0.39	0.02	0.05	0.33	0.05	0.04	0.00	0.08	0.03	0.01
ITALY	0.03	0.07	0.06	0.06	0.06	0.08	0.21	0.09	0.07	0.03	0.03	0.04	0.00
JAPAN	0.03	0.07	0.11	0.08	0.22	0.62	0.47	0.32	0.05	0.10	0.21	0.04	0.01
NETHRLD	0.12	0.29	0.16	0.27	0.46	0.46	0.86	0.36	0.34	0.18	0.15	0.19	0.05
SINGAPO	0.03	0.07	0.18	0.29	0.20	0.79	0.50	0.50	0.19	0.05	0.26	0.10	0.04
SPAIN	0.34	0.27	0.21	0.23	0.09	0.24	0.25	0.15	0.07	0.05	0.08	0.03	0.03
SWEDEN	0.17	0.20	0.15	0.22	0.12	0.23	0.39	0.20	0.12	0.06	0.09	0.06	0.03
SWITZRL	0.10	0.54	0.27	0.40	0.18	0.20	0.71	0.24	0.30	0.06	0.10	0.17	0.04
UK	0.04	0.15	0.10	0.21	0.28	0.69	0.83	0.49	0.29	0.12	0.24	0.17	0.04
US	0.03	0.08	0.05	0.07	0.13	0.12	0.84	0.38	0.42	0.09	0.08	0.27	0.04

Part 2 : returns in US dollars

Series	-r6	-r5	-r4	-r3	-r2	-r1	r0	r1	r2	r3	r4	r5	r6
DENMARK	0.00	0.00	-0.01	0.00	0.05	0.21	0.20	0.11	0.02	0.02	0.07	0.00	-0.01
GERMANY	0.05	0.10	0.08	0.11	0.11	0.09	0.24	0.09	0.10	0.03	0.02	0.03	0.02
FRANCE	0.13	0.17	0.08	0.10	0.10	0.06	0.26	0.12	0.14	0.04	0.03	0.06	0.02
HONGKONG	0.03	0.37	0.26	0.23	0.02	0.02	0.17	0.03	0.02	0.00	0.05	0.02	0.00
ITALY	0.01	0.04	0.02	0.05	0.03	0.05	0.11	0.06	0.02	0.01	0.02	0.00	-0.01
JAPAN	0.03	0.04	0.04	0.05	0.10	0.34	0.32	0.18	0.02	0.04	0.11	0.02	0.00
NETHRLD	0.02	0.03	0.03	0.05	0.08	0.10	0.15	0.07	0.05	0.02	0.02	0.02	0.01
SINGAPO	0.01	0.03	0.09	0.16	0.13	0.41	0.33	0.29	0.10	0.02	0.12	0.05	0.01
SPAIN	0.17	0.10	0.08	0.10	0.04	0.11	0.15	0.07	0.02	0.01	0.02	0.01	0.01
SWEDEN	0.04	0.04	0.03	0.05	0.03	0.07	0.12	0.06	0.03	0.01	0.02	0.01	0.00
SWITZRL	0.03	0.15	0.11	0.13	0.07	0.08	0.24	0.11	0.09	0.02	0.03	0.04	0.01
UK	0.02	0.09	0.08	0.11	0.15	0.30	0.45	0.28	0.17	0.05	0.10	0.08	0.02
US	0.01	0.04	0.03	0.04	0.06	0.08	0.43	0.28	0.24	0.05	0.03	0.13	0.02

Changes in national equity indices, like single stocks, may be uncorrelated in the means, but may still be related through their second moments. Table 3.6 reports the cross and lagged cross correlations of squared returns between each national portfolio and the world index. A high coefficient of current and/or leading (lagged) squared returns reveals that an increase in volatility in the world portfolio tends to be followed by an increase in current and/or past (future) volatility on a local portfolio. A *lagged* strong coefficient further indicates that shocks in the world market will affect price movements on domestic markets over the next few days. However, the response of the local portfolio does not necessarily have to be of the same manner (sign) with the changes in price of the world market. The econometric model reported in this chapter has been designed to capture the conditional volatility for both world and local markets jointly with their cross-dependence.

Table 3.7 reports the OLS estimates and residual analysis of the linear SIM in equation (1.4). In the second column from the right are reported the Ljung-Box statistic for (non) serial correlation of order 6. Here again, we can observe on almost all residual series a significant serial correlation. The last column reports the Jarque-Bera test for normality. We found strong deviations from the Gaussian distribution on almost all national equity portfolios, both in local currency and US dollars. This is not surprising since the model of (1.4) fits a linear relationship between local portfolios and market return and thus will not reveal any non-linear relationship which might affect the second moments of the series.

Table 3.7. OLS estimates of Single Index Model (1.4)

	returns in local currency					returns in US$				
Country	α	β	R^2	Q(6)	JB	α	β	R^2	Q(6)	JB
Denmark	0.000	0.347	0.125	87.640	255.40	0.000	0.371	0.093	24.999	17.66
	(0.45)	(19.33)		(0.00)	(0.00)	(0.89)	(16.39)		(0.00)	(0.00)
Germany	0.000	0.893	0.266	17.802	160.19	0.000	0.720	0.198	21.817	29.42
	(0.37)	(30.73)		(0.01)	(0.00)	(0.45)	(25.38)		(0.00)	(0.00)
France	0.000	0.812	0.261	12.501	18.14	0.000	0.668	0.203	6.350	0.13
	(0.28)	(30.38)		(0.05)	(0.00)	(0.9	(25.78)		(0.39)	(0.94)
Hongkong	0.000	0.865	0.141	67.253	11559.07	0.000	0.588	0.088	43.661	13175.68
	(1.37)	(20.68)		(0.00)	(0.00)	(1.41)	(15.85)		(0.00)	(0.00)
Italy	0.000	0.578	0.107	35.210	379.07	0.000	0.559	0.101	22.274	246.90
	(0.32)	(17.68)		(0.00)	(0.00)	(0.40)	(17.13)		(0.00)	(0.00)
Japan	0.000	1.318	0.487	15.222	356.08	0.000	1.179	0.416	16.272	1369.22
	(1.24)	(49.80)		(0.02)	(0.00)	(0.27)	(43.13)		(0.01)	(0.00)
Nethrld	0.000	0.785	0.367	52.969	1.53	0.000	0.584	0.190	50.967	25777.02
	(1.37)	(38.91)		(0.00)	(0.47)	(1.61)	(24.75)		0.00	0.00
Singapore	0.000	0.673	0.129	27.242	2955.11	0.000	0.495	0.089	31.429	2318.21
	(1.11)	(19.61)		(0.00)	(0.00)	(1.65)	(15.94)		(0.00)	(0.00)
Spain	0.000	0.587	0.142	65.948	73.96	0.000	0.535	0.119	23.429	55.10
	(1.32)	(20.74)		(0.00)	(0.00)	(1.25)	(18.72)		(0.00)	(0.00)
Sweden	0.000	0.755	0.191	24.449	0.07	0.000	0.612	0.130	18.024	636.85
	(1.45)	(24.82)		(0.00)	(0.96)	(1.34)	(19.77)		(0.01)	(0.00)
Switzerl	0.000	0.766	0.308	19.567	338.81	0.000	0.628	0.186	23.063	1205.93
	(0.61)	(34.09)		(0.00)	(0.00)	(0.47)	(24.42)		(0.00)	(0.00)
Uk	0.000	0.824	0.392	6.192	62.45	0.000	0.364	0.064	18.806	19.96
	(0.83)	(40.96)		(0.40)	(0.00)	(0.85)	(13.32)		(0.01)	(0.00)
Us	0.000	1.004	0.456	110.391	267.56	0.000	0.686	0.288	44.480	2023.04
	(1.01)	(46.72)		(0.00)	(0.00)	(1.03)	(32.46)		(0.00)	(0.00)

Note: For columns α and β t-values are shown in parenthesis. For the columns entitled Q(6) and JB, p-values are shown.

3.2 Empirical Results

3.2.1 Modelling the Conditional Betas.

To compute the portfolio DEaR and VaR, it is necessary to get daily estimates of the variance-covariance matrix Ω. The portfolio risk as estimated in (1.2) relies upon a very strong assumption; the mean, m, and variance, σ^2, that the series of returns, Y, does not change over the measurement period. However, as we have seen in chapter two, empirical studies have found that asset variances and covariances are not constant but change over time, *e.g.* Christie (1982).

Multivariate GARCH models are ideal to capture changes on the joint second moments of a set of security returns. However, the need to estimate a large number of parameters on a 13 variable system makes multivariate GARCH computationally infeasible. As mentioned in chapter two an alternative solution is provided by using thirteen bivariate GARCH(1,1) systems, each one consisting of an individual local equity portfolio, and the world index. If Y_1, Y_2 are the series of returns for the locally diversified portfolio and the world index, the bivariate GARCH system can be written as[4]:

$$Y_{1,t} = \Phi_1 \varepsilon_{1,t-1} + \varepsilon_{1,t} \tag{3.1.a}$$

$$Y_{2,t} = \Phi_2 \varepsilon_{2,t-1} + \varepsilon_{2,t} \tag{3.1.b}$$

$$\begin{vmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{vmatrix} \sim N(0, H_t) \quad \text{where} \quad H_t \equiv [H_{ij,t}] \quad \text{is a 2x2 symmetric matrix:}$$

The terms Φ_1 and Φ_2 in the mean equation are set to capture any serial correlation which may be caused by thin trading (see Pagan and Schwert (1990) p. 271). For reasons explained in chapter two, it is more appropri-

[4] The mean specification has been based on a number of diagnostic tests. The ARMA(0,1) account for serial dependencies which were present in most series. As we will latter this mean specification leaves white noise residuals. We also considered a higher MA or AR order but the likelihood ratio tests rejected this hypothesis.

ate to restrict the variance to its diagonal specification, see Engle *et al.* (1984). This is similar to the specification in (2.16). The specification of the conditional variance covariance equation is the following:

$$h_{11,t} = \omega_{11} + a_{11}\, \varepsilon_{1,t-1}^{2} + b_{11}\, h_{11,t-1} \tag{2.16.a}$$

$$h_{12,t} = \omega_{12} + a_{12}\, \varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{12}\, h_{12,t-1} \tag{2.16.b}$$

$$h_{22,t} = \omega_{22} + a_{22}\, \varepsilon^{2}_{2,t-1} + b_{22}\, h_{22,t-1} \tag{2.16.c}$$

The conditional variance, $h_{11,t}$ and $h_{22,t}$, are expressed as past realisations of each return series squared disturbances. The covariance of the two return series, $h_{12,t}$, is a function of the cross product between past disturbances in the two markets. The ratio $\frac{h_{12,t}}{h_{22,t}}$ forms the local portfolio's time-varying *beta*. The conditional *beta* for each local portfolio with the market is estimated individually from the rest of the series.

3.2.2 The Advantanges of our Factor Model

The above model can be seen as the conditional parameterisation of the SIM. From a statistical perspective, the SIM in (1.4) arises from the assumption that the return on an individual asset and the return on the market have a bivariate normal distribution. The bivariate GARCH is based on the same assumption but allows the second moments of the joint distribution to be conditional on past returns. Another distinctive feature of this model is that it captures the shifts in time not only of the systematic component of a security's risk but also of its unsystematic counterpart. Thus, the above proposed model may provide an answer to questions such as *it' is true whether any change in a security's volatility is due to its systematic or specific counterpart* or *whether changes in various security betas are correlated.*

King *et al.* (1994) used a multifactor model to explain the changes in variances and covariances on monthly returns of sixteen national stock markets. They model the variance-covariance matrix of excess market returns as a function of the innovations in a set of both observable[5] economic variables (or factors) and unobservable factors. The unobservable factors are correlated with market returns but are constructed to be orthogonal with the observable factors. Both types of factors are zero mean and their conditional variances are modelled as GARCH(1,1) process.

The King *et al.* model has several similarities with the one followed in our study, of equations (2.16) and (3.1). Both models allows for national markets interdependence through their common movement with observable factor(s)[6]. In addition the variance of these factors, as in our model, is modelled conditionally as a GARCH(1,1) process. Similarly, the King *et al.* model divides risk into systematic and specific time-varying components. Moreover, in their investigation, King *et al.*, the systematic risk is partitioned to further counterparts (which are due to observable and unobservable factors).

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[5] These factors however are not literally observable. They are mapped out from economic variables.

[6] One observable factor in our model.

However, our approach differs in some aspects from that of King *et al.* in an attempt to overcome the limitations of their investigation. For instance, King et al. use monthly data from 1970-1988 whereas in this study we employ daily data (approximately 2400 more observations for each returns series) collected during more recent period; hence, our model fits better with the current dynamics that govern the global market prices[7]. Furthermore, our study also differs in the identification of the factors and the way factors are allowed to influence the conditional mean of the data. Our approach involves the use of the market model to explain the common movement of the different national equities. It aims to simplify the computation of a time-varying correlation matrix. It is not a pricing or arbitrage model. On the other hand, the King *et al.* model is more general since it includes unobservable factors as well observable ones. Furthermore, the King *et al.* study is based on an equilibrium pricing model which formulates the conditional mean of the market excess returns to be a linear combination of the risk premia associated with each factor. It, however, restricts the "price of the risk" (that is the sensitivity of each market to the changes of the variance of the corresponding factor) on each market to be constant across time. However, Harvey *et al.* (1992) shown that the "price of the risk" does change over different periods. In our model the conditional mean, on both market index and local portfolio returns, depends only on its own past returns.

The major advantage of our approach is that it allows different national markets to be linked through a single factor. This factor is observable (not notional) and its values are changing daily. In the King *et al.* study the (economic) factors are notional; they are calculated as unobservable factors (using Kalman filter) but correlated to economic variables, hence

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[7] Yet, both studies cover the 1987 crash.

being denoted as observable. Given that these are economic variables they do not change each day. Our model is computationally feasible and can be applied to portfolios of any size (since in our model the number of unknown parameters grows linearly with the number of assets in the portfolio). The King *et al.* model is computationally intensive and relies on factors that change monthly; hence it is unsuited in the VaR analysis. On the other hand the King *et al.* is an equilibrium model and it can be used for arbitrage trading.

3.2.3 Testing for ARCH

The non-linear system in (2.16) has a large number of parameters, so hypothesis tests are necessary before any estimation is attempted. A test for ARCH has been proposed by Engle (1982) and is based on the Lagrange Multiplier (LM) principle. It is straightforward since it only requires estimates of the homoskedastic model. Engle *et al.* (1984) extended the test to deal with bivariate ARCH models.

For the diagonal model, the test consists of three times the sample size times the sum of the R^2 of the three following regressions.

$$\hat{\varepsilon}^2_{1,t} = \varpi_{1,0} + \sum_{i=1}^{p} \varpi_{1,i}\, \hat{\varepsilon}^2_{1,t-i} + e_{1,t} \tag{3.2.a}$$

$$\hat{\varepsilon}^2_{2,t} = \varpi_{2,0} + \sum_{i=1}^{p} \varpi_{2,i}\, \hat{\varepsilon}^2_{2,t-i} + e_{2,t} \tag{3.2.b}$$

$$\hat{\varepsilon}_{1,t}\, \hat{\varepsilon}_{2,t} = \varpi_{12,0} + \sum_{i=1}^{p} \varpi_{12,i}\, \hat{\varepsilon}_{1,t-i}\, \hat{\varepsilon}_{2,t-i} + e_{12,t} \tag{3.2.c}$$

Where $\hat{\varepsilon}_{1,t}$ and $\hat{\varepsilon}_{2,t}$ are the residuals from the autoregression of Y_1 and Y_2 series respectively.

Table 3.8 LM test for ARCH.

COUNTRY	return in local currency	return in US$
DENMARK	1770.622	194.348
	(0.00)	(0.00)
GERMANY	3392.561	263.828
	(0.00)	(0.00)
FRANCE	4899.281	367.580
	(0.00)	(0.00)
HONGKONG	4152.499	335.350
	(0.00)	(0.00)
ITALY	2888.320	708.908
	(0.00)	(0.00)
JAPAN	1786.135	83.005
	(0.00)	(0.00)
NETHRLD	2952.367	20.140
	(0.00)	(0.512)
SINGAPORE	6556.192	2626.180
	(0.00)	(0.00)
SPAIN	5101.521	1474.213
	(0.00)	(0.00)
SWEDEN	3126.162	140.659
	(0.00)	(0.00)
SWITZERL	3784.999	73.221
	(0.00)	(0.00)
UK	2211.626	2058.508
	(0.00)	(0.00)
US	3124.471	968.839
	(0.00)	(0.00)

Note: Table shows the X^2 statistics with probability values in parenthesis.

When H_0 for homoskedasticity is tested against an ARCH(p) process then the LM statistic is asymptotically equivalent to $3N(R_1^2 + R_2^2 + R_{12}^2)$, where R^2 is the coefficient of determination in each equation. In this case the LM statistic has a $\chi^2(3p)$ asymptotic distribution, where p is the number of lagged regressors in each equation.

The test was performed on all thirteen local portfolios for a lag order of six and the results are shown in table 3.8. The test results reject the null hypothesis of non ARCH errors at the 99% confidence level, across all bivariate sets for both local and dollar returns. The LM tests here affirm the earlier hypothesis that stock return variances follow an autoregressive process. Furthermore, the above modified test indicates that each local portfolio's covariance with the world market, and so its *beta*, will probably follow an ARCH process.

3.2.4 Estimation Issues

A bivariate GARCH was constructed for each of the thirteen local portfolios with the world market index as formulated in (3.1) and (2.7). Numerical maximisation of the likelihood function for each individual bivariate set was achieved using the Berndt *et al.* (1974) algorithm. One condition that needs always to be satisfied when estimating multivariate GARCH models is that the determinant of the variance covariance matrix **H** be positive at each function valuation. Two methods are commonly used to achieve this. The first, introduced by Bollerslev (1990), treats the correlation coefficient, ρ, between $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ as an unknown (constant) parameter. The conditional variances $h_{1,t}$ and $h_{2,t}$ are modelled as a GARCH(p,q) process and so the covariance $h_{12,t}$ estimated as conditional on $h_{1,t}$ and $h_{2,t}$ must guarantee the restriction $\rho = \frac{h_{12,t}}{\sqrt{h_{1,t} h_{2,t}}}$ at all periods.

The constant correlation method has been used in many bivariate specifications, *e.g.* Baillie and Bollerslev (1990), Kroner and Sultan (1991).

This bivariate specification guarantees that **H** is positive definite at each t. However, it relies on strong assumptions which have some adverse consequences on the results such as imposing the covariance $h_{12,t}$ to be always of the same sign. An alternative solution has been suggested by Baba *et al.* (1990). Nevertheless, this method imposes some restrictions on the sign of the parameters and does not always guarantee positive definitiveness in H_t.

Rather than restricting the parameters in our calculations in any way, we imposed a penalty on the likelihood function to guarantee that H_t is always positive definite. Initial values for the parameters have been obtained by maximising the likelihood with the downhill simplex algorithm. This algorithm is almost insensitive to bad starting values and robust to discontinuities which may arise because of the penalty imposed in the likelihood function.

3.2.5 Parameter Estimates

Parameter estimates with the *-t* statistics for each bivariate GARCH are reported in table 3.9. The *-t* statistics on the conditional variances and covariance equations strongly support the existence of ARCH effects and appear to validate the model specification. The coefficients, a_{11}, a_{22} and a_{12} measure the impact of last period's squared innovation $\varepsilon_{i,t-1}^2$, $\varepsilon_{j,t-1}^2$ on conditional variances $h_{1,t}$, $h_{2,t}$ and the cross product, $\varepsilon_{i,t-1}\varepsilon_{j,t-1}$, on conditional covariance $h_{12,t}$. These are positive and significant in each bivariate set.

The coefficients b_{11}, b_{22} and b_{12} which measure the long run joint GARCH process among each of the local equity portfolios and the world index are positive and significant across all thirteen systems of equations. In addition, the condition $a+b<1$ holds for every variance and covariance equation indicating a stationary GARCH process. Furthermore, for every conditional variance and covariance equation we have $a+b>0$. Thus, the homoskedastic model, which is a special case when $a+b=0$, can be strongly rejected in all cases. In particular, the coefficient b_{12}, which accounts for the conditional covariance between national and world markets, is positive and statistically significant across each bivariate system, supporting the existence of a strong interaction between domestic stock prices and the rest of the world. Thus, a strong and positive relationship between past errors and conditional variances is found to hold between every local market and the world index. Equally significant results are obtained when returns are translated to US dollars. However the values of the likelihood functions are now smaller across all 13 sets of equations indicating the difficulty of fitting a conditional model when returns are contaminated with exchange rates.

The results indicate that the bivariate GARCH model captures sizeable changes over time not only in stocks' overall risk but also in the two risk components as well. Such a model parameterisation overcomes previous criticisms of the linear SIM for not allowing assets' risk to change over time. The model proposed here treats both systematic risk and total risk as time varying. In contrast to smoothing techniques, described in the first chapter, which just filter the series from a noise term, our bivariate GARCH approach exploits the information contained in past realisations of returns which affect the whole structure of the variance-covariance relationship between an asset and the market. Thus, it allows each national market's volatility to be a function of the past disturbances arising in its own and the world market's return.

TABL 3.9 Coefficient estimates for the bivariate GARCH in model of (2.1) and (3.1)
Part 1 returns in local currency

COUNTRY	Φ_1	Φ_2	ω_1	ω_2	ω_{12}	a_1	a_2	a_{12}	b_1	b_2	b_{12}	ML
DENMARK	0.314	0.206	3.977	1.705	0.529	0.148	0.089	0.045	0.743	0.858	0.882	-13852.89
	(15.543)	(10.448)	(8.643)	(5.128)	(3.191)	(9.374)	(7.302)	(4.254)	(32.047)	(44.895)	(30.797)	
GERMANY	0.053	0.182	1.915	1.226	0.572	0.059	0.069	0.036	0.923	0.892	0.937	-15178.73
	(2.757)	(9.761)	(4.918)	(5.211)	(4.139)	(8.241)	(7.510)	(6.379)	(100.394)	(63.585)	(94.383)	
FRANCE	0.081	0.199	3.939	1.220	0.923	0.072	0.068	0.044	0.889	0.893	0.918	-15053.34
	(4.208)	(10.892)	(5.013)	(5.445)	(4.560)	(7.743)	(7.952)	(6.887)	(62.451)	(67.201)	(76.212)	
HONG KONG	0.127	0.188	4.813	1.206	0.365	0.081	0.071	0.037	0.887	0.891	0.940	-15725.02
	(6.795)	(9.712)	(5.819)	(5.108)	(2.876)	(9.553)	(7.324)	(5.483)	(79.859)	(61.201)	(81.975)	
ITALY	0.237	0.213	1.243	1.498	0.410	0.051	0.085	0.034	0.939	0.868	0.934	-15442.20
	(12.366)	(10.969)	(4.131)	(5.201)	(2.651)	(9.236)	(7.453)	(4.758)	(136.577)	(50.381)	(56.872)	
JAPAN	0.084	0.152	1.343	0.561	0.548	0.069	0.048	0.052	0.919	0.932	0.931	-14756.29
	(5.145)	(9.406)	(6.219)	(5.616)	(5.572)	(12.578)	(9.477)	(11.409)	(159.213)	(131.669)	(163.195)	
NETHRLD	0.015	0.157	1.165	1.036	0.499	0.054	0.065	0.036	0.925	0.902	0.936	-14197.01
	(0.825)	(8.704)	(4.273)	(5.408)	(4.330)	(8.046)	(8.262)	(7.142)	(93.434)	(77.235)	(102.432)	
SINGAPORE	0.128	0.210	13.942	1.540	0.416	0.165	0.089	0.034	0.685	0.862	0.919	-15138.58
	(6.024)	(10.763)	(7.280)	(5.182)	(2.893)	(8.990)	(7.475)	(3.850)	(21.246)	(47.567)	(48.643)	
SPAIN	0.206	0.209	2.328	0.911	0.391	0.086	0.063	0.048	0.890	0.908	0.926	-14980.70
	(10.474)	(10.967)	(7.489)	(5.049)	(3.885)	(10.878)	(8.151)	(7.647)	(92.378)	(81.598)	(97.524)	
SWEDEN	0.137	0.195	2.873	1.360	0.651	0.085	0.078	0.056	0.893	0.878	0.906	-15308.43
	(6.958)	(10.297)	(5.288)	(5.135)	(3.987)	(9.502)	(7.188)	(6.985)	(79.120)	(54.209)	(65.551)	
SWISS	0.083	0.187	4.454	1.283	0.829	0.089	0.070	0.056	0.841	0.888	0.891	-14540.67
	(4.337)	(10.063)	(5.833)	(4.945)	(4.782)	(8.531)	(7.337)	(7.154)	(45.057)	(57.118)	(57.352)	
UK	0.030	0.201	2.386	1.380	1.251	0.056	0.072	0.043	0.907	0.884	0.903	-14351.97
	(1.679)	(11.721)	(3.693)	(5.883)	(3.676)	(5.794)	(8.770)	(5.854)	(53.286)	(66.596)	(45.684)	
US	0.061	0.220	1.264	1.913	0.932	0.037	0.101	0.039	0.943	0.838	0.928	-13972.58
	(3.710)	(13.519)	(5.401)	(7.897)	(7.175)	(7.005)	(11.999)	(8.085)	(122.210)	(67.689)	(121.505)	

ote t statistic in parentheses .

Part 2 returns in US dollars

COUNTRY	Φ_1	Φ_2	ω_1	ω_2	ω_{12}	a_1	a_2	a_{12}	b_1	b_2	b_{12}	ML
DENMARK	0.134	0.153	2.381	2.338	1.090	0.054	0.083	0.062	0.917	0.861	0.859	-15356.01
	(7.113)	(7.971)	(4.454)	(4.592)	(2.828)	(7.324)	(6.645)	(5.618)	(77.997)	(41.428)	(25.881)	
GERMANY	0.025	0.123	2.456	1.766	0.508	0.044	0.066	0.028	0.938	0.892	0.953	-15939.88
	(-1.321)	(6.732)	(3.385)	(4.780)	(3.623)	(6.274)	(6.627)	(5.833)	(90.888)	(56.571)	(126.042)	
FRANCE	0.032	0.131	4.733	1.495	0.623	0.063	0.059	0.037	0.900	0.905	0.942	-15770.15
	(1.659)	(7.294)	(4.150)	(4.743)	(3.535)	(7.043)	(6.836)	(6.764)	(60.162)	(65.698)	(107.955)	
HONGKONG	0.122	0.133	4.729	1.973	0.281	0.080	0.072	0.029	0.888	0.881	0.950	-16163.75
	(6.408)	(6.763)	(5.667)	(4.367)	(2.326)	(9.224)	(6.188)	(4.509)	(77.120)	(46.259)	(85.067)	
ITALY	0.145	0.144	2.454	1.787	0.524	0.054	0.072	0.031	0.931	0.886	0.945	-16270.82
	(7.493)	(7.592)	(3.828)	(4.460)	(2.445)	(7.826)	(6.612)	(4.669)	(102.445)	(52.181)	(71.127)	
JAPAN	0.033	0.088	1.834	0.433	0.540	0.057	0.041	0.045	0.932	0.948	0.944	-15602.35
	(1.955)	(5.489)	(5.174)	(3.724)	(4.215)	(11.261)	(9.848)	(11.277)	(164.561)	(159.272)	(185.830)	
NETHRLD	-0.040	0.094	2.057	1.516	0.367	0.045	0.064	0.027	0.926	0.899	0.956	-15016.88
	(2.208)	(5.204)	(4.156)	(5.072)	(3.756)	(7.259)	(7.456)	(6.564)	(83.511)	(69.540)	(145.925)	
SINGAPORE	0.117	0.153	15.282	2.404	8.049	0.157	0.086	0.064	0.691	0.857	0.164	-15660.91
	(5.476)	(7.741)	(6.842)	(4.383)	(2.658)	(8.564)	(6.424)	(3.431)	(20.332)	(38.120)	(0.574)	
SPAIN	0.085	0.129	5.600	1.717	0.599	0.077	0.069	0.032	0.882	0.890	0.942	-15952.20
	(4.538)	(6.728)	(4.953)	(4.691)	(3.268)	(8.224)	(6.900)	(5.028)	(58.982)	(56.520)	(79.738)	
SWEDEN	0.081	0.126	2.538	1.971	0.867	0.064	0.069	0.053	0.921	0.883	0.908	-16071.95
	(4.273)	(6.647)	(4.250)	(4.519)	(3.655)	(9.518)	(6.354)	(7.473)	(103.665)	(48.777)	(65.904)	
SWISS	-0.025	0.129	6.459	2.138	0.886	0.068	0.073	0.049	0.873	0.875	0.911	-15671.77
	(1.324)	(6.957)	(4.460)	(4.566)	(3.635)	(6.645)	(6.359)	(6.515)	(44.543)	(44.831)	(63.339)	
UK	0.028	0.163	2.662	1.798	0.316	0.038	0.072	0.037	0.939	0.885	0.913	-15942.97
	(1.388)	(8.337)	(2.736)	(4.206)	(2.065)	(5.214)	(6.256)	(4.738)	(71.261)	(48.193)	(38.358)	
US	-0.041	0.149	0.343	1.003	0.223	0.025	0.054	0.022	0.969	0.922	0.969	-14738.08
	(2.440)	(8.665)	(3.750)	(4.765)	(4.033)	(6.924)	(9.385)	(7.562)	(217.445)	(107.567)	(242.105)	

Note: *t* statistic in parentheses

Such a model specification is consistent with some stylised facts which characterise asset returns such as the clustering in time of large forecast errors. Thus, large past values in the error term ε will lead to a covariance matrix $\mathbf{H}$ with larger elements and to a greater likelihood of another large value of ε in the immediate future.

Almost all the parameter estimates were statistically significant in each set of GARCH equations therefore suggesting that our specification computes more efficient and more efficient risk estimates than those obtained using the linear least squares method. In addition, the GARCH parameterisation presented here captures the time path of the risk and its components rather than their average value as is the case with historical risk analysis or with the various adjustment or smoothing methods described in chapter two.

3.2.6 Diagnostic Tests

Diagnostic tests have been carried out on the fitted residuals, $\hat{\varepsilon}$, derived from each bivariate system. It is anticipated that the GARCH parameterisation of the SIM will remove any heteroskedasticity and serial correlation from the series and will leave white noise residuals. Failure of this test may reveal model miss-specification. Table 3.10 contains estimates of the regression:

$$\hat{\varepsilon}^2_{1,t} = \alpha + \beta \hat{h}_{1,t} \tag{3.3}$$

with heteroskedasticity-consistent, White (1980), -t statistics given in parentheses. As Pagan and Ullah (1988) shows, if the forecasts of h_1 are unbiased then $\alpha=0$ and $\beta=1$. This hypothesis is accepted for all local portfolios except Switzerland and US when returns are expressed in local terms and for the Netherlands when returns are in US dollars. The coefficient of determination in (3.3), R^2, measures the fraction of the total variation of everyday returns explained by the estimated conditional variance. Given the difficulty of forecasting daily stock market data, the R^2 coefficients are quite large, indicating a high degree of predictability in next day's volatility. When returns are in US dollars the GARCH model can on average predict one third of next day's (squared) price movements. Given the difficulty which arises with predicting the daily changes of speculative prices, this is a remarkable result.

A second test has been carried out to detect the presence of serial correlation and higher order ARCH effects. This is based on the Ljung-Box statistic and has been applied to the standardised residuals and squared standardised residuals for the local market returns equation. The results are reported in the same table together with the p-values. As can be seen, the serial correlation has been removed for all series but four, Italy, Spain, Switzerland and US. Recalling that, based on the OLS residuals, the same test statistics were much higher this suggests that our ARMA specification has removed time dependence among the price changes. In the next column are reported the Ljung-Box statistics of order six on the squared standardised residuals.

TABL 3.10 Diagnostic test for the GARCH residuals

SERIES	returns in local currency						returns in US $					
	α	β	R^2	Q(6)-1	Q(6)-2	JB	α	β	R^2	Q(6)-1	Q(6)-2	JB
Denmark	3.08	0.90	0.264	10.422	6.428	72.380	0.23	0.99	0.307	3.523	10.871	6.310
	(0.91)	(0.94)		(0.10)	(0.37)	(0.00)	(0.03)	(0.12)		(0.74)	(0.09)	(0.04)
Germany	-7.05	1.06	0.367	8.012	12.720	7.790	-12.89	1.09	0.353	7.92	6.716	18.330
	(0.85)	(0.72)		(0.24)	(0.05)	(0.02)	(0.95)	(0.90)		(0.24)	(0.35)	(0.00)
France	-11.51	1.11	0.350	8.749	7.583	17.680	-11.58	1.09	0.339	10.629	12.717	9.500
	(1.12)	(1.0)		(0.18)	(0.27)	(0.00)	(0.80)	(0.71)		(0.10)	(0.05)	(0.01)
HongKong	-10.83	1.09	0.337	8.312	19.486	3.250	-10.39	1.09	0.336	8.287	18.932	3.260
	(0.89)	(0.98)		(0.21)	(0.00)	(0.20)	(0.85)	(0.96)		(0.22)	(0.00)	(0.20)
Italy	0.41	1.00	0.334	13.118	12.979	37.820	1.30	1.00	0.339	17.828	9.873	10.390
	(0.05)	(0.03)		(0.04)	(0.04)	(0.00)	(0.10)	(0.01)		(0.01)	(0.13)	(0.01)
Japan	6.97	0.99	0.344	3.599	14.994	3.050	4.77	0.99	0.342	5.253	20.135	0.210
	(0.92)	(0.10)		(0.73)	(0.02)	(0.22)	(0.41)	(0.08)		(0.51)	(0.00)	(0.90)
Nethrld	-10.27	1.18	0.360	6.594	14.585	71.540	-21.40	1.31	0.351	6.783	12.458	14.690
	(1.73)	(1.61)		(0.36)	(0.02)	(0.00)	(2.28)	(2.20)		(0.34)	(0.05)	(0.00)
Singapore	-12.53	1.14	0.332	8.966	4.343	2.280	-13.06	1.14	0.315	7.890	4.142	7.990
	(1.43)	(1.39)		(0.18)	(0.63)	(0.32)	(1.27)	(1.22)		(0.25)	(0.66)	(0.02)
Spain	2.65	0.98	0.323	22.830	32.926	63.150	-12.98	1.11	0.321	24.555	18.177	10.480
	(0.40)	(0.29)		(0.00)	(0.00)	(0.00)	(0.97)	(1.00)		(0.00)	(0.01)	(0.01)
Sweden	-1.76	0.99	0.354	12.122	6.700	29.990	1.01	0.97	0.338	18.425	16.544	39.670
	(0.19)	(0.09)		(0.06)	(0.35)	(0.00)	(0.08)	(0.33)		(0.01)	(0.01)	(0.00)
Switzrl	-20.08	1.33	0.349	17.169	9.484	68.110	-18.65	1.18	0.330	8.268	10.258	18.830
	(2.66)	(2.58)		(0.01)	(0.15)	(0.00)	(1.26)	(1.24)		(0.22)	(0.11)	(0.00)
UK	-11.17	1.17	0.376	5.727	7.428	14.370	-15.78	1.14	0.334	6.076	4.355	12.250
	(1.45)	(1.40)		(0.45)	(0.28)	(0.00)	(0.97)	(0.96)		(0.42)	(0.63)	(0.00)
US	-17.39	1.29	0.317	30.568	3.400	16.100	-5.10	1.09	0.312	21.646	3.714	29.930
	(2.26)	(2.31)		(0.00)	(0.76)	(0.00)	(0.79)	(0.87)		(0.00)	(0.72)	(0.00)

Note: The Q statistics, of order 6, and significance levels for the standardised residuals and the squared standardised residuals (local portfolio) are in the fourth and fifth columns respectively. In parenthesis, below the estimates, α and β, are: first column, the t-statistics for testing α=0, second column the t-statistic for the hypothesis β=1. The last column shows the Jarque-Bera normality test.

The GARCH effect have now been removed for most series. However, in some cases, *i.e.* Hong Kong, Japan and Spain, there is a need to encompass a GARCH process of higher order. Finally, under the column JB reports the Jarque-Bera statistics for normality. When returns are measured in US dollars, the null hypothesis of normality cannot be rejected for two local portfolios, Hong-Kong and Japan. Nevertheless, in all other cases the statistic is much lower than the OLS model, indicating that most of the excess kurtosis has been removed. The Jarque-Bera test, however, shows that it is easy to accept normality when the variance is non constant.

Clearly, the diagnostic test results are very satisfactory and when compared with the equivalent OLS tests then allow us to reach the conclusion that the GARCH parameterisation followed in this chapter (although very general and simple), has removed the non-linearities for the majority of the local portfolios examined. Furthermore, it has indicated the common origins of risk between national stocks and the world factor. However, it remains to be seen how effective these beta estimates are in measuring portfolio risk.

3.2.7 The Time Changes in the National Markets Risk

By defining a portfolio's residual variance to be the difference between total variance, $h_{11,t}$, and the market related variance, $\left(\frac{h_{12,t}}{h_{22,t}}\right)^2 h_{22,t}$ the time path for each local portfolio's idiosyncratic variance can be obtained. The series together with the systematic (conditional) risk are graphically presented in the figures 3A.I to 3A.XIII, found in appendix 3A. In each page, the chart on the top shows the two *beta* series, the time varying (conditional) and the constant (historical). Given the significance and sign of coefficients a_{12} and b_{12}, the decision to allow the covariance to be time varying rather than restrict it to be a constant is justified. Conditional *beta* indicates the way each local portfolio responds to current (world) market conditions. Historical *beta* restricts a markets' response to be equal across different periods. Our study provides evidence against the constant beta hypothesis. We can notice that they are many similarities and differences in the way various conditional *betas* behave over time. Understanding how national stocks may respond to current world market conditions may provide insights into the way volatility is transmitted from one national market to another. Such information will be very valuable in predicting portfolio risk. Questions like *did the volatility during the 1987 crash increase because of a rise in the systematic or the specific component of a local market's risk? are the changes in national markets' systematic and specific risk related to, or independent of, the analogous changes in the rest of the national markets*? Such questions may be answered by studying the time structure of the conditional risk series. The analysis of the national conditional betas and the search for any patterns in the way volatility changes will be the focus of chapter five.

These questions can be answered partly in a less scientific way by observing the chart in the middle on each page. This shows the share of systematic in total risk. It is clear that both the volume of each risk component and its share in the total volatility vary across countries. The share of systematic in overall risk remains unstable over time, indicating that an increase in volatility may arise from either, domestic uncertainty or international turmoil. For most country portfolios, on average less than a quarter of the volatility can be attributed to the world market factor while the remaining three quarters is due to that country's specific events. By contrast, for Japan and US equities, the systematic part accounts for more than half of total risk. Given the relatively low volatility of US equities, it can be argued that purely domestic US factors have had a relatively small impact on US stocks' volatility. However, considering that together US and Japanese stocks count for over 70% of the world index capitalisation, it can be argued that their specific and world systematic risk cannot easily be segregated.

The lower chart on each page exhibits the time path of both systematic and specific conditional risk for each national stock market. The diagram shows the total (conditional) volatility at the end of each week with the two counterpart as overstacking series.

3.3 Estimating Portfolio DEaR

To illustrate how our methodology can be used to monitor portfolio risk we constructed a hypothetical portfolio, diversified across all thirteen national markets of our data sample. We then calculated its DEaR over the 10 year data period and used the exponential smoothing (ES) risk estimates to compare our GARCH model's effectiveness in monitoring portfolio risk.

To form this hypothetical portfolio we weighted each national index in proportion to its capitalisation in the world index as on December 1995. The portfolio weights are reported in the next table:

Table 3.11 Portfolio weights at Dec. 95

country	our portfolio	world index
DENMARK	0.004854	0.004528
FRANCE	0.038444	0.035857
GERMANY	0.041905	0.039086
HONG KONG	0.018918	0.017645
ITALY	0.013626	0.012709
JAPAN	0.250371	0.233527
NETHERLAND	0.024552	0.022900
SINGAPORE	0.007147	0.006667
SPAIN	0.010993	0.010254
SWEDEN	0.012406	0.011571
SWITZERLAND	0.036343	0.033898
UK	0.103207	0.096264
US	0.437233	0.407818

The 10 year historical returns of the thirteen national indexes have been weighted according to the above numbers to form the returns of our hypothetical portfolio. Since portfolio losses need to be measured in one currency, we expressed all local returns in US dollars and then formed the portfolio's historical returns. Table 3.12 reports the portfolio's descriptive statistics together with the Jarque-Bera normality test. The last column is the probability that our portfolio returns are generated from a normal distribution.

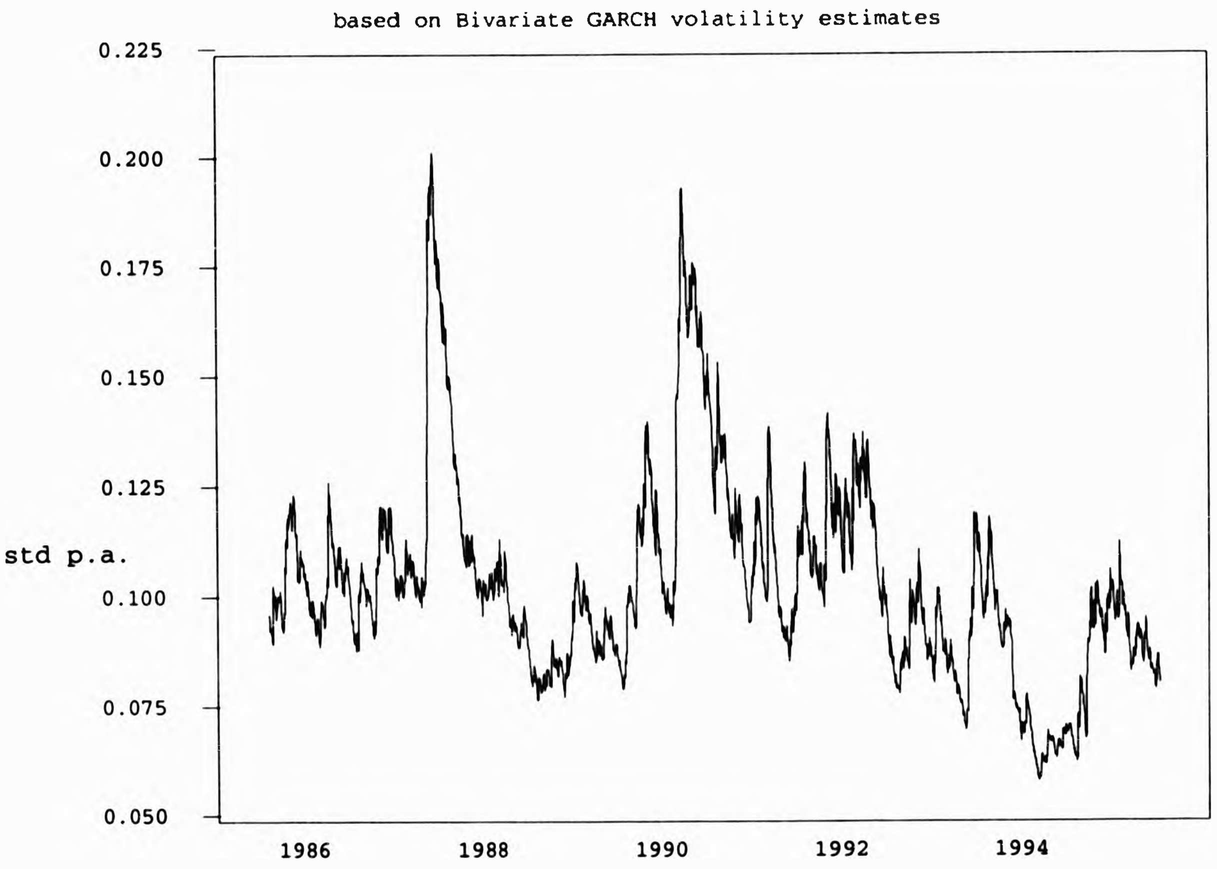
Table 3.12 Descriptive statistics of the portfolio historical returns

mean (p.a.)	std. dev (p.a.)	skewness	kurtosis	JB test	p value
10.92%	12.34%	-2.828	62.362	3474.39	0.000

Note: The test for normality is the Jarque-Bera test, $N((\sigma^3)^2/6 + (\sigma^4-3)^2/24)$. The last column is the significance level.

Figure 3.1 shows our hypothetical portfolio's conditional volatility over the ten year period. The conditional beta risk estimates have been employed to compute the time varying variance-covariance of this portfolio[8]. Therefore, the portfolio's daily volatility is computed as in (1.2). It is clear the increase in portfolio volatility occurred during the 1987 crash and 1990 Gulf invasion.

Fig 3.1 Portfolio Volatility



3.3.1 Evaluating the Effectiveness of GARCH Volatility Estimates

The bivariate GARCH methodology used so far has been complicated and computationally intensive. Its use can only be justified if we can demonstrate that it generates results superior to less intensive methods. Thus, to evaluate the effectiveness of the bivariate GARCH model at

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[8] The off diagonal elements of the variance-covariance matrix $\mathbf{\Omega}$ are estimated as in Sharp (1963) SIM.

monitoring portfolio risk, we used a series of different criteria (tests) and compared the results with a benchmark methodology widely used by the industry, the exponential smoothing (ES).

We formed a 13x13 variance covariance matrix by using the thirteen bivariate ES systems[9], each consisting of the world index and one of the local portfolios. The smoothing factor λ has been set equal to 0.94 for all variance and covariance equations in each bivariate ES system[10]. However, to keep the similarities in the mean equation with the bivariate GARCH systems, we modelled the mean equation as an ARMA(0,1) or ARMA(1,1) and then estimated the time-varying joint second moments on these residuals.

The statistical advantages in modelling the portfolio returns, using the GARCH estimates of variances and correlations, can be seen by examining the distributional properties of the residuals. The normality test on the portfolio standardised residuals has a value of 276.33, much lower than the value of the test on the unconditional distribution[11]. This indicates that the bivariate GARCH volatility estimates have removed most of the excess kurtosis from the data.

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[9] The univariate ES, described in chapter two, can be expanded to a bivariate version to model the joint second moments of two return series.

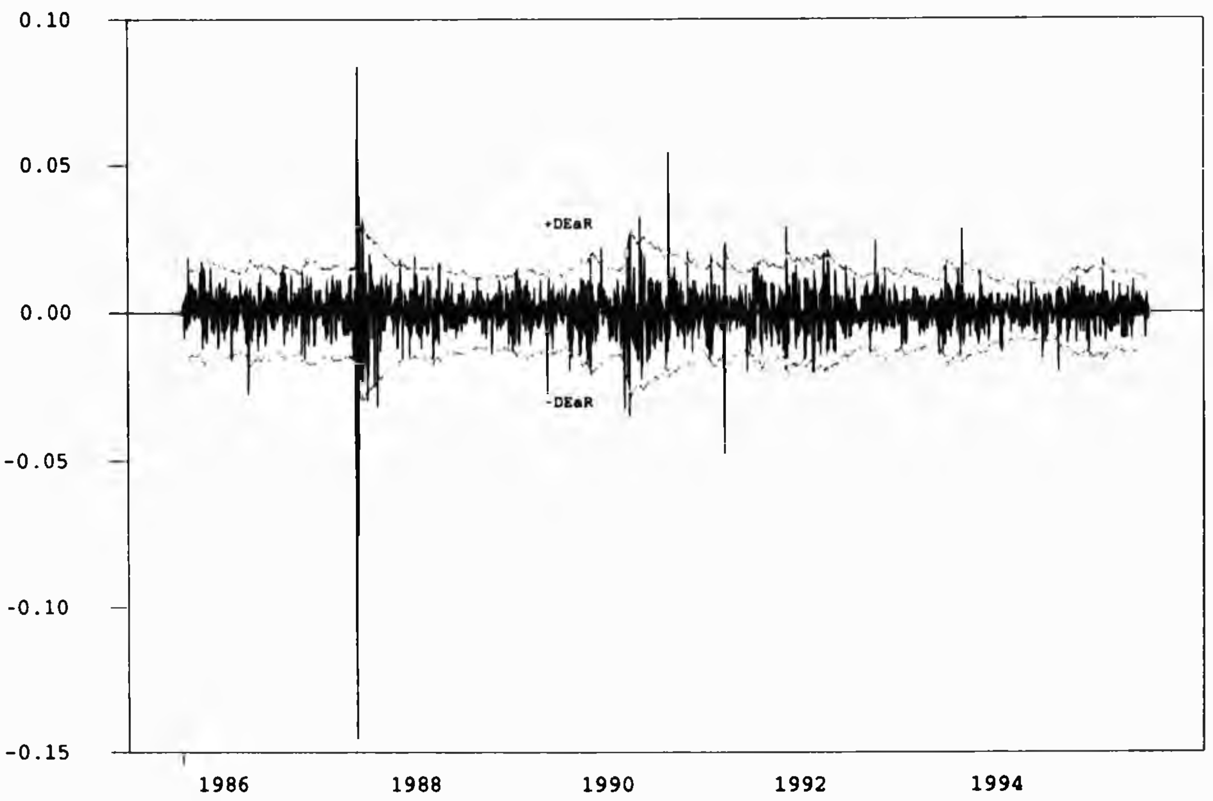
[10] The decision to keep the same value of λ across all variance and covariance equations has been made because we want to keep computation efforts to minimum. The value of 0.94 is recommended by JP Morgan and is widely accepted by the industry (see Riskmetricks (1995) p 80. When λ is allowed to take different values in each equation then optimal values can be found by maximising a likelihood function. This method however is equally intensive as the multivariate GARCH but is less accurate since it imposes a zero mean process and a restrictive GARCH process.

[11] We have used the residuals from an ARMA(0,1) fitted on the portfolio returns.

We compared the DEaR estimates on the theoretical portfolio from our bivariate GARCH model against the realised daily losses and profits over the ten year period. This is illustrated in figure 3.2 where +DEaR and -DEaR form a confidence cone in which 98% of the daily profits and losses are expected to lie.

Fig 3.2 Portfolio DEAR

based on bivariate GARCH volatilities



Ideally, we expect to see that 1% of daily profits, which is 25 days out of our 10 year sample, and the same amount of losses to exceed each of the DEaR estimates. The number of portfolio daily profits and losses that exceed the DEaR are shown in table 3.13. The last two columns report how many times the portfolio's losses, exceed the ES smoothing based DEaR. Obviously the GARCH based DEaR underestimates daily profits more often than the normal probability predicts. However, in risk management we are interested in monitoring the downside risk. Upside risk (on long positions) is always desirable and we should only be concerned for the number of portfolio daily losses that exceed the DEaR threshold. As we see, portfolio losses exceed the GARCH based DEaR on 24 days, which is almost equal to what the normal distribution predicts. By contrast, the portfolio losses are greater than those predicted by the ES in only 18 days, which means that the ES overestimates the portfolio's risk.

Table 3.13 Number of time portfolio return exceed DEaR

	Bivariate GARCH DEaR	ES DEaR
profits	41	24
losses	26	18

The ability of the bivariate GARCH to calculate portfolio risk can be evaluated further by applying the Pagan and Ullah test of (3.3). Table 3.14 contains the estimates from the regression of the portfolio squared residuals against a constant and the bivariate GARCH variance estimates. Clearly there is little power in the portfolio conditional variance (as computed from the bivariate GARCH estimates) to predict the portfolio volatility. Unlike in the local portfolio returns where the GARCH variance estimate explains about one third of the squared return variation, the R^2 on the above regression is only 2.58%. Furthermore, the GARCH volatility estimates seem to underestimate systematically the portfolio variance. The Ljung-Box statistic carried out on the standardised residuals of the hypothetical portfolio returns provides strong evidence that the portfolio conditional variance computed as the quadratic product of the bivariate GARCH variances and covariances did not remove all the volatility clusters present in the portfolio returns.

In the same table we report the analogous test carried out for the benchmark risk estimate, the ES. Although the ES volatility estimates do not show evidence of systematic bias in its ability to explain the squared residuals, the R^2 is still too small to justify the employment of ES in modelling portfolio volatility.

Table 3.14 Evaluating Variance estimates

conditional variance	α	β	R^2	Q(6)-1
GARCH	0.002	2.141	0.0258	154.378
	(1.441)	(2.725)		0.000
ES	0.000	0.655	0.0514	2.707
	(1.542)	(1.499)		0.844

Note: In parenthesis, below the estimates, α and β, are: first column, the t-statistics for testing $\alpha=0$, second column the t-statistic for the hypothesis $\beta=1$. The Q statistics, of order 6, and significance levels for the squared standardised residuals are in the last column.

3.4 Conclusions

The objective of this chapter has been to estimate portfolio VaR using conditional time series techniques to model changes in portfolios volatility over different periods. As we have seen in the earlier chapters, security returns do not exhibit constant second moments, and this is one of the factors which causes the fat tails observed in the empirical distribution of security returns. We studied the unconditional empirical distribution of daily returns on thirteen developed equity portfolios across the world over a period of ten years and tested the hypothesis that their volatility is constant over different periods. We found that the distributions of returns on all national market returns we examined are non-normal. Further investigation revealed that the returns are conditionally heteroskedastic which can explain the excess kurtosis. The negative large skewnes observed in most markets may be caused by changes in the means. Furthermore, we found evidence that the way national equities are linked with each other, changes over time.

We used multivariate GARCH to model daily changes in variances and covariances of the thirteen national portfolios in our sample. The GARCH methodology is ideal for capturing changes in the data volatility and for removing most of the excess kurtosis. In contrast to unconditional methods, such as historical volatility and least squares analysis, which restrict variables to be homoskedastic, the GARCH models regard homoskedasticity as a special case. We preferred GARCH models over other time varying models because they allow the conditional variance to be autoregressive, a property known to characterise daily security returns since the early days of modern finance[12].

Because the empirical use of multivariate GARCH is restricted to a few series at a time we adopted the SIM algorithm to overcome the problem. This is a approach that is similar to the multifactor model of King et al. (1994) in that it allows for national markets to interdepend through their common movement with observable factor(s). Furthermore, the variability of these factors, is conditionally modelled as a GARCH(1,1) process and that it divides risk to systematic and specific time-varying components. Our approach involves first partitioning the variance-covariance matrix into N off-diagonal elements and then capturing the joint dynamics of the second moments of each local equity index with a common index, *i.e.* the world market portfolio. Then all possible pair-wise combi-

[12] Multivariate stochastic volatility may be an alternative method but for reasons we mention in chapter two we prefer the use of multivariate GARCH.

nations of the N asset returns can be constructed as in Sharpe (1963) and the portfolio volatility of current investment holdings be computed as in (1.2). In this method the number of unknown parameters grows linearly with the number of assets in the portfolio and because each beta is estimated independently from the rest we overcome computational and other convergence problems which may arise in a higher order multivariate GARCH. At the same time we can estimate the time-varying variance-covariance matrix on a large number of series, as long there exists an index to which all assets are correlated. Hence, the portfolio's VaR was determined by using the magnitudes of past changes in the market factor and the sensitivity of each asset in the portfolio to the daily changes of the market factor.

The GARCH methodology enabled us to make better use of the information set available at each period to estimate portfolios market risk. The results indicate that most of the excess kurtosis has been removed from the data and the distribution of portfolio residual returns is very close to normal. The portfolio losses which exceed the DEaR calculated upon the GARCH-SIM risk estimates are in line with what the normal distribution predicts. However, further tests have shown that the portfolio variance based upon the bivariate GARCH is rather a biased estimate of the portfolio daily volatility and has little predictive power to foresee any changes in it. One of the reasons for that may be the way we estimated the portfolio's conditional variance-covariance matrix. The breakdown of this matrix into factorised non-linear bivariate systems does not guarantee either the orthogonality conditions on the residuals or the multivariate properties of the data. In the next chapter we will see how this problem may be overcome by simplifying the way portfolio volatility is estimated but without sacrificing volatility precision.

In addition, the GARCH-SIM approach allowed us to estimate both systematic and specific volatility as time varying. The results suggests areas for further study concerning the time path of risk among individual assets or classes of assets. Studying the time structure of risk will significantly help us to understand the dynamics which link volatility waves between national stock markets and improve volatility timing analysis between leading local markets. For example, between the US and smaller markets. The time-varying modelling of assets' systematic risk should also have implications for other investment decisions. For example, in portfolio analysis, if the asset returns have time-varying second moments, the mean/variance trade-off of optimal portfolios is not constant. Consequently, the portfolio weights are subject to frequent adjustment due to non-synchronous changes in assets' systematic and specific risk components. Serious implications may also arise in asset pricing, especially for those assets, such as options, whose prices are very sensitive to risk changes.

Appendix 3A

Fig 3A.I DENMARK - returns in local currency

beta estimates (conditional vs unconditional)

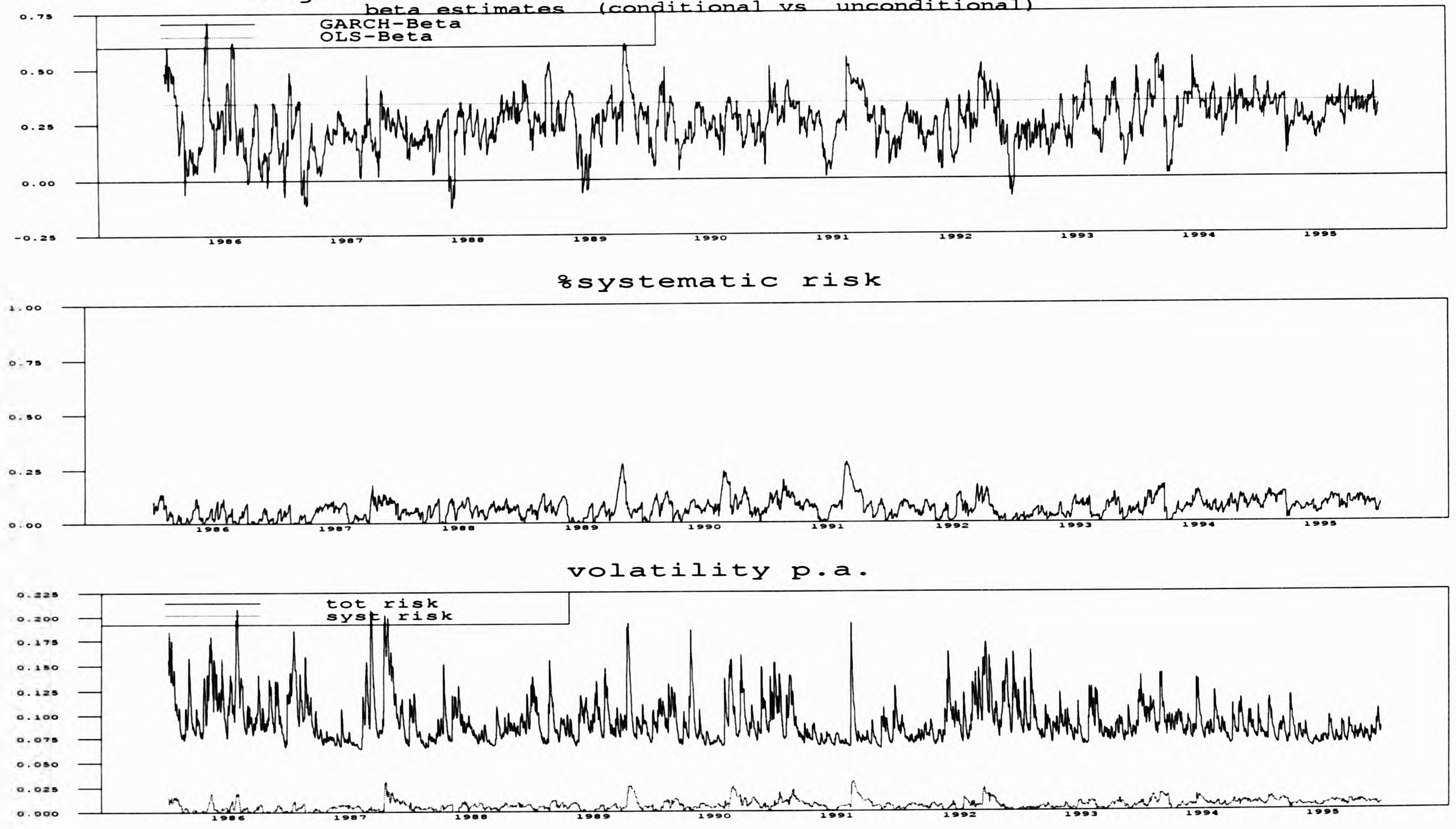


Fig 3A.II GERMANY - returns in local currency

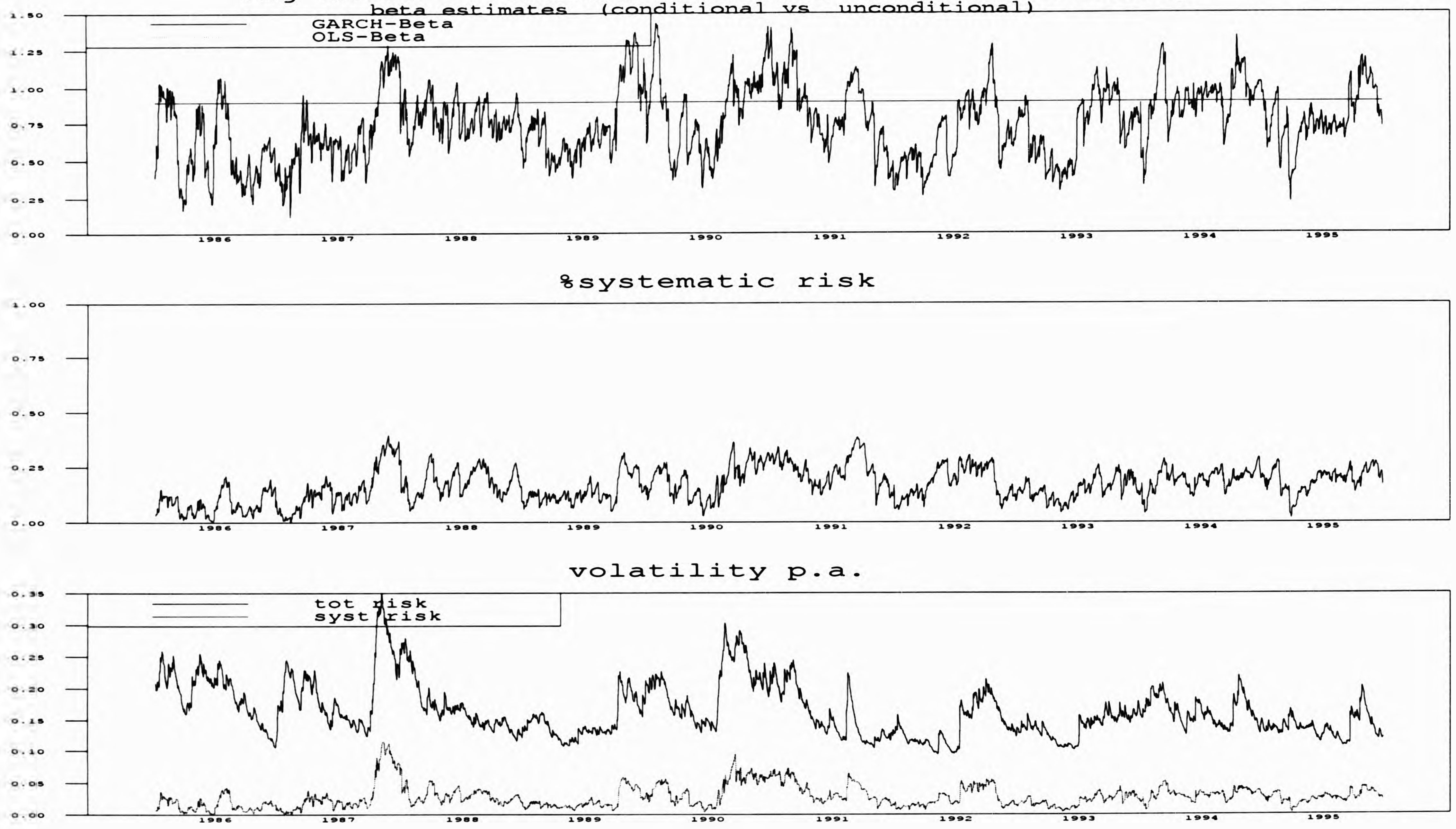


Fig 3A.III FRANCE - returns in local currency

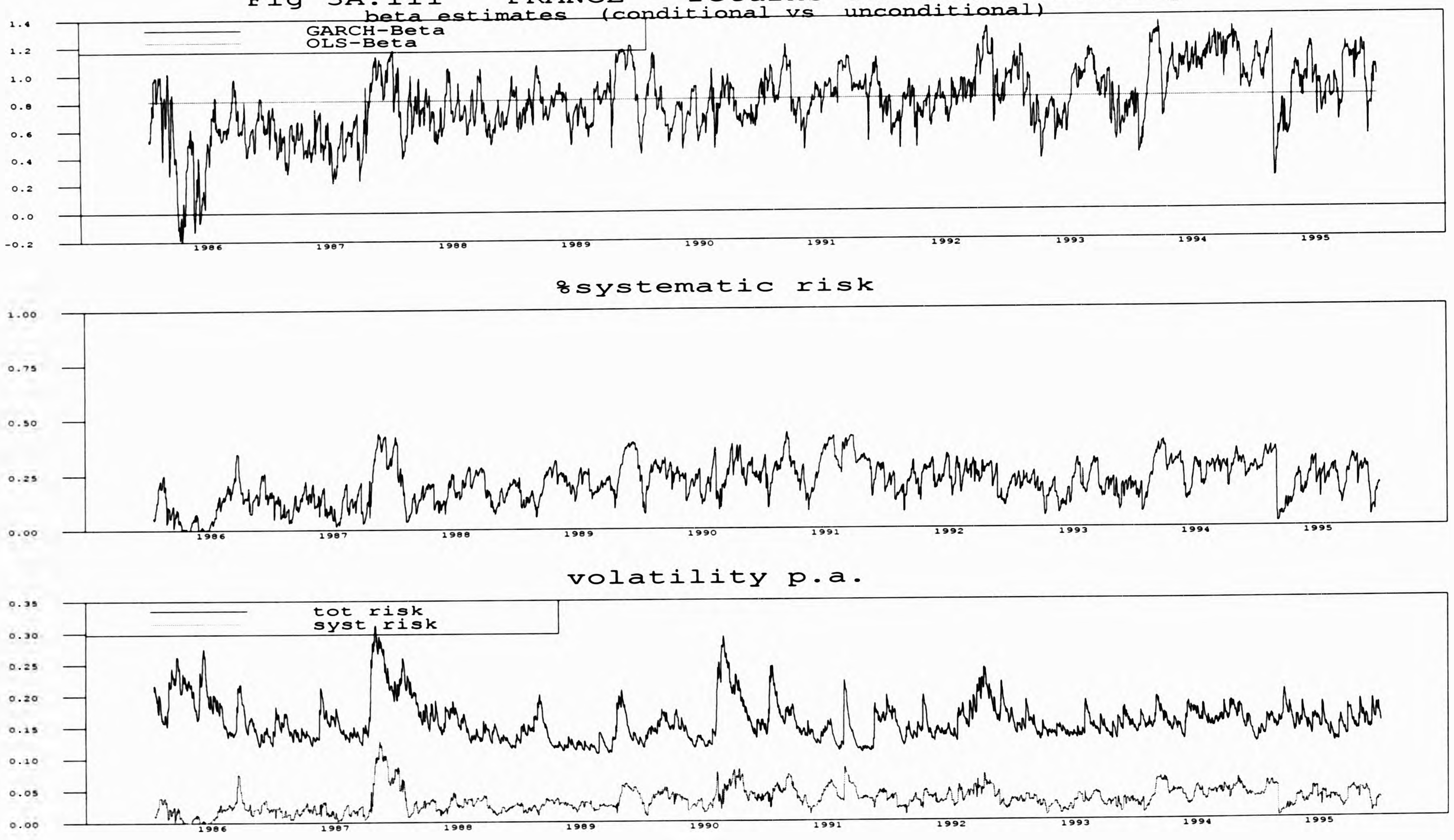


Fig 3A.IV HONG KONG - returns in local currency

beta estimates (conditional vs unconditional)

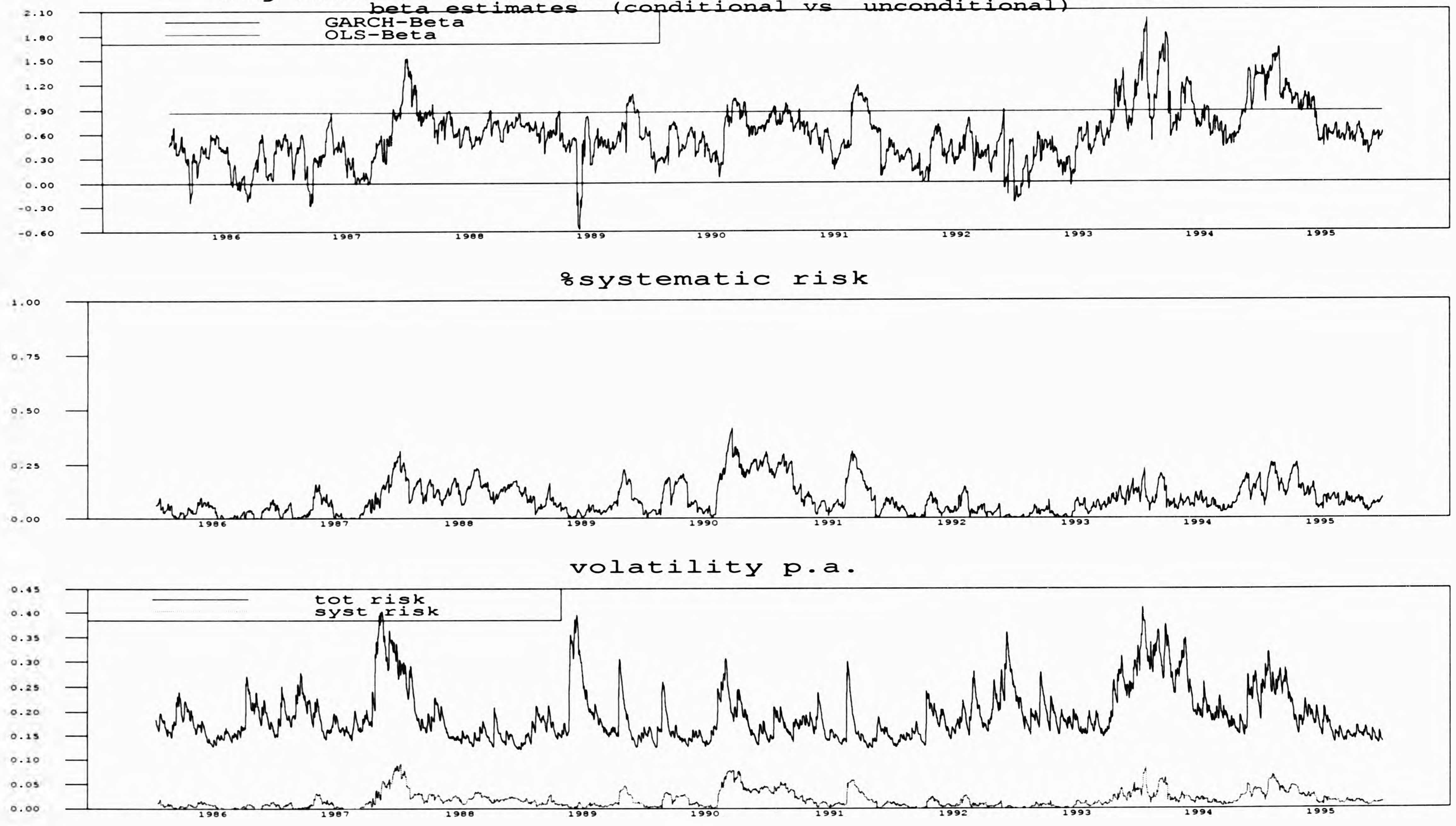


Fig 3A.V ITALY - returns in local currency

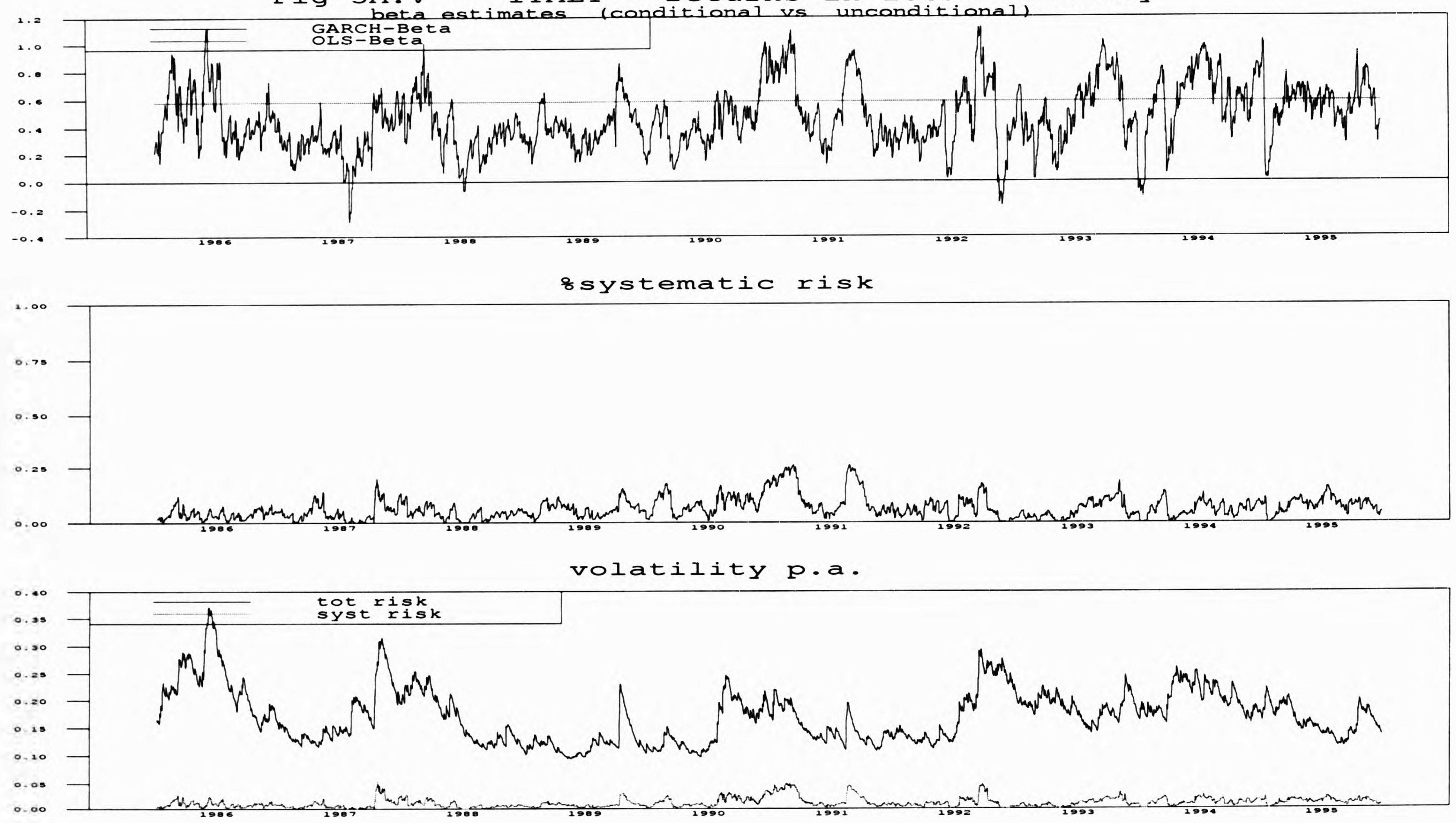


Fig 3A.VI JAPAN - returns in local currency

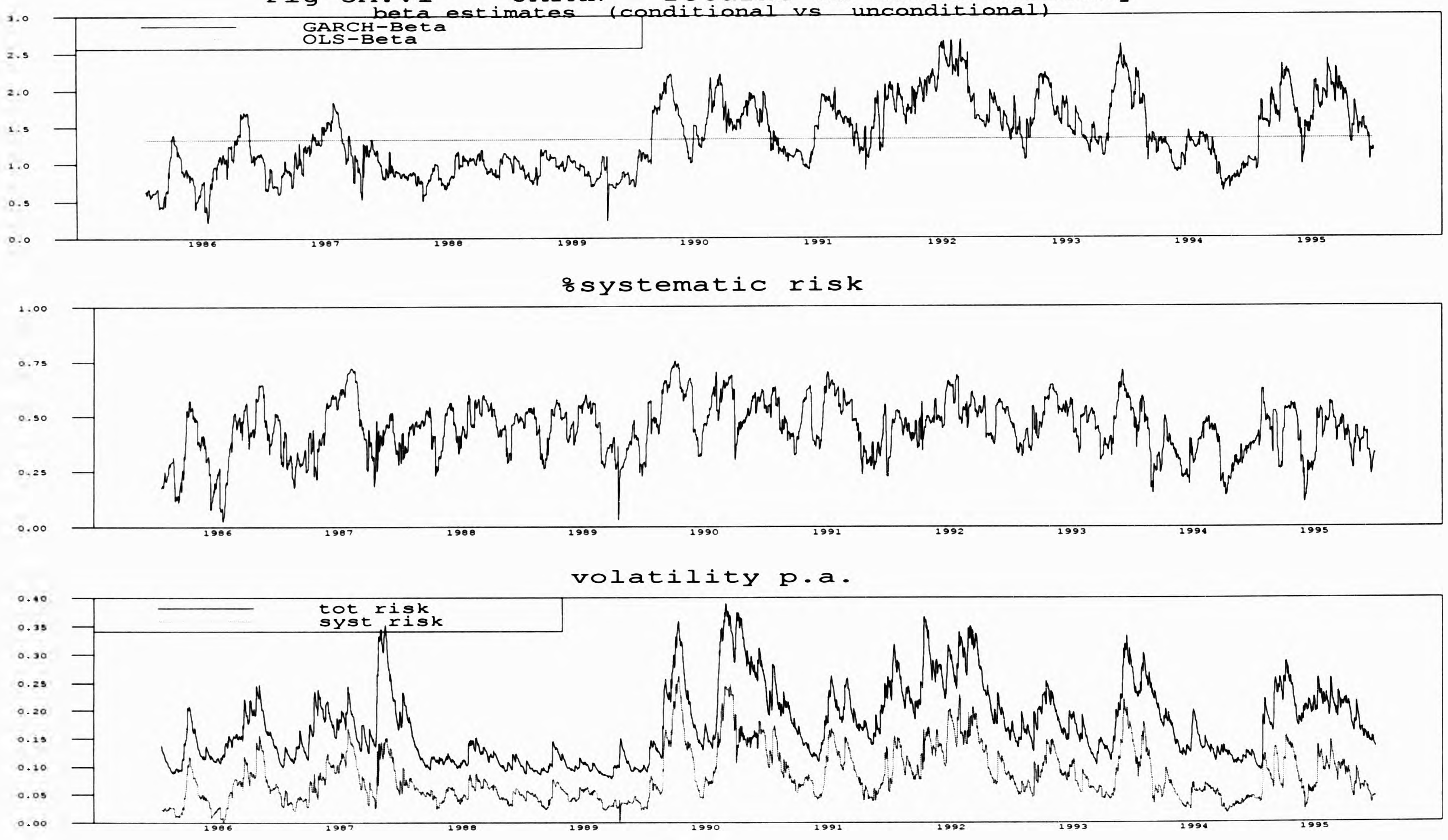


Fig 3A.VII NETHERLAND - returns in local currency

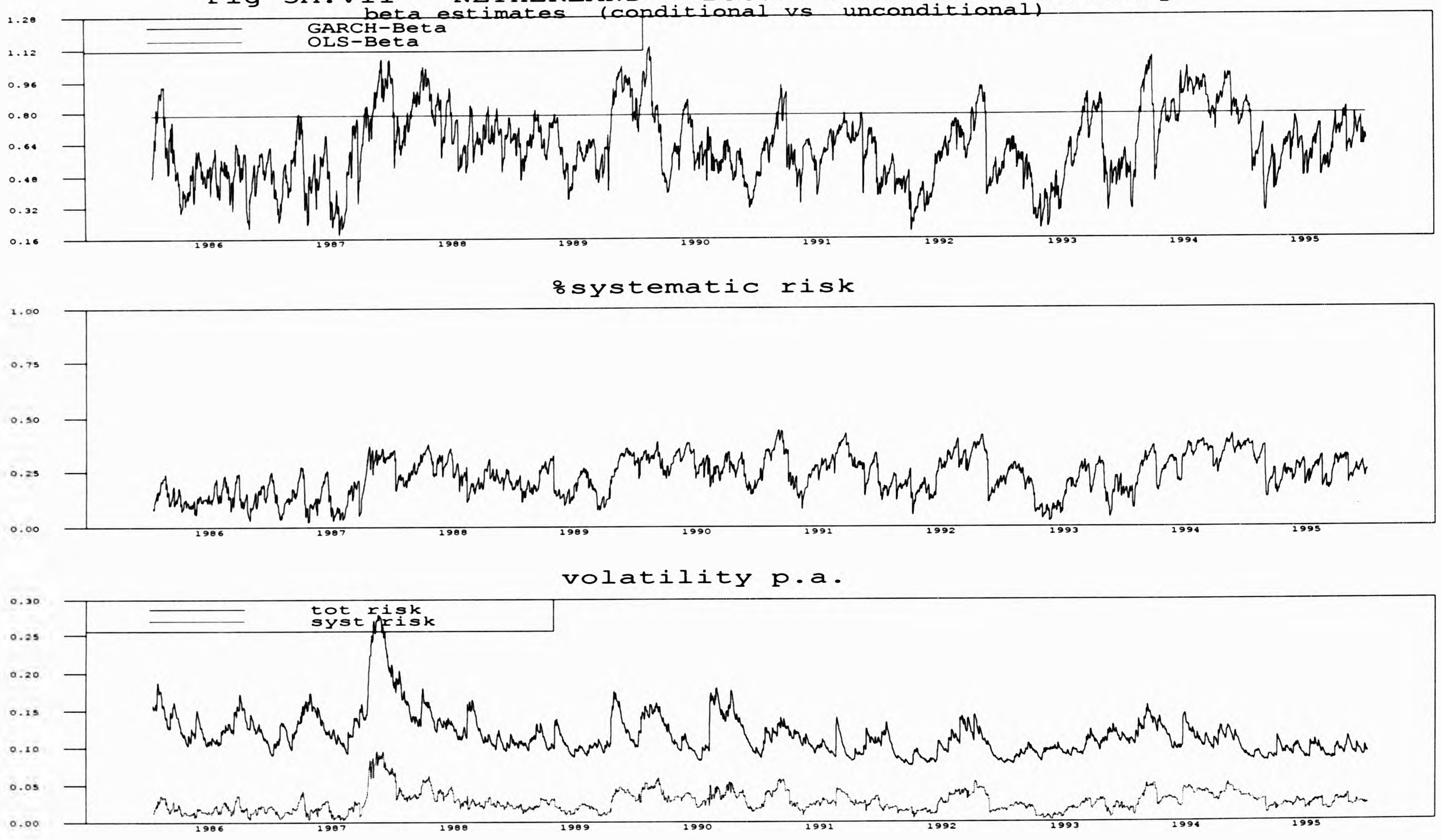


Fig 3A.VIII SINGAPORE - returns in local currency

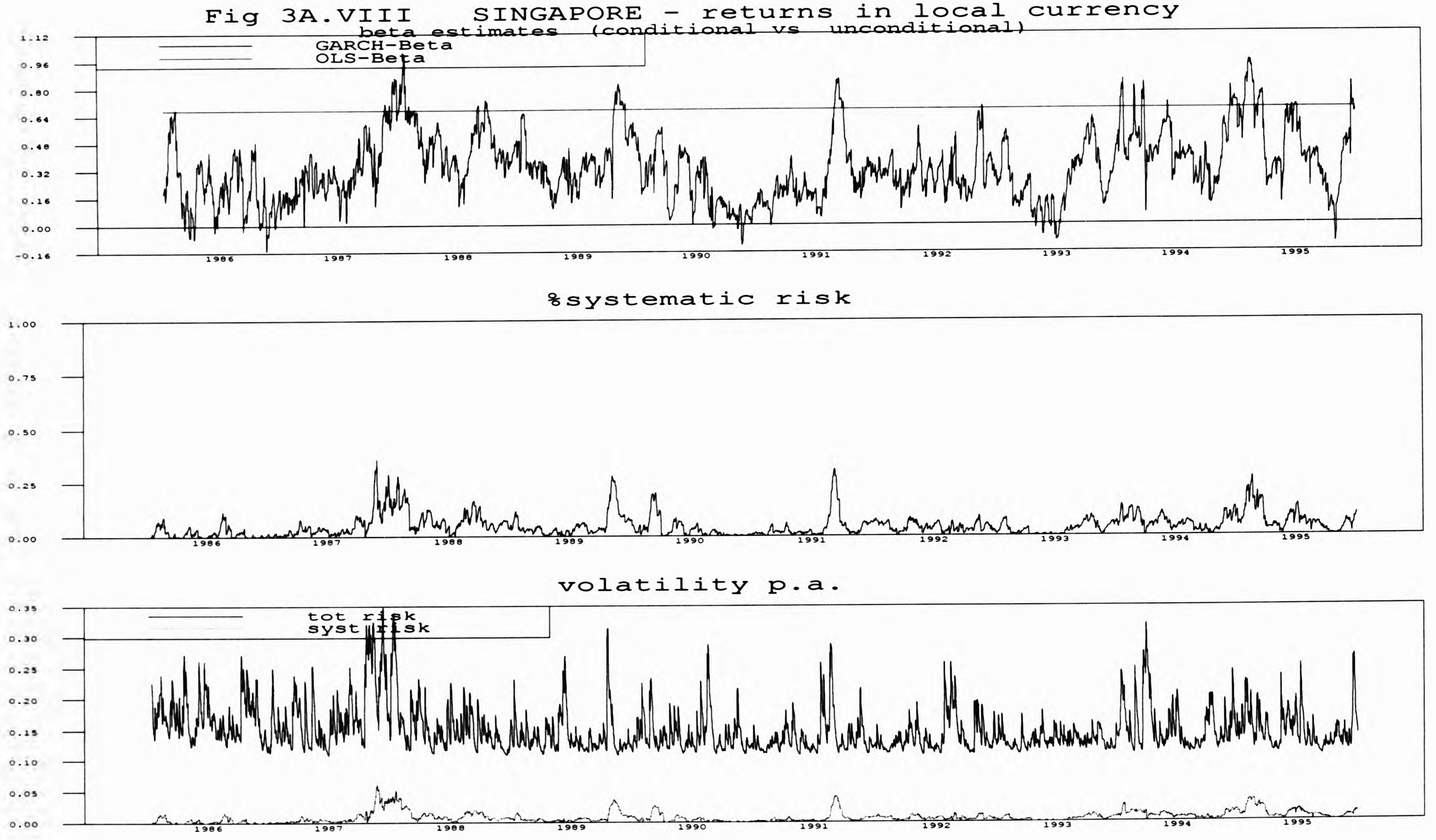


Fig 3A.IX SPAIN - returns in local currency

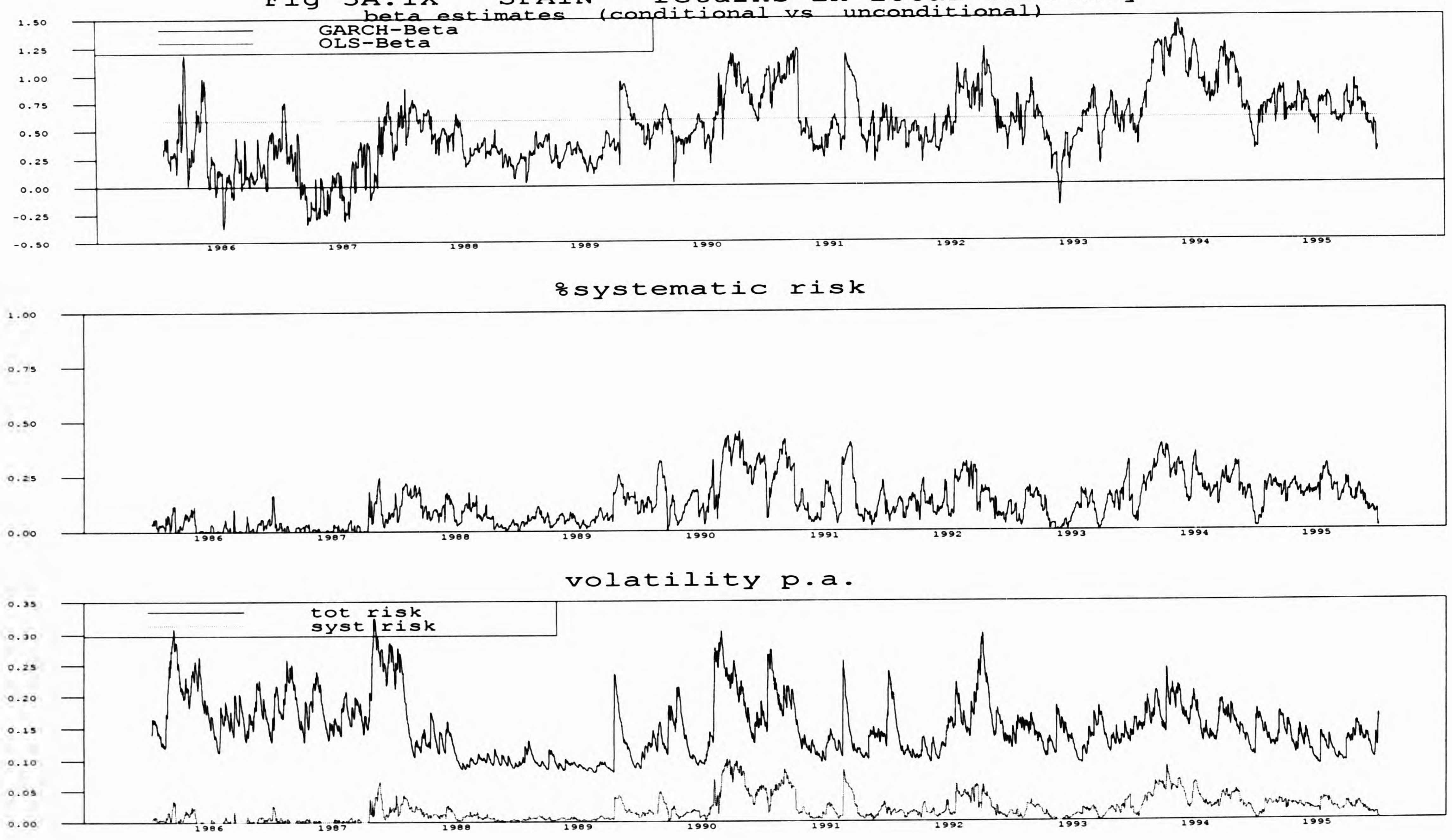


Fig 3A.X SWEDEN - returns in local currency

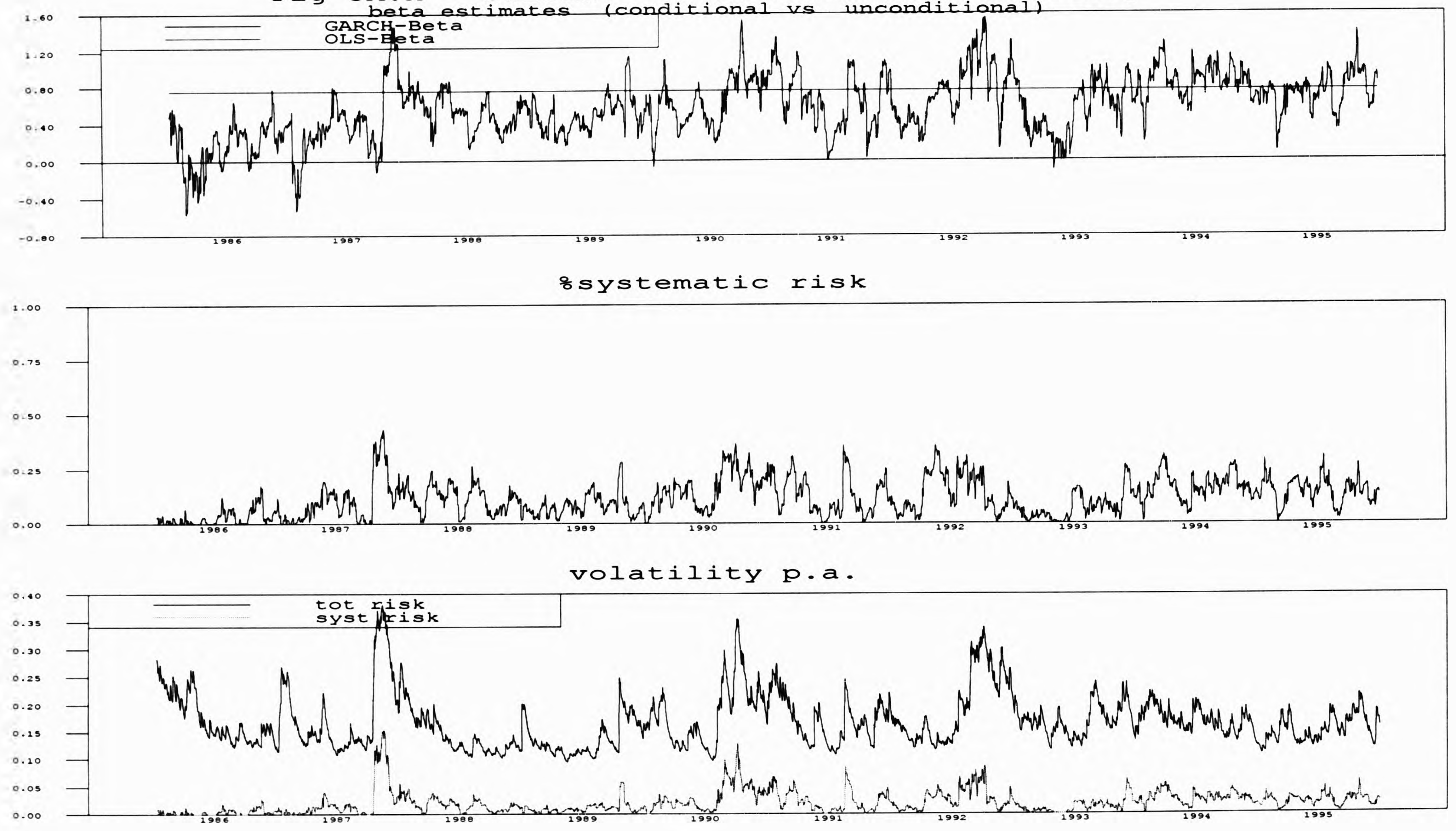


Fig 3A.XI SWITZERLAND - returns in local currency

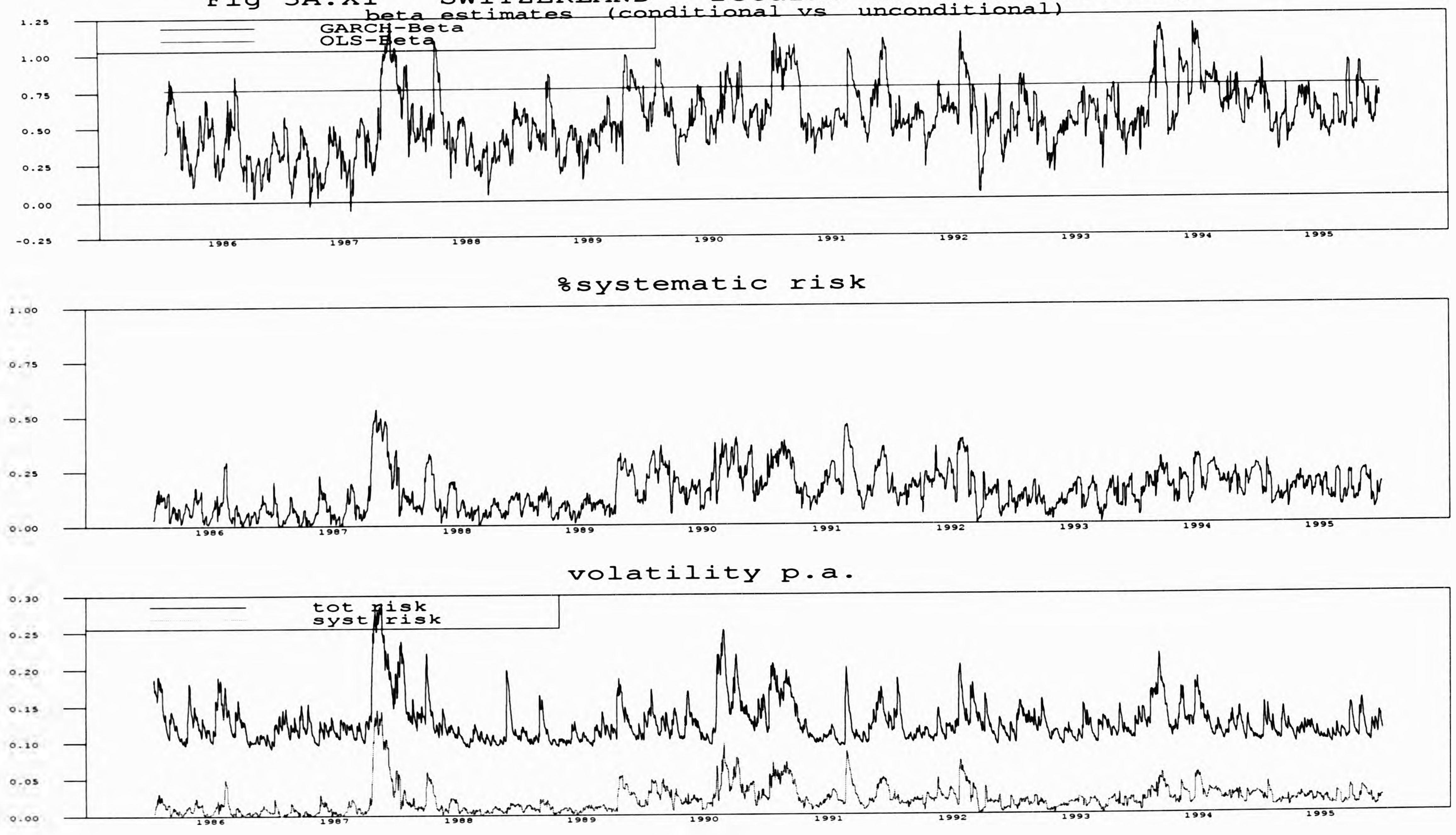


Fig 3A.XII UK - returns in local currency

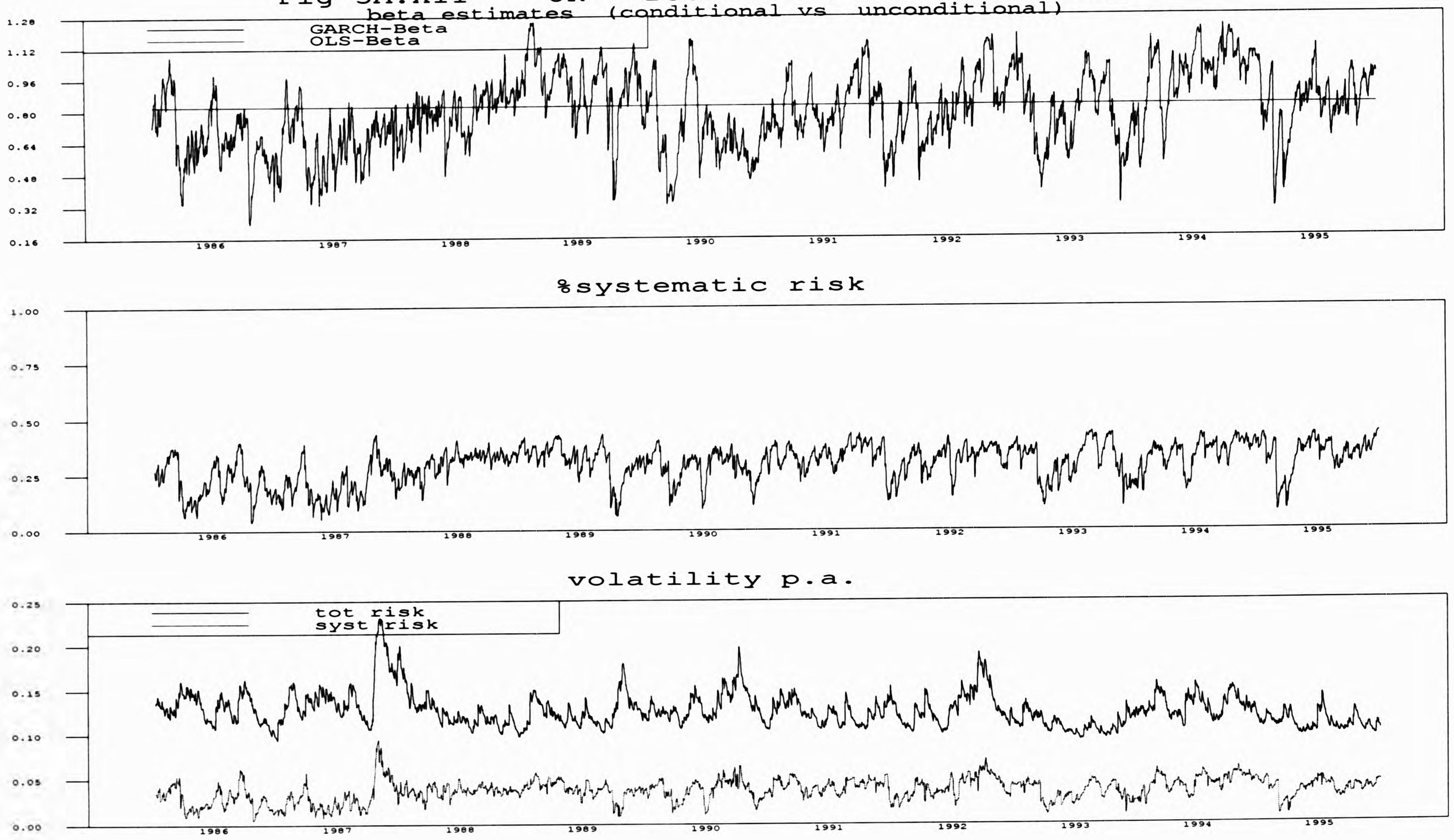
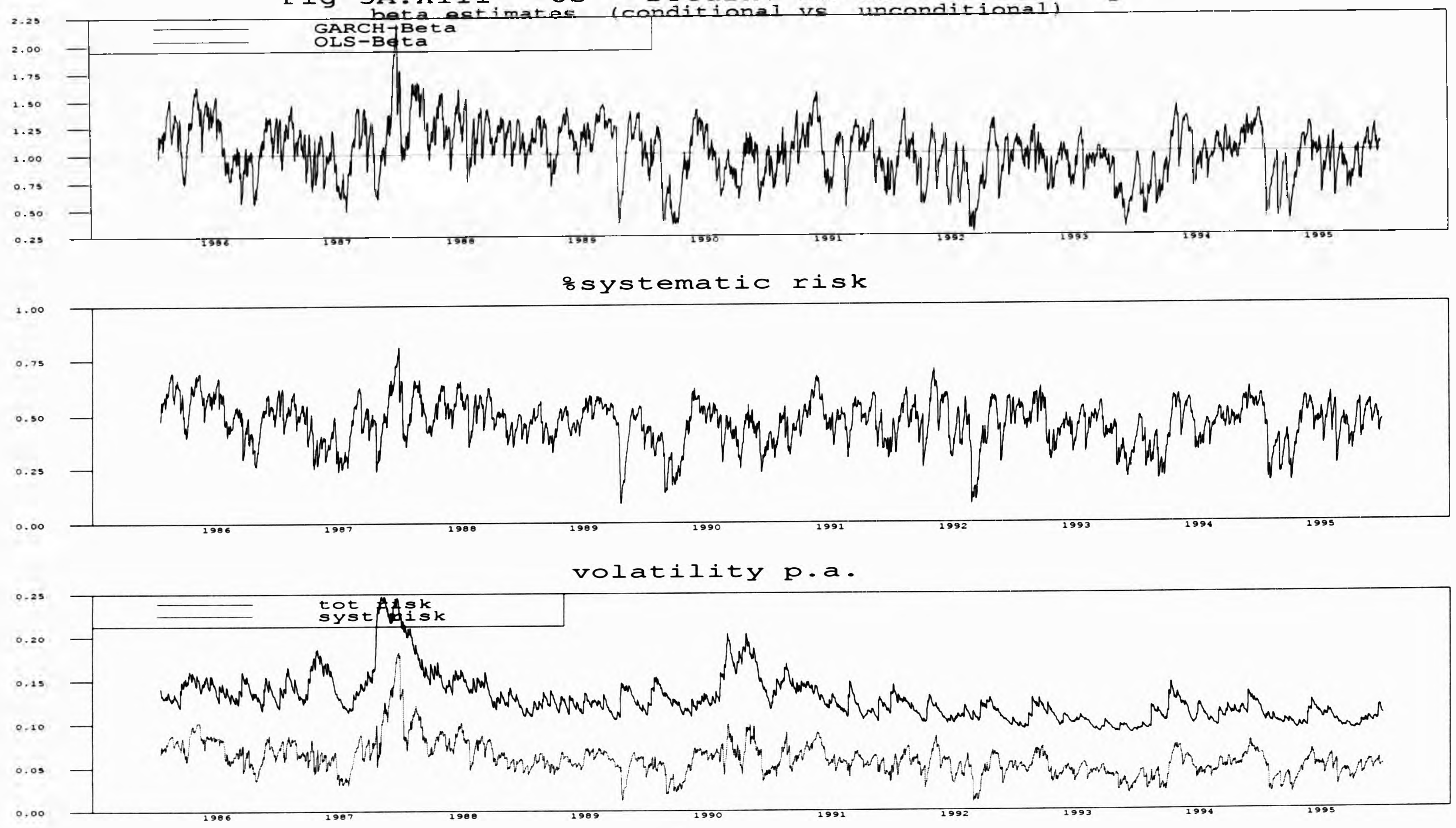


Fig 3A.XIII US - returns in local currency



Appendix 3B

Fig 3B.I DENMARK - returns in US$

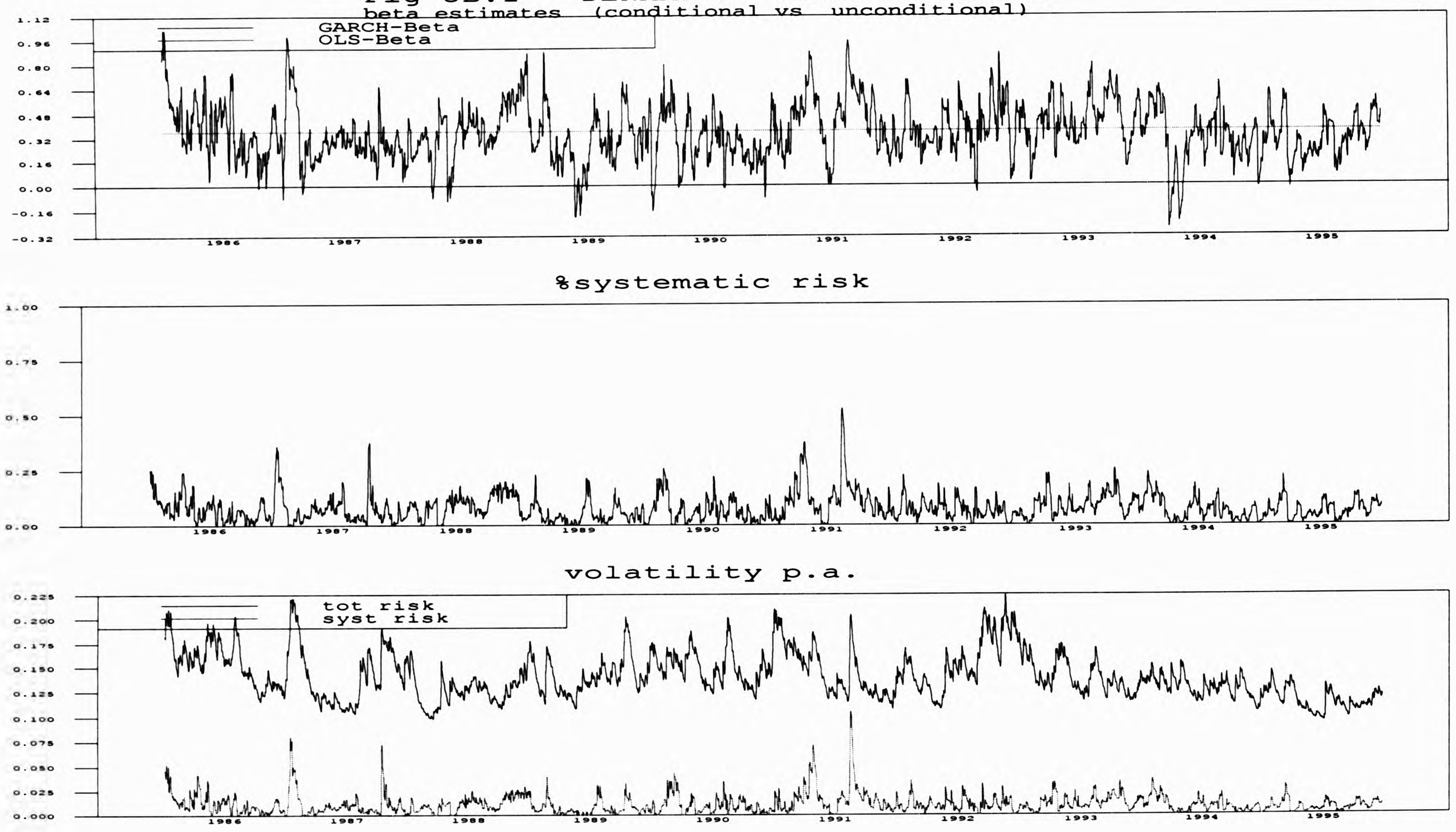


Fig 3B.II GERMANY - returns in US$

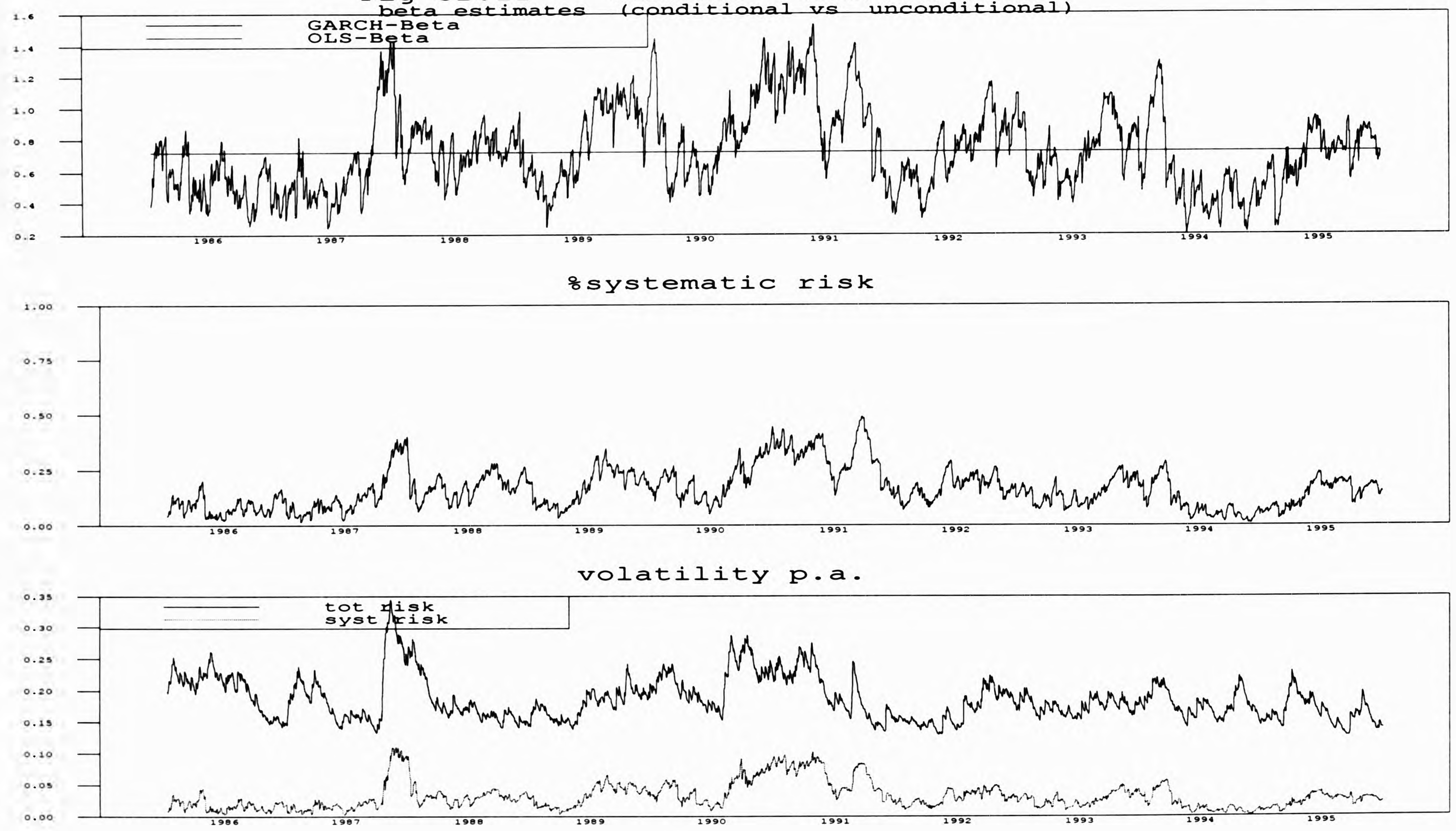


Fig 3B.III FRANCE - returns in US$

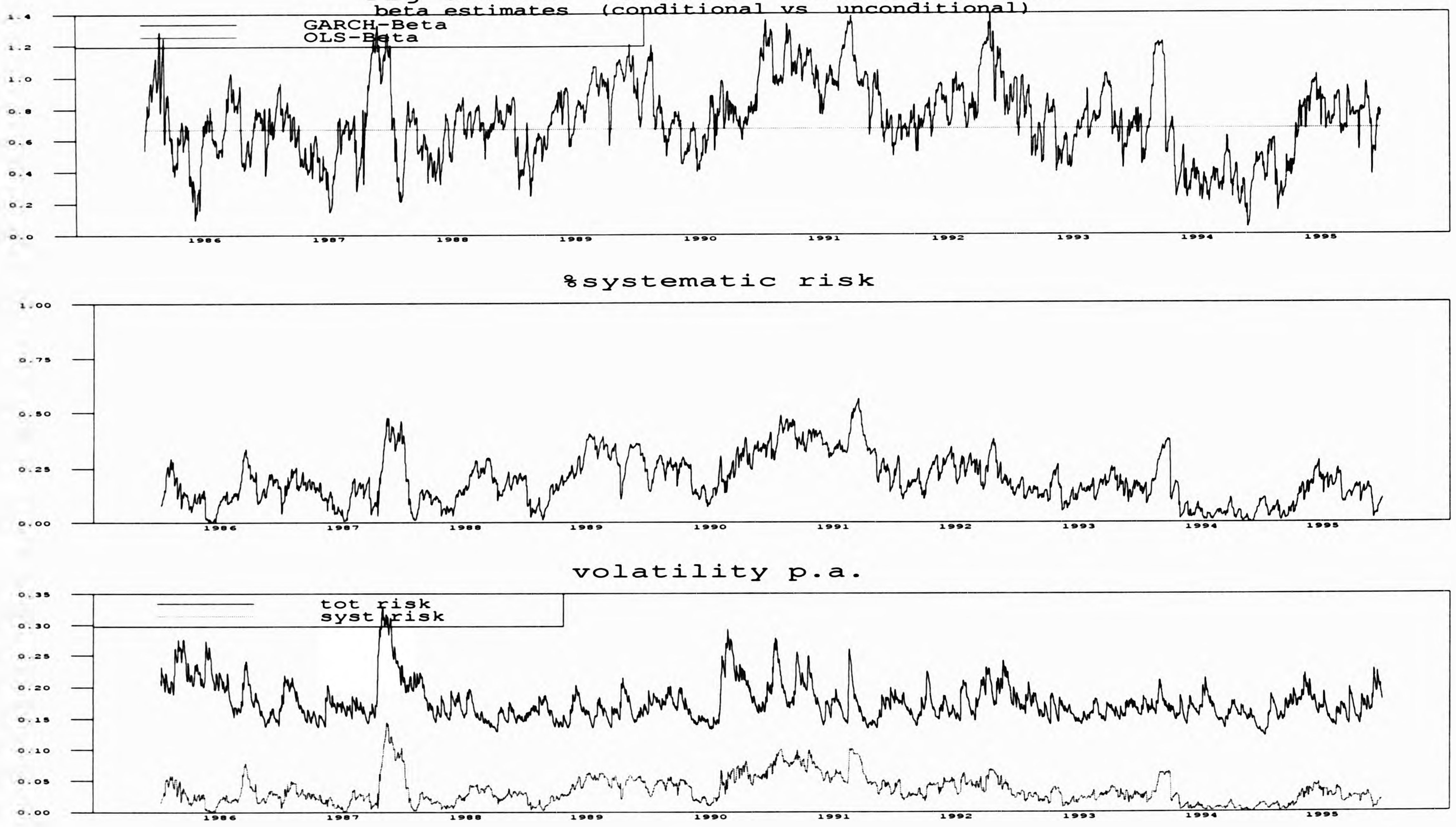


Fig 3B.IV HONG KONG - returns in US$

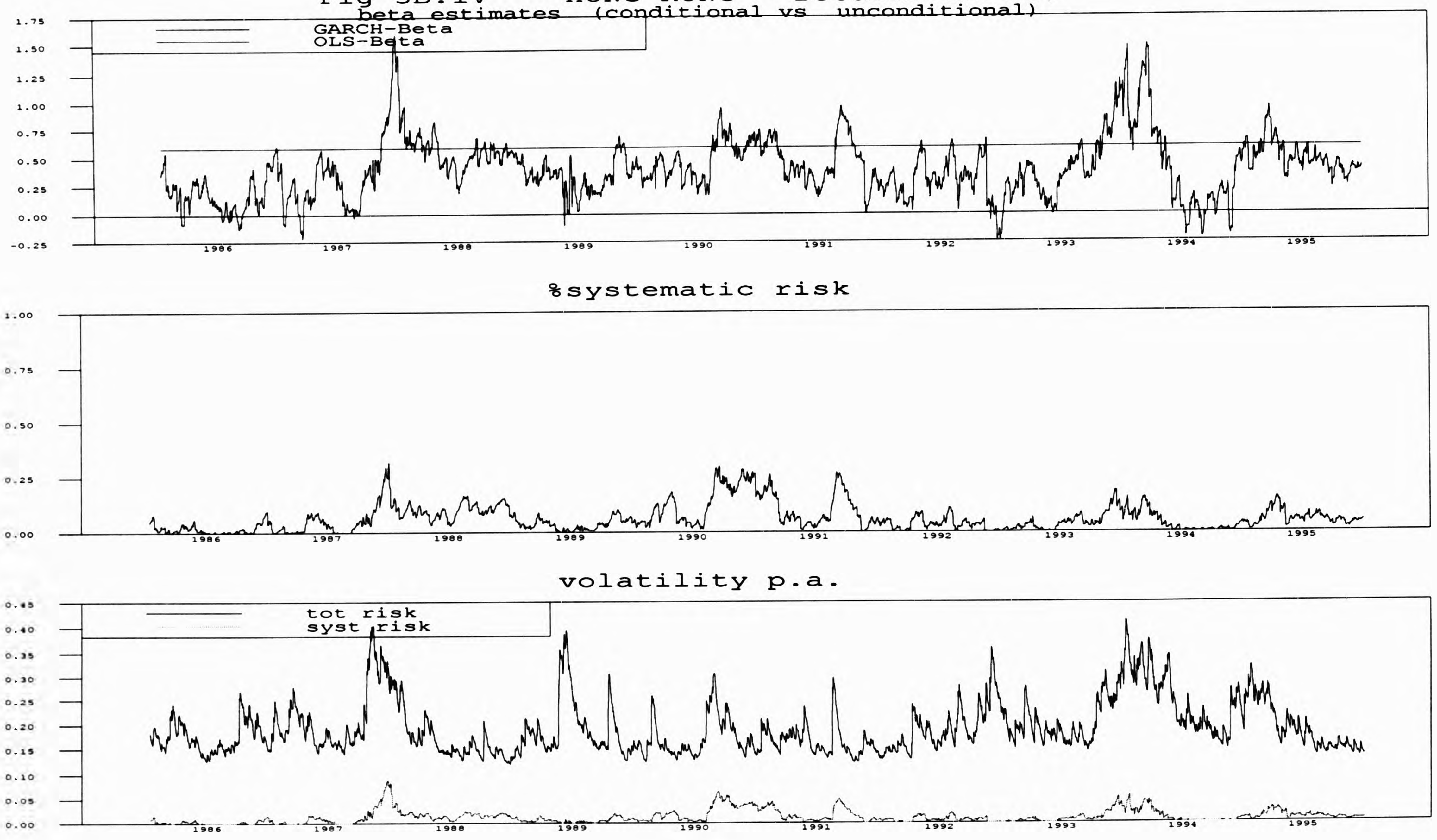


Fig 3B.V ITALY - returns in US$

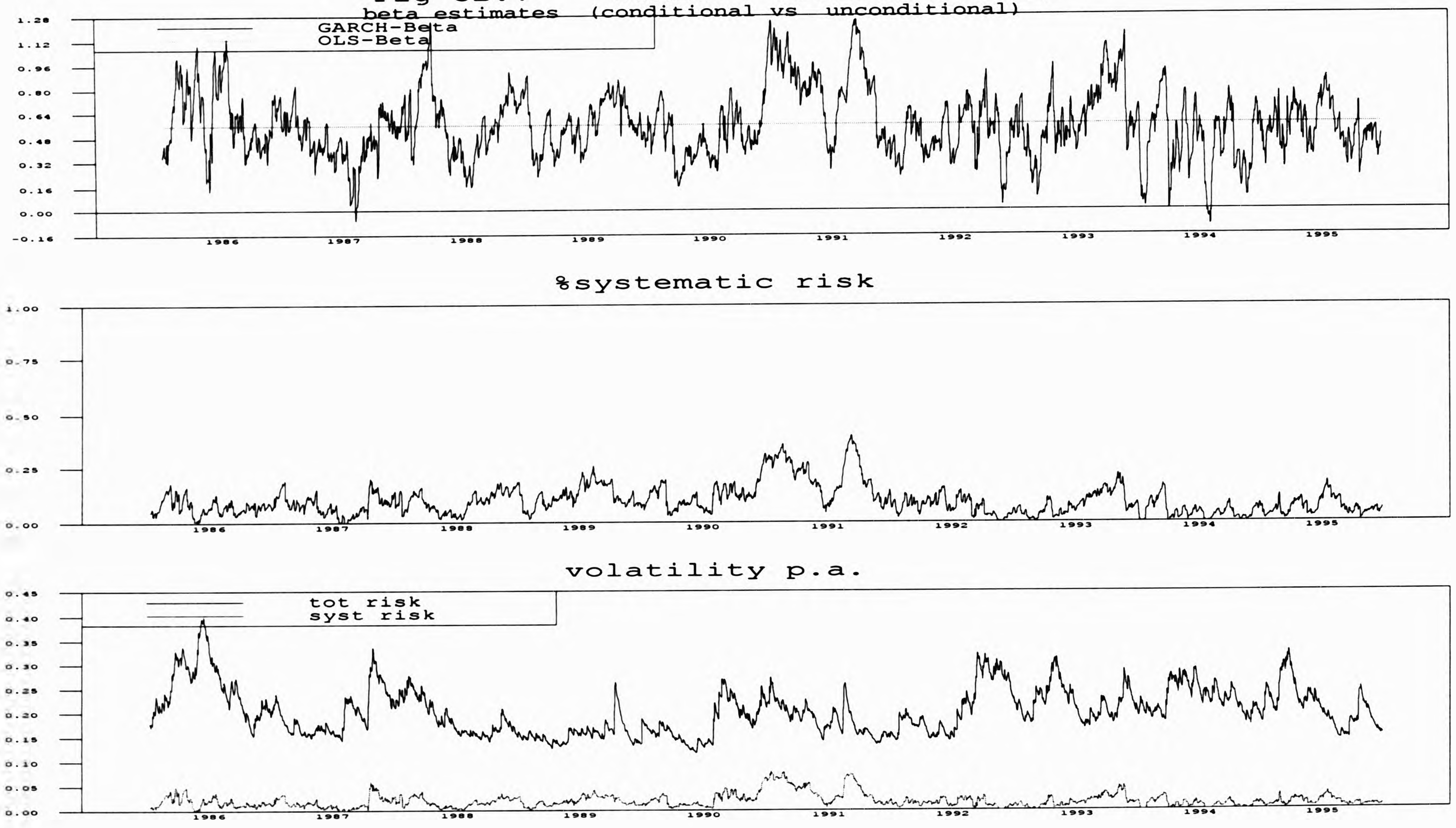


Fig 3B.VI JAPAN - returns in US$

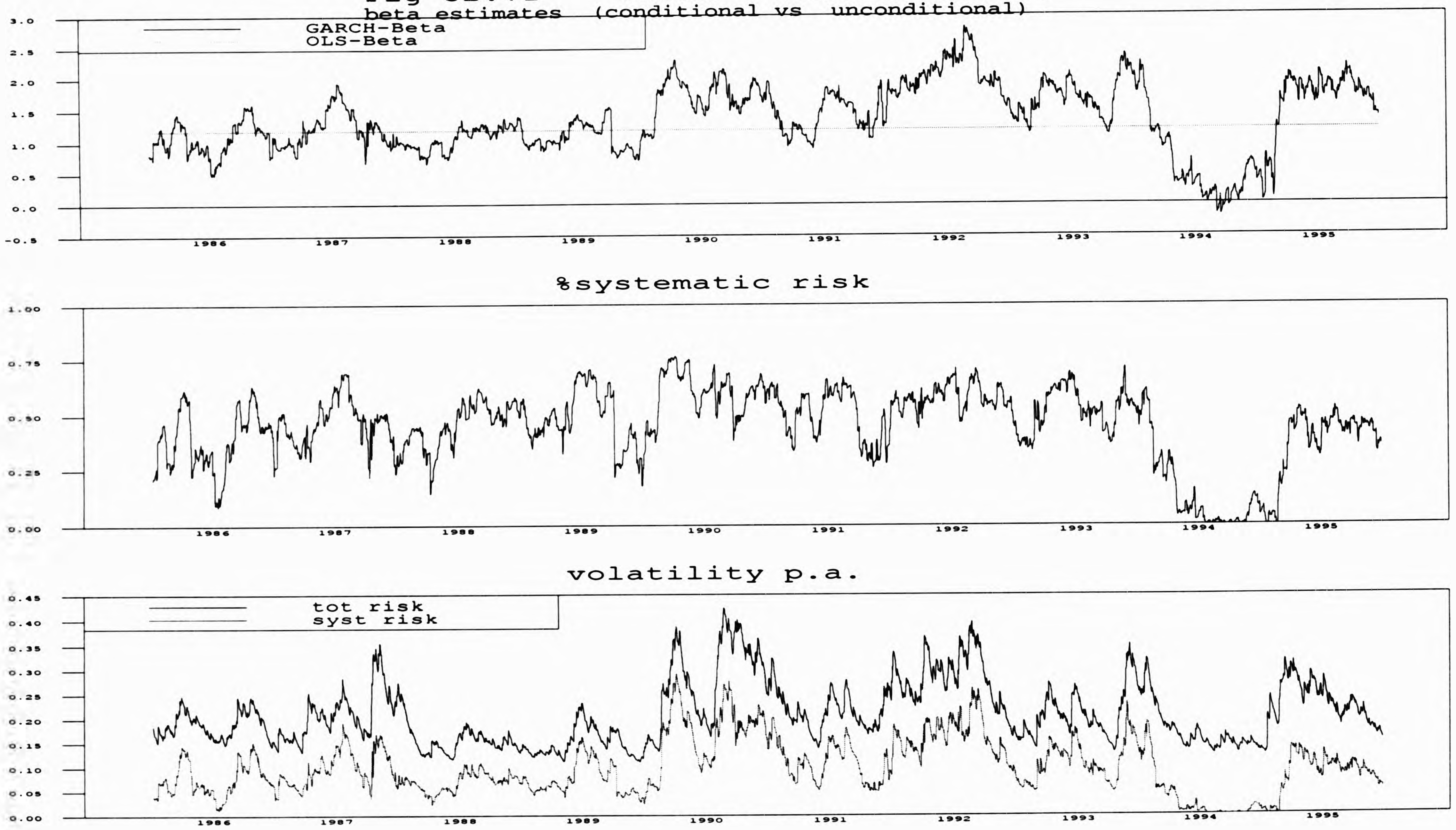


Fig 3B.VII NETHERLAND - returns in US$

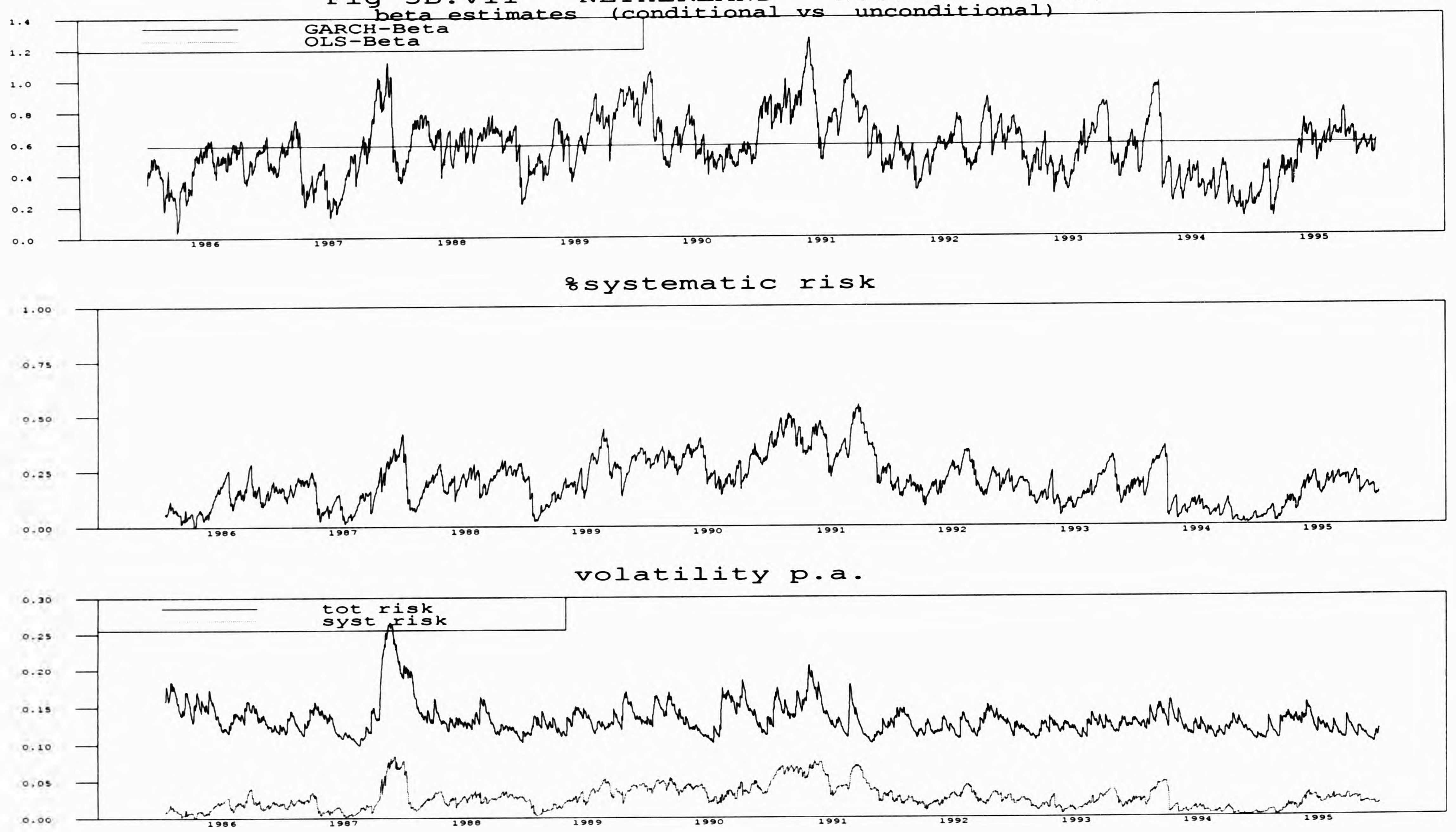


Fig 3B.VIII SINGAPORE - returns in US$

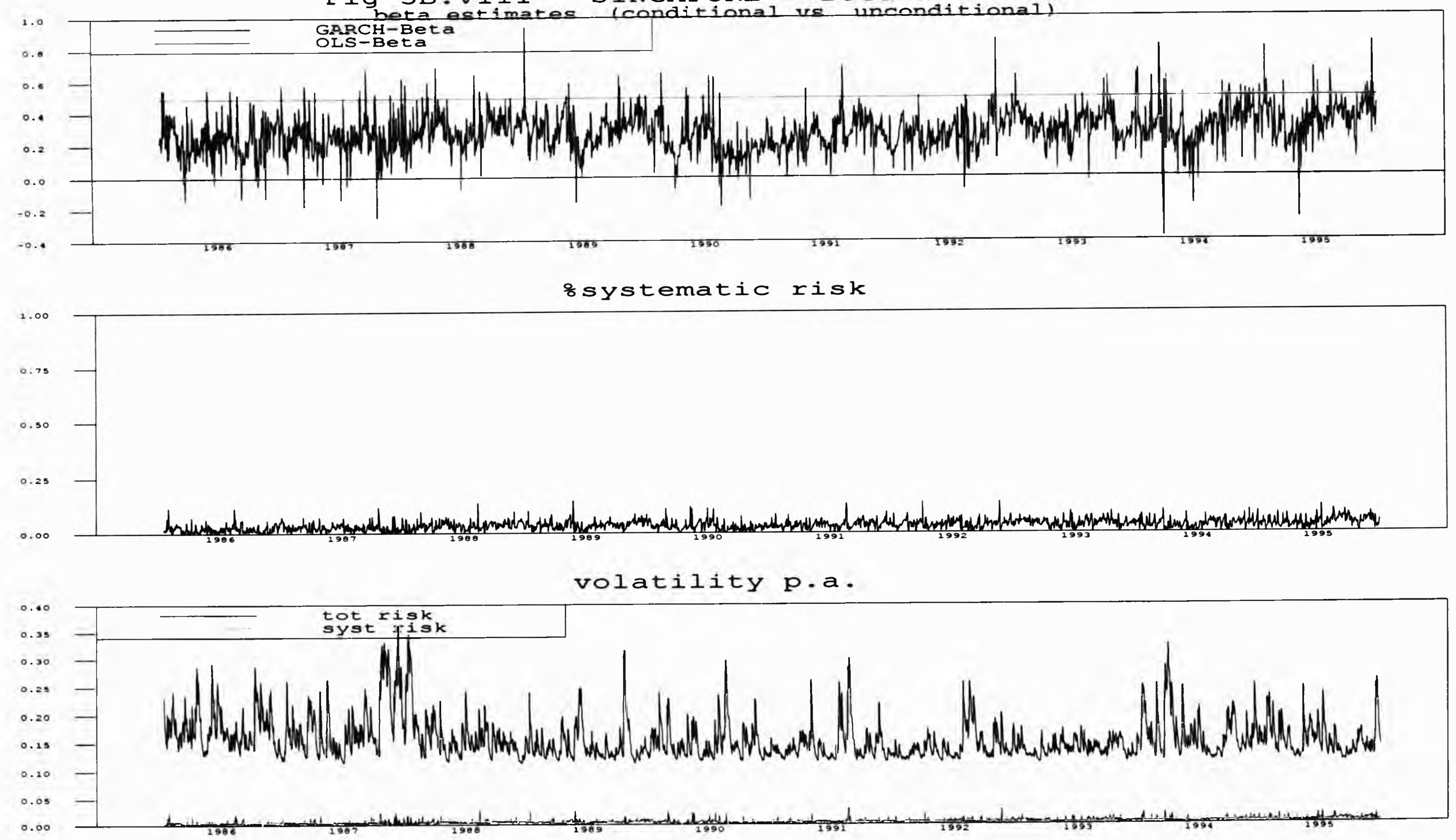


Fig 3B.IX SPAIN - returns in US$

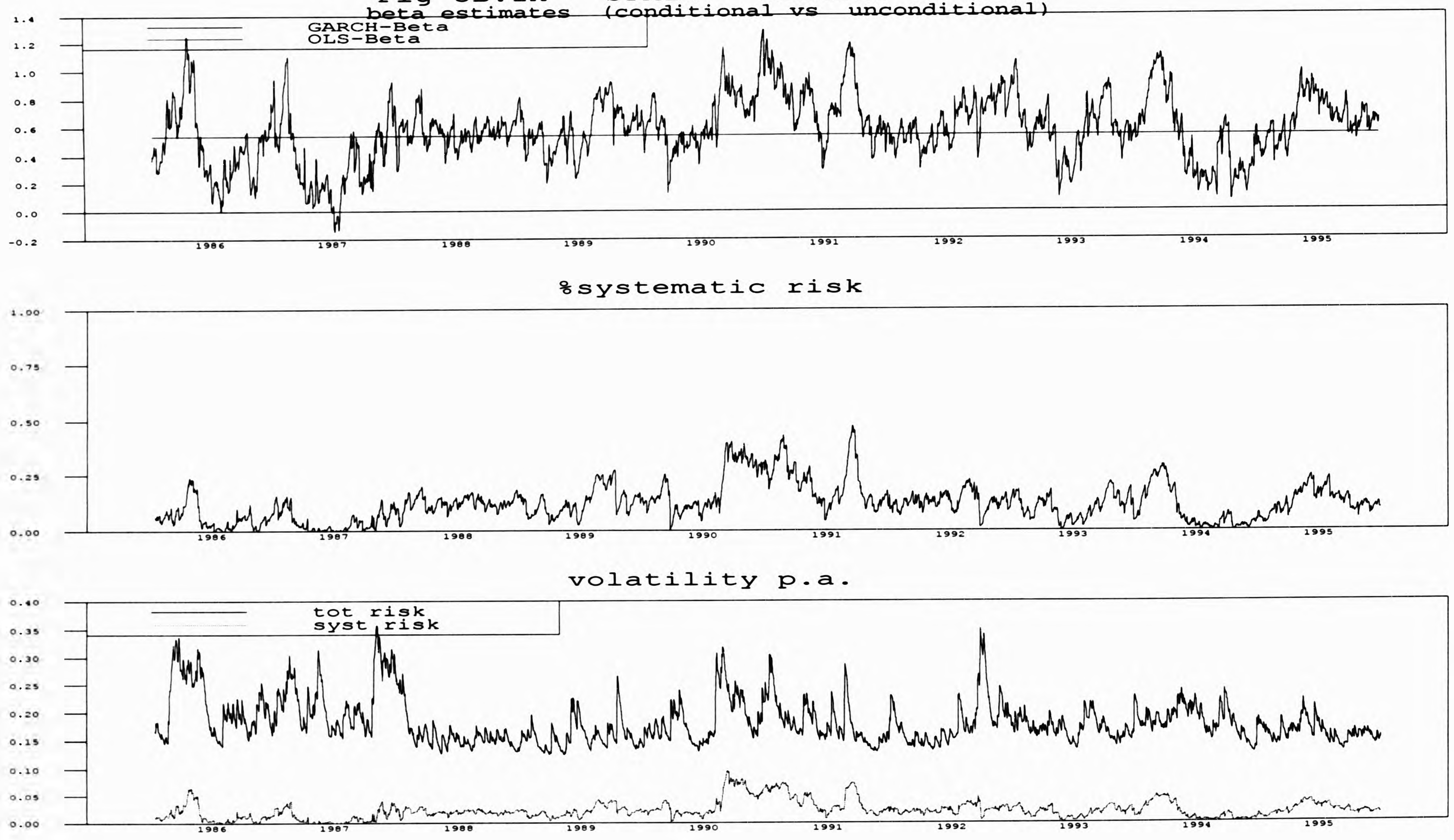


Fig 3B.X SWEDEN - returns in US$

beta estimates (conditional vs unconditional)

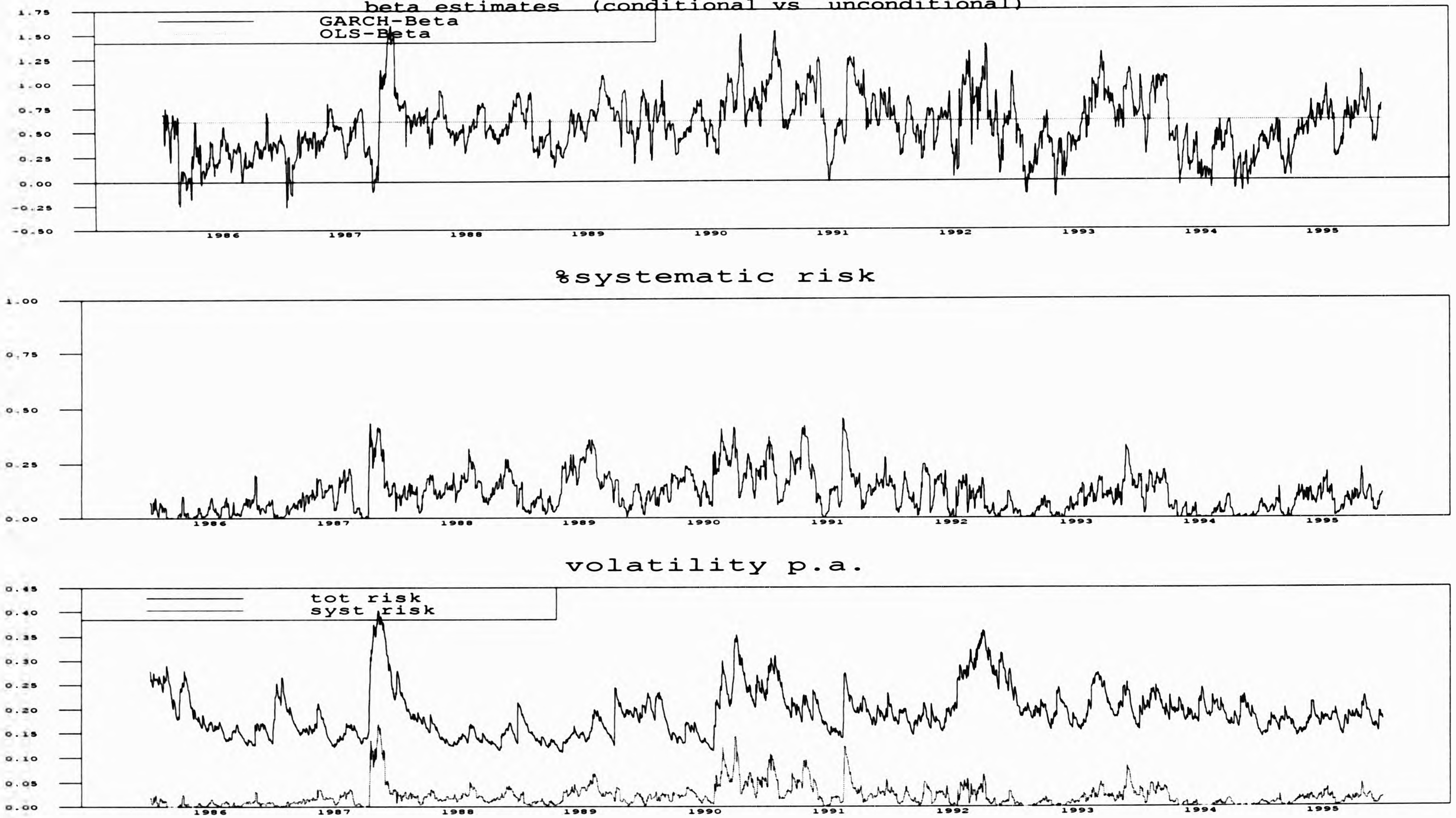


Fig 3B.XI SWITZERLAND - returns in US$

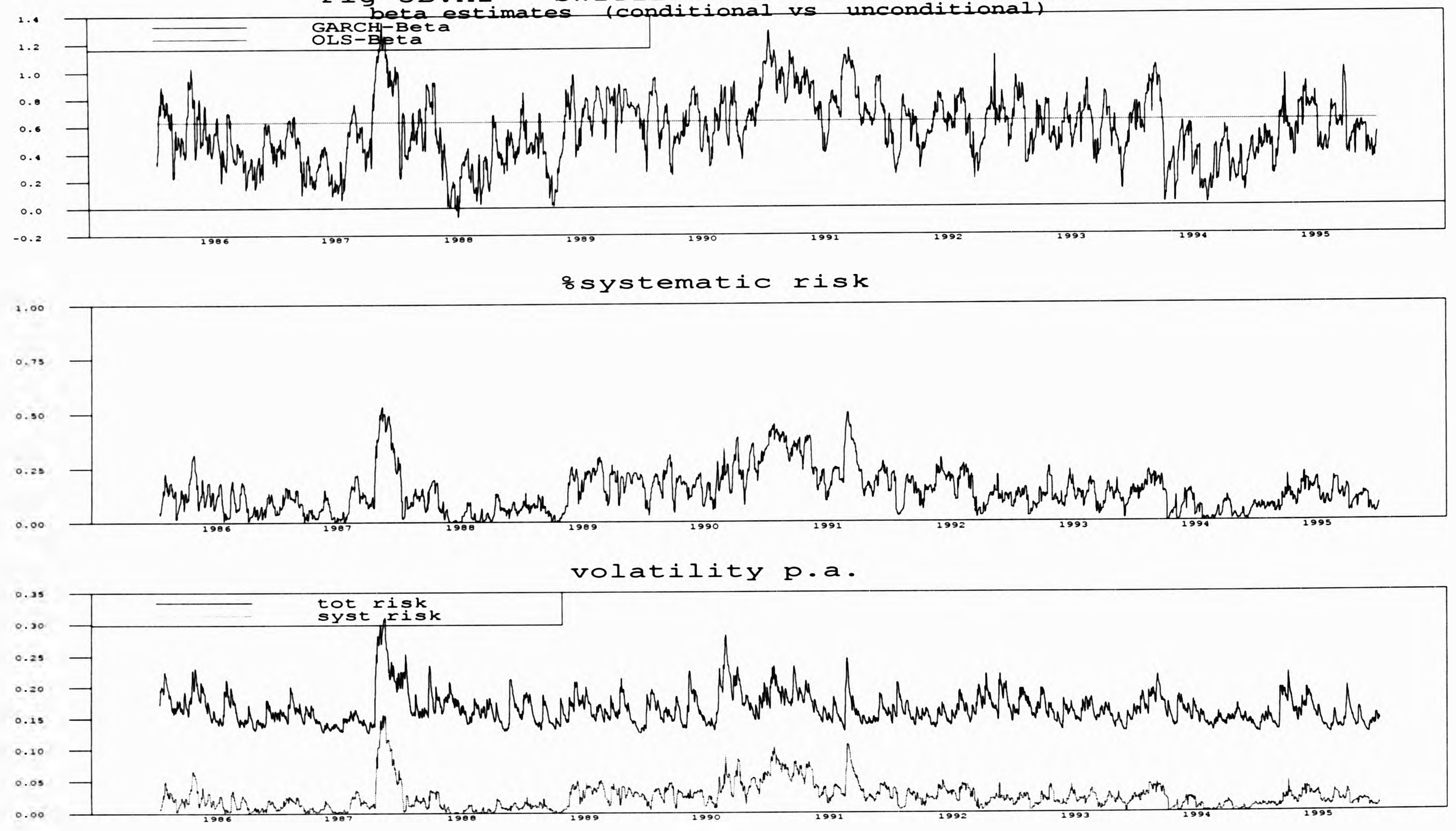


Fig 3B.XII UK - returns in US$

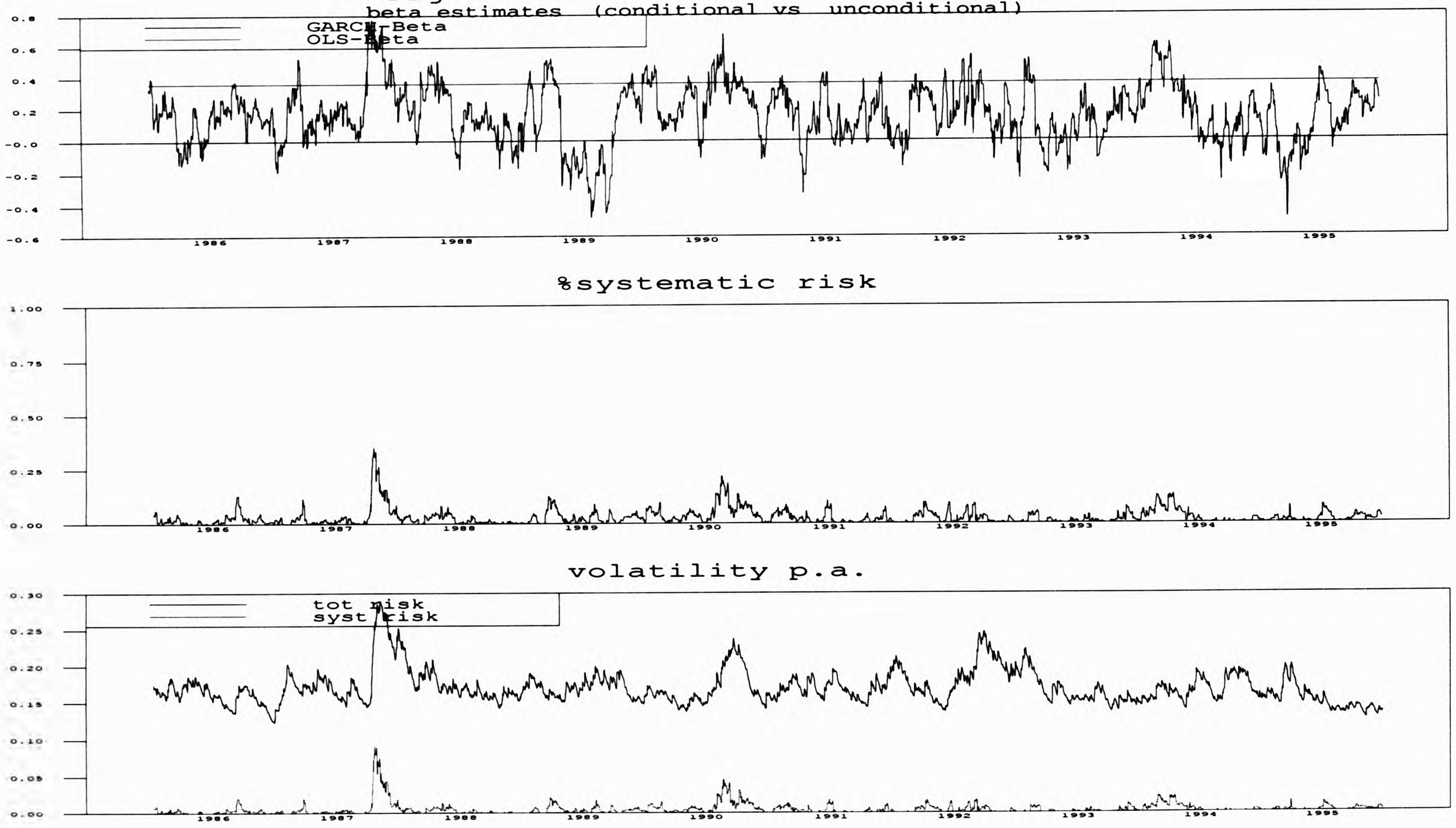
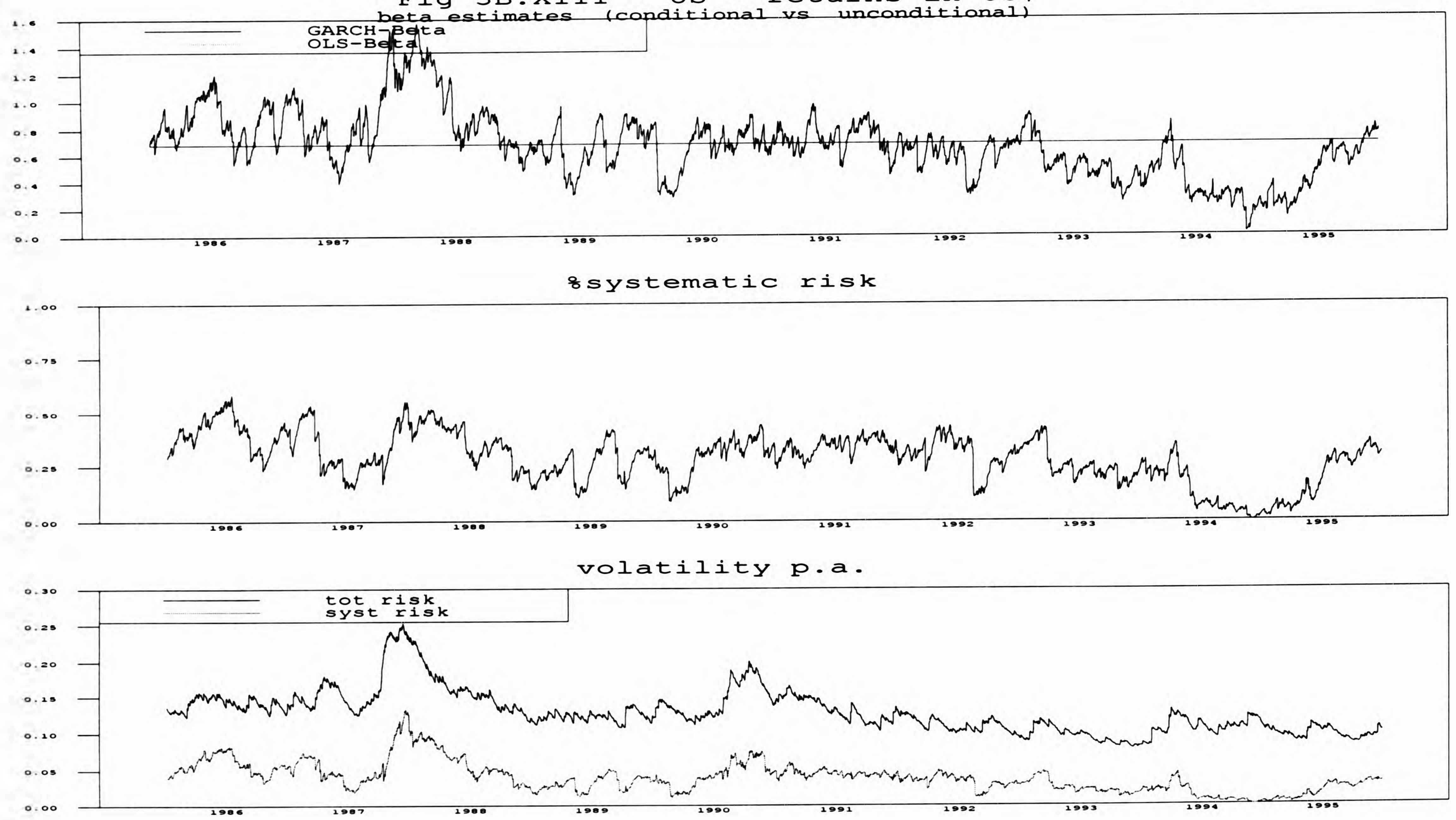


Fig 3B.XIII US - returns in US$



Chapter 4

A Simplified Approach to the Conditional Estimation of Value at Risk (VaR)[1]

Emerging risk-management techniques use VaR to assess the market risk of a portfolio. We propose a relatively simple method to estimate VaR to reflect conditionally new information about the volatility of securities held in a portfolio. While portfolio holdings might aim at diversifying risk, this risk is subject to continuous changes. In the previous chapter we used a multivariate GARCH methodology to estimate past, current, and to predicted future risk levels of our current position. The diagnostic tests showed that although the number of portfolio losses which exceed a DEaR of 99% probability were similar to what the normal distribution says, our estimates of portfolio risk were systematically biased. This can

[1] The ideas in this chapter are the by-product of joint research work with Professor Giovanni Barone-Adesi. The simplified approach to portfolio VaR (section 4.1.1) is based on the model published in the 1996 joint article. This thesis, however, applies the same methodology to a complete new data set. I would like to thank the participants at the 5th annual conference of the European Financial Management Association, Innsbruck, 1996 for helpful comments.

translate into poor risk management when a lower DEaR threshold is used or when risk management is combined with asset allocation.

In this chapter we will use a different approach to estimate portfolio VaR. We will show that the use of historical returns of portfolio components and current weights can produce accurate estimates of current risk for a portfolio of traded securities. Information on the time series properties of returns of the portfolio components is transformed into a conditional estimate of current portfolio volatility without needing to use complex time series procedures. Stress testing and correlation stability are discussed in this framework.

The hypothetical portfolio of chapter three (table 3.11) is employed here to assess the VaR methodology presented in this chapter. In addition to being faster to compute, and more flexible in the econometric specification, the results suggest that this "simplified approach" to VaR is superior to the correlation based model in chapter three.

4.1 A Simplified Way to Compute the Portfolio's VaR

In the previous chapters we have seen that the implementation of VaR models requires risk estimates for the portfolio holdings. Historical volatilities are ill-behaved measures of risk because they presume that the statistical moments of the security returns remain constant over different time periods. Conditional multivariate time series techniques are more appropriate since they use past information in a more efficient way to compute current variances and covariances. One such model which fits well with financial data is the multivariate GARCH. Its use, however, is restricted to a few assets at the time. In chapter three we have shown that it is possible to simplify the computation by adopting the SIM. However, this method requires the existence of one common factor capable of explaining a large part of the assets' variance and leave their residual risk being orthogonal. When a portfolio is diversified across a wide mix of different types of financial assets, *e.g.* equities, commodities, fixed income securities etc., a unique factor with the above properties may not exist[2].

A further limitation with this method arises from the way the variance-covariance matrix is partitioned. Unlike the full multivariate GARCH, where all variances and covariances are estimated jointly, in the SIM multivariate GARCH there is no guarantee that the resulting variance-covariance matrix comes from a NxN multivariate distribution. For example the matrix may not be positive definite or may not describe well the changes in all covariances. Hence, portfolio variance estimates are quite likely to be wrongly estimated. As we have seen in chapter three, the SIM-GARCH produces biased risk estimates for portfolios with low R^2. But is the variance-covariance necessary when portfolio risk needs to be estimated? Are there other ways of estimating portfolio volatility? This is the issue to be investigated in this chapter.

4.1.1 Our Approach to Conditional VaR

A simple procedure to overcome the difficulties of inferring current portfolio volatility from past data, is to utilise the knowledge of current portfolio weights and historical returns of the portfolio components in order to construct a hypothetical series of the returns that the portfolio would

[2] Obviously, a multi-factor model it may be more appropriate but this will increase the number of parameters in each GARCH system and it will make its use unwieldy.

have earned if its current weights had been kept constant in the past. Let $\mathbf{R}_t$ be the Nx1 vector $(R_{1,t}, R_{2,t}, .., R_{n,t})$ where $R_{i,t}$ is the return on the *ith* asset over the period (t-1,t) and let W be the Nx1 vector of the portfolio weights over the same period. The historical returns of our current portfolio holdings are given by:

$$Y_t = W^T R_t \tag{4.1}$$

In investment management, if W represents actual investment holdings, the series Y can be seen as the historical path of the portfolio returns[3]. Following Markowitz (1952, 1956) the portfolio's risk and return trade-off can be expressed in terms of the statistical moments of the multivariate distribution of the weighted investments as:

$$E(Y_t) = E(W^T R) = m \tag{4.2.a}$$

$$\mathrm{var}(Y_t) = W^T \Omega W = \sigma^2 \tag{4.2.b}$$

where Ω is the unconditional variance-covariance matrix of the returns of the N assets.

A simplified way to find the portfolio's risk and return characteristics is by estimating the first two moments of Y.

$$E(Y) = m \tag{4.3.a}$$

[3] When W represents an investment holding under consideration, Y describes the behaviour of this hypothetical portfolio over the past.

$$\mathrm{var}(Y) = E[Y - E(Y)]^2 = \sigma^2 \qquad (4.3.b)$$

Hence, if historical returns, are known the portfolios mean and variance can be found as in (4.3). This is easier than (4.2.) and still yields identical results.

The method in (4.3.b) can easily be deployed in risk management to compute the value at risk at any given time t. However, σ^2 will only characterise current conditional volatility if W has not changed. If positions are being modified, the series of past returns, Y, needs to be reconstructed and σ^2, the volatility of the new position, needs to be re-estimated as in (4.3.b).

This approach has many advantages. It is simple, easy to compute and overcomes the dimensionality and bias problems that arise when the NxN covariance matrix is being estimated. On the other hand, the portfolio's past returns contain all the necessary information about the dynamics that govern aggregate current investment holdings. In this chapter we will use this approach to make the best use of this information[4]. For example, it might be possible to capture the time path of portfolio (conditional) volatility using conditional models like GARCH.

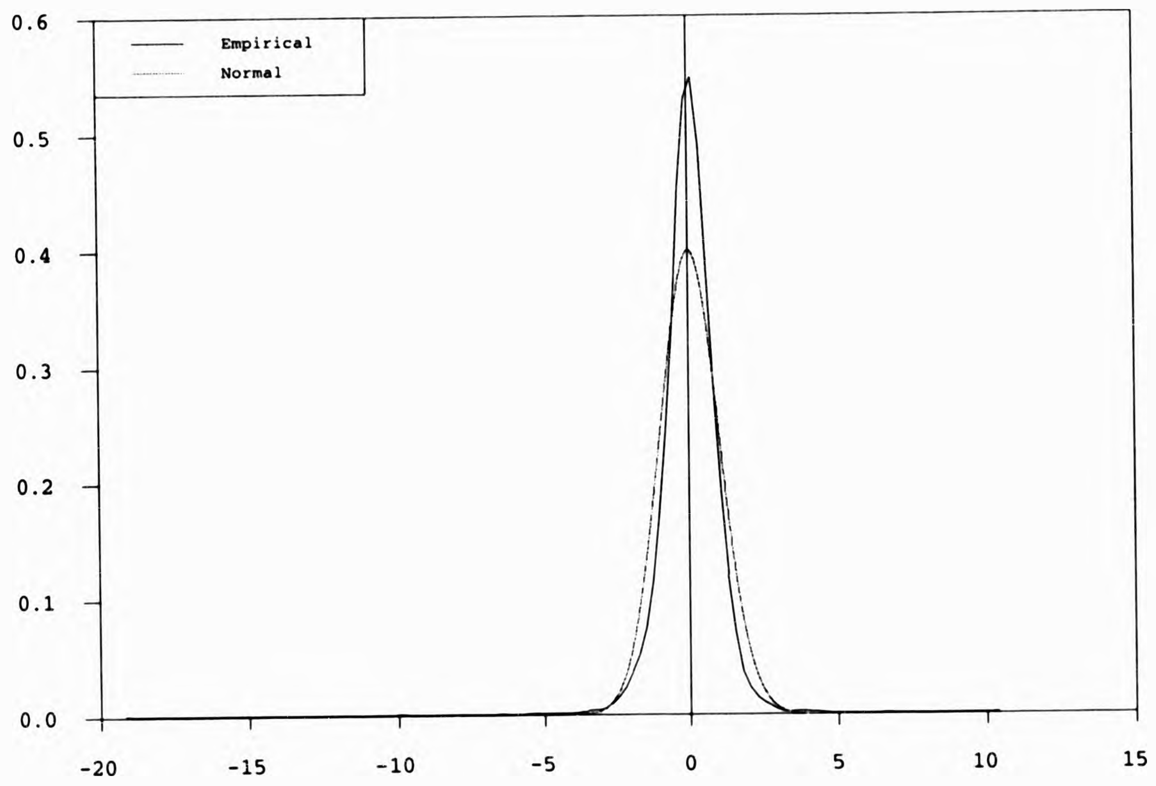
4.2 An Empirical Investigation

To illustrate how our procedure can be used in measuring portfolio VaR and in order to evaluate its effectiveness, we will use the same hypo-

[4] Markowitz (1952, 1956) incorporates equation (4.2.b) in the objective function of his portfolio selection problem because his aim was to find the optimal vector of weights W. However if W is known a priori then the portfolio's (unconditional) volatility can be computed more easily as in (4.3.b).

thetical portfolio returns constructed in the previous chapter. As has been shown, the empirical distribution of this portfolio is characterised by excess kurtosis. Further evidence of this is provided by examining figure 4.1 where the standardised empirical distribution of this portfolio returns is shown.

Fig 4.1 Empirical Distribution of Standardised Portfolio Returns



The area under the continuos line represents the standardised empirical distribution of our hypothetical portfolio[5]. The dashed line shows the shape of the distribution if returns were normally distributed. The values on the horizontal axis are far above and below the (3.0,-3.0) range which is due to very large daily portfolio gains and losses[6].

---

[5] Throughout this thesis with the term "empirical distribution" we are referring to the Kernel estimators as described in Siverman (1986).

[6] The statistical moments of the distribution are reported in table 3.12.

4.2.1 Modelling Portfolio Volatility

The excess kurtosis in this portfolio is likely to be caused by changes in its variance. We can capture these shifts in the variance by employing GARCH modelling. For a portfolio diversified across a wide range of assets, the non-constant volatility hypothesis is an open issue[7]. The LM test and the Ljung-Box statistic are employed to test this hypothesis. The test statistics with significance levels are reported in table 4.1.

Table 4.1 Testing for ARCH

	LM test(6)	Ljung-Box (6)
test statistic	352.84	640.64
p-value	(0.00)	(0.00)

Both tests are highly significant, indicating that the portfolio's volatility is not constant over different days and the squares of the portfolio returns are serially correlated[8].

One of the advantages that the model in (4.3) has is that it simplifies the econometric modelling on the portfolio variance. Because we only have to model a single series of returns we can select a conditional volatility model that bests fits the data. As we have seen in chapter two there are two families of models, the GARCH and SV, which are particularly suited to capturing changes in volatility of financial time series. To model the hypothetical portfolio volatility, we use GARCH modelling because it offers wide flexibility in the mean and variance specifications

[7] In a widely diversified portfolio, which may contain different types of assets, the null hypothesis of non-ARCH may not be rejected even if each asset follows a GARCH process itself.

[8] If the null hypothesis had not been rejected, then portfolio volatility could be estimated as a constant.

and its success in modelling conditional volatility has been well documented in the financial literature. That will also enable us to make a comparison between the full variance-covariance model of chapter three and the univariate approach of (4.3).

We tested for a number of different GARCH parameterisations and found that an asymmetric GARCH(1,1)-ARMA(0,1) specification best fits[9] our hypothetical portfolio (see table 3.11). This is defined as :

$$Y_t = \Phi\varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \tag{4.4.a}$$

$$h_t = \omega + \alpha\,(\varepsilon_{t-1} + \gamma)^2 + \beta\, h_{t-1} \tag{4.4.b}$$

The parameter estimates reported in table 4.2 are all highly significant, confirming that portfolio volatility can be better modelled as conditionally heteroskedastic. The coefficient α that measures the impact of last period's squared innovation, ε, on today's variance is found to be positive and significant; In addition, $\frac{\omega + \alpha\gamma^2}{1-\alpha-\beta} > 0$ indicating that the unconditional variance is constant.

---

[9] A number of different GARCH parameterisations and lag orders have been tested. Among these conditional variance parameterisations are the GARCH, exponential GARCH, threshold GARCH and GARCH with t distribution in the likelihood function. We used a number of diagnostic tests, i.e. serial correlation, no further GARCH effect, significant t-statistics. The final choice for the model in (4.4) is the unbiassednes in conditional variance estimates as tested by the Pagan-Ullah test of (3.3) and absence of serial correlation in the residual returns. Non-parametric estimates of conditional mean functions, employed latter, support this assumption.

Table 4.2 Parameter estimates of equation (4.4)

series	Φ	ω	α	β	γ	Likelihood
estimate	0.013	1.949	0.086	0.842	-3.393	-8339.79
t-statistic	(2.25)	(3.15)	(6.44)	(29.20)	(5.31)	

Moreover, the constant volatility model, which is the special case of $\alpha=\beta=0$, can be rejected. The coefficient γ that captures any asymmetries in volatility that might exist is significant and negative, indicating that volatility tends to be higher when the portfolio's values are falling.

4.3 Diagnostics and Stress analysis

Correct model specification requires that diagnostic tests be carried out on the fitted residual, $\hat{\varepsilon}$. Table 4.3 contains estimates of the regression:

$$\hat{\varepsilon}_t^2 = a + b\hat{h}_t \tag{3.3}$$

with heteroskedasticity-consistent, White (1980), -*t* statistics given in parentheses.

Table 4.3 Diagnostics on the GARCH residuals

	a	b	R^2	Q(6) on $\hat{\varepsilon}_t$	Q(6) on $\hat{\varepsilon}_t^2$	JB
statistic	-7.054	1.224	0.373	3.72	10.13	468.89
significance	(1.73)	(1.87)		(0.71)	(0.12)	(0.00)

The hypotheses that a=0 and b=1 cannot be rejected at 95% confidence level indicating that our GARCH model produces a consistent estimator for the portfolio's time-varying variance. The uncentered coefficient of determination, R^2 in (3.3), which measures the fraction of the total variation of everyday returns explained by the estimated conditional variance, and has a value 37.3%. Since the portfolio conditional variance uses the information set available from the previous day, the above result indicates that our model, on average, can predict more than one third of next day's squared price movement. The next two columns in table 4.3 contain the Ljung-box statistic of order six for the residuals and squared residuals. Both null hypothesises, for serial correlation and further GARCH effect, cannot be rejected indicating that our model has removed the volatility clusters from the portfolio returns and left white noise residuals. The last column contains the Jarque-Bera normality test on the standardised residuals. Although these residuals still deviate from the normal distribution, most of the excess kurtosis has been removed, indicating that our model describes the portfolio returns better than the historical volatility model.

In figure 4.2 the standardised innovations of portfolio returns are shown. The upper and lower horizontal lines represent the 2.33 standard deviations (0.01 probability) threshold. We can see that returns are moving randomly net of any volatility clusters. Figure 4.3 shows the kernel distribution of these standardised innovations against the normal distribution. It is apparent that the distribution of these scaled innovations is rather non-normal with values reaching up to fourteen standard deviations. However, when the outliers to the left, (which reflect the large losses during the 1987 crash), are omitted, the empirical distribution of the portfolio residual returns matches that of a Gaussian.

Fig 4.2 Portfolio Stress Analysis

(standardised conditional residuals)

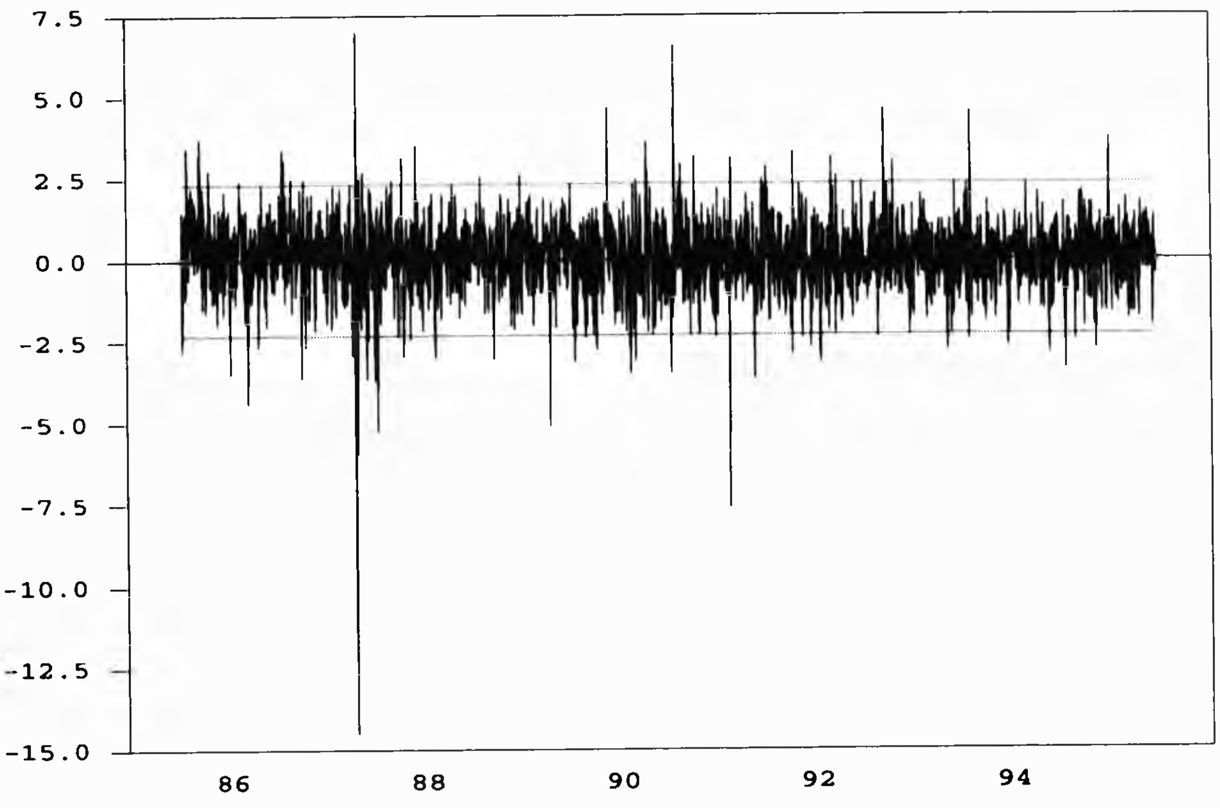
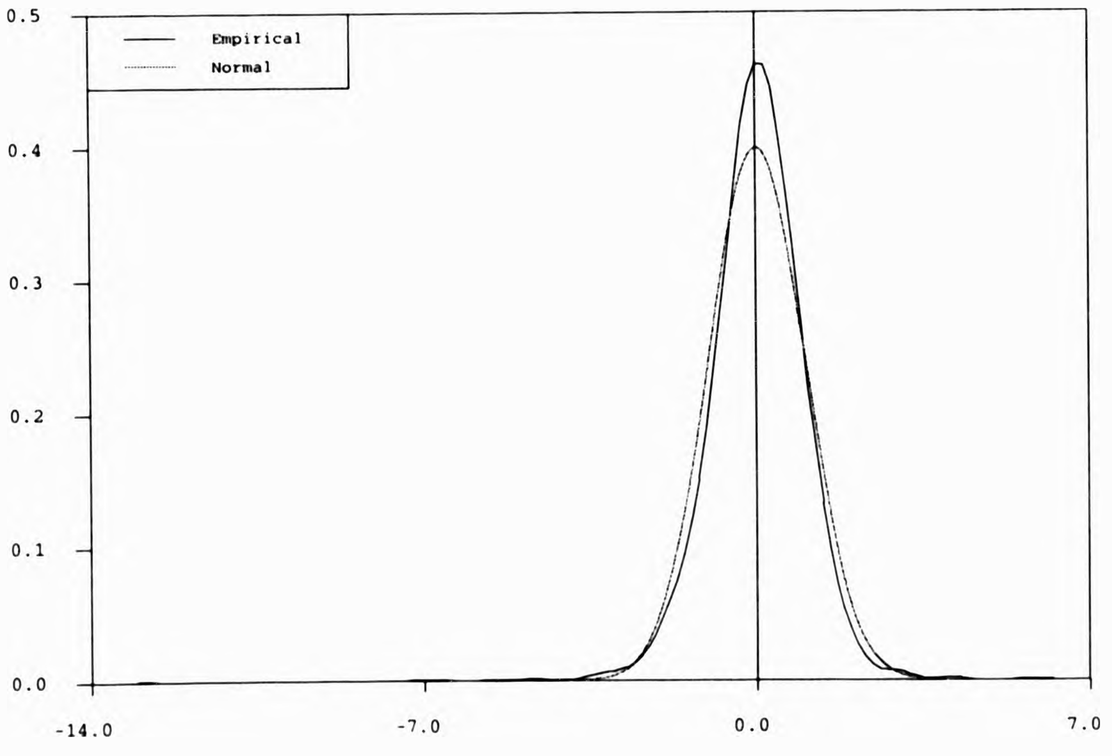


Fig 4.3 Empirical Distribution of Portfolio Conditional Distribution



These results substantiate the superiority of the volatility model in (4.4) in monitoring portfolio risk. Our model captures all volatility clusters present in the portfolio returns, removes a large part of the excess kurtosis and leaves residuals approximately normal. Furthermore, our method for estimating portfolio volatility using only one series of past returns is much faster to compute than the variance-covariance method and provides unbiased volatility estimates with higher explanatory power.

4.4 Correlation Stability and Diversification Benefits

Conditional VaR models which use the quadratic equation (1.2) to update portfolio volatility, *e.g.* RiskMetrics (1995), need first to estimate all the possible pair-wise covariances. In a widely diversified portfolio, *e.g.* containing 100 assets, there are 4950 conditional covariances and 100 variances to be estimated. Furthermore, any model used to update the covariances must keep the multivariate features of the joint distribution. With a large matrix like that, and is unlikely to get unbiased estimates[10] for all 4950 covariances and at the same time guarantee that the joint multivariate distribution still holds. Obviously, errors in covariances as well as in variances will affect the accuracy of our portfolio's VaR estimate and will lead to wrong risk management decisions. Our approach

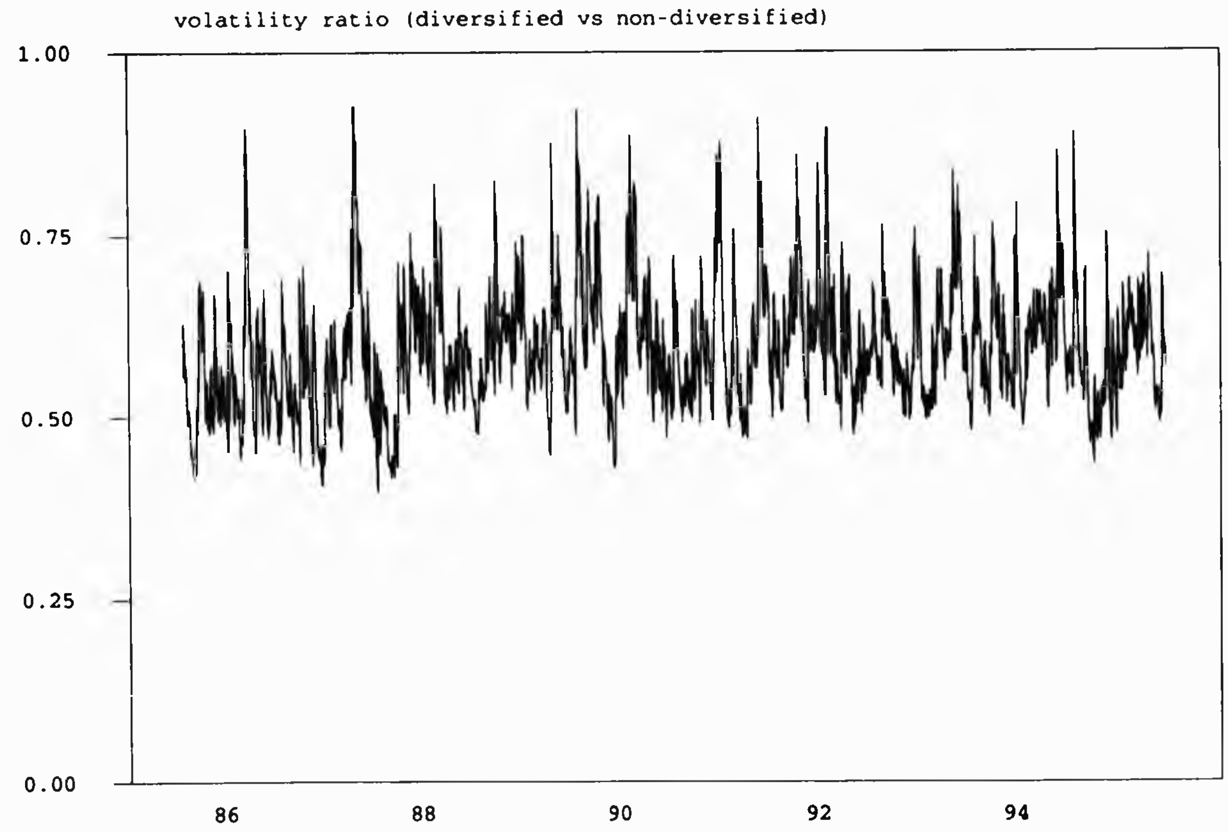
[10] The Pagan-Ullah (1988) test can also be applied to measure the goodness of fit of a conditional covariance model. This stands on regressing the cross product of the two residual series against a constant and the covariance estimates. The unbiasedness hypothesis requires the constant to be zero and the slope to be one. The uncentered coefficient of determination of the regression tells us the forecasting power of the model. Unfortunately, even with daily observations, for most financial time series the coefficient of determination tends to be very low, pointing to the great difficulty of getting good covariance estimates.

estimates conditionally the volatility of only one univariate time series, the portfolio's historical returns, and so overcomes all of the above problems. Furthermore, since it does not require the estimation of the variance-covariance matrix, it can be easily computed and can handle an unlimited number of assets. On the other hand it takes into account all changes in assets' variances and covariances.

Another appealing property of our approach is to disclose the impact that the overall changes in correlations have on portfolio volatility. It can tell us what proportion an increase/decrease in the portfolio's VaR is due to changes in asset variances or correlations. We will refer to this as *correlation stability.*

It is known that each correlation coefficient is subject to changes at any time. Nevertheless, changes across the correlation matrix may not be correlated and their impact on the overall portfolio risk may be diminished. Our conditional VaR approach allows to attribute any changes in the portfolio's conditional volatility to two main components; changes in asset volatilities and changes in asset correlations. If h_t is the portfolio's conditional variance, as estimated in (4.2), its time-varying volatility is $\sigma_t = \sqrt{h_t}$. This is the volatility estimate of a diversified portfolio at period t. By setting all pair-wise correlation coefficients in each period equal to 1.0, the portfolio's volatility becomes the weighted volatility of its asset components. Conditional volatilities of the individual asset components can be obtained by fitting a GARCH type model for each return series. We denote the volatility of this undiversified portfolio as s_t. The quantity $\left(1-\frac{\sigma_t}{s_t}\right)$ tells us what proportion of portfolio volatility has been diversified away because of non perfect correlations. If that quantity does not change significantly over time, then the weighted overall effect of time-varying correlations is invariant and we have *correlation stability*. The correlation stability shown in figure 4.4 can be used to measure the risk manager's ability to diversify portfolio's risk. On a well diversified (constantly weighted) portfolio, the quantity $\left(1-\frac{\sigma_t}{s_t}\right)$ should be invariant over different periods. Barone-Adesi and Giannopoulos (1996) have shown that a portfolio invested only in bonds is subject to greater correlation risk than a portfolio containing commodities and equities, because of the tendency of bonds to fall in step in the presence of large market moves.

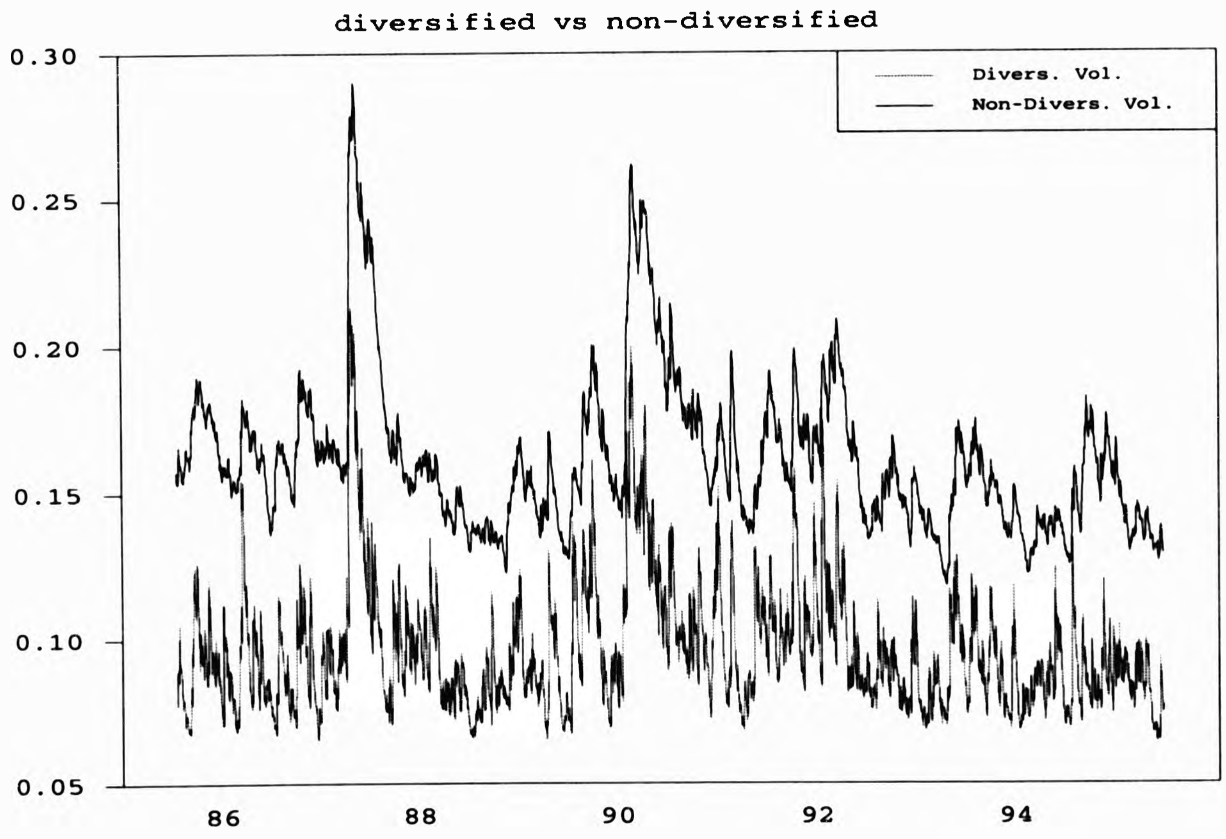
Fig 4.4 Portfolio Correlation Stability



The "weighted" effect of changes in correlations can also be shown by observing the diversified against the undiversified portfolio risk. Figure

4.5 illustrates how the daily annualised standard deviation of our hypothetical portfolio behaves over the tested period. The upper line shows the volatility of an undiversified portfolio; this is the volatility the same portfolio would have if all pair-wise correlation coefficients of the assets invested were 1.0 at all times. The undiversified portfolio's volatility is simply the weighted average of the conditional volatilities of each asset included in the portfolio. Risk managers who rely on the average standard historical risk measures will be surprised by the extreme values of volatility a portfolio may produce in a crash. Our conditional volatility estimates provide early warnings about this risk increase and therefore are a useful supplement to existing risk management systems.

Fig 4.5 Portfolio Conditional Volatility



Descriptive statistics for diversified and undiversified portfolio risk are reported in table 4.4. These range of volatility are those that would have been observed had the portfolio weights been effective over the whole sample period. Due to the diversification of risk, the portfolio's volatility is reduced by an average of 40%[11]. During the highly volatile period of the 1987 crash, the risk is reduced by a quarter.

Table 4.4 Portfolio risk statistic

portfolio risk	minimum	maximum	mean
Diversified	0.0644	0.2134	0.0962
undiversified	0.01192	0.2978	0.1632

4.5 Portfolio VaR and "Worst Case" Scenario

4.5.1 Portfolio VaR

Using the hypothetical portfolio of the previous chapter, the volatility during the last trading day of 1995 (29 December) estimated by the GARCH model of (4.4) was 7.7063% p.a. Using this estimate in (1.1) the portfolio's DEaR for that day is 2.85% of its value. By substituting the above volatility estimate in (1.3), the portfolio VaR over the next few trading days can be estimated. However, the VaR estimate in (1.3) assumes that the volatility will remain invariant during the period over which the VaR is estimated. But volatility prevailing at the end of 1995 was very low indeed; below historical average and about one third of the peak levels observed during the crash and Iraqi invasion of Kuwait. One major advantage that our methodology has is that it forecasts portfolio

[11] That is the average volatility of a diversified over the average of an undiversified portfolio.

volatility recursively upon the previous day's volatility. Then it uses these volatility forecasts to calculate the **VaR** over the next days. Below, we discuss how this method is implemented.

By substituting the last day's residual return and variance in (4.4.b) we can estimate the portfolio's volatility for day **t+1** and by taking the expectation, we can estimate recursively the forecast for longer periods. Hence, portfolio volatility forecast over the next 10 days is

$$h_{t+i} = \omega + \alpha\,(\varepsilon_t + \gamma)^2 + \beta\, h_t \qquad \text{if } i = 1 \qquad (4.5.a)$$

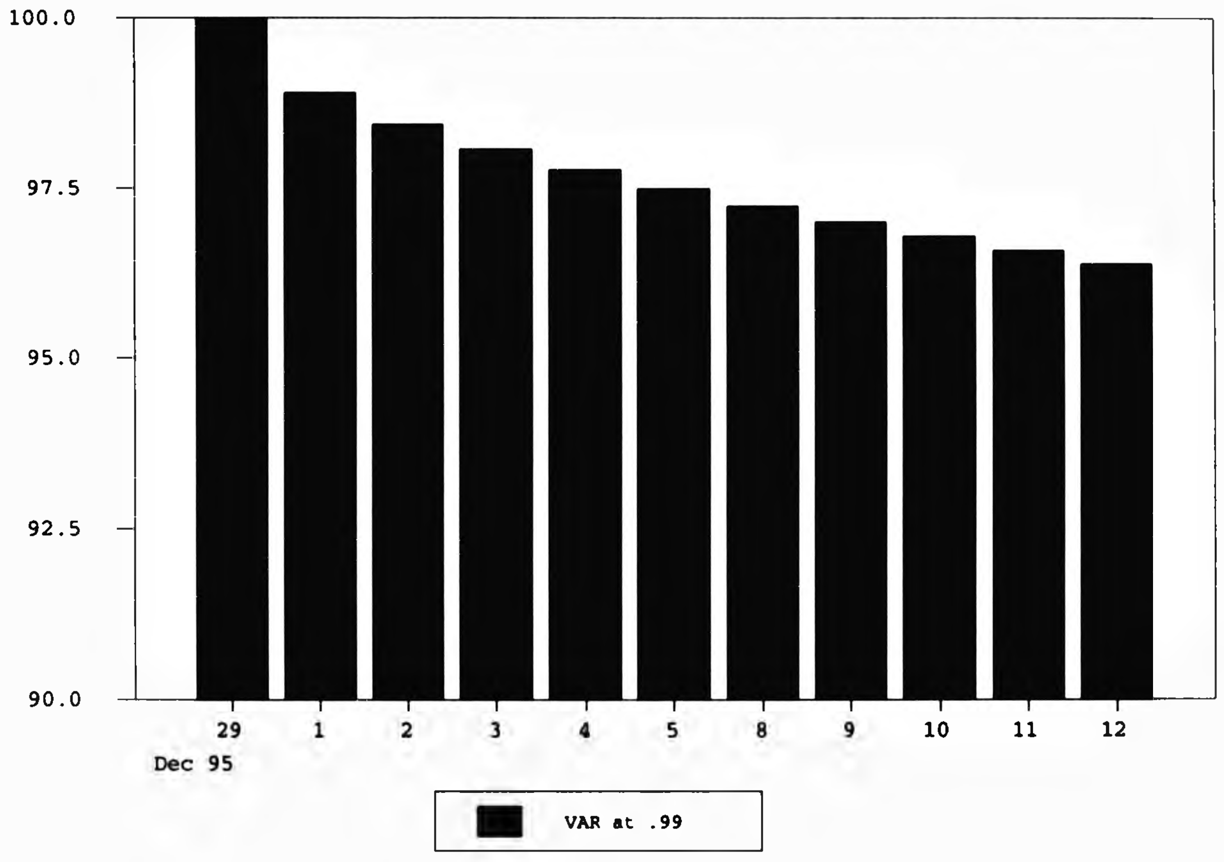
$$h_{t+i/t} = \omega + \alpha\gamma^2\ \ (\alpha+\beta)h_{t+i-1/t} \qquad \text{if } i > 1 \qquad (4.5.b)$$

Therefore, when portfolio volatility is below average levels, the forecast values will be rising[12]. The portfolio **VaR** that will be calculated on these forecasts will be more realistic about possible future losses.

Figure 4.6 shows our hypothetical portfolio's **VaR** for 10 periods of length between one and 10 days. The portfolio **VaR** is estimated at the close business on 29 December 1995. To estimate the **VaR** we obtain volatility forecasts for each of the next business days, as in (4.5). The **DEaR** is 1.104% while the 10 days **VaR** is 3.62%.

[12] The forecast of the portfolio variance converges to a constant, $(\omega+\alpha\gamma^2)/(1-\alpha-\beta)$, which is also the mean of the portfolio's conditional volatility.

Fig 4.6 Portfolio VAR



4.5.2 Worst Case Scenario

VaR measures the market risk of a portfolio in terms of the frequency that a specific loss will be exceeded. In risk management, however, it is important to know the size of the loss rather than the number of times the losses will exceed a pre-defined threshold. The type of analysis which tells us the worst that can happen to a portfolio's value over a given period is known as the "worst-case scenario" (WCS). Hence, the WCS is concerned with the prediction of uncommon events which by definition are bound to happen. The WCS will answer the question, how badly will it hit?

For a VaR model, the probability of exceeding a loss at the end of a short period, is a function of the last day's volatility and the square root of time (assuming no serial correlation). Here, however, the issue of fat tails arises. It is unlikely, that there exists a volatility model that predicts the likelihood and size of extreme price moves. For example, in this study we observed that the GARCH model removes most of the kurtosis but still leaves residuals equal to several standard deviations. Given that extreme events, such as the 1987 crash, have a realistic probability of occurring again at any time, any reliable risk management system must account for them.

The WCS is commonly calculated by using structured Monte-Carlo simulation (SMC)[13]. This method aims to simulate the volatilities and correlations of all assets in the portfolio by using a series of random draws of the factor shocks (ε_{t+1}). At each simulation run, the value of the portfolio is projected over the VaR period. By repeating the process several thousand times, the portfolio returns density function is found and the WCS is calculated as the loss that corresponds to a very small probability under that area. There are three major weaknesses with this analysis. Firstly, there is a dimensionality problem which also translates to computation time. To overcome this, RiskMetrics (1995, p 98) proposes to simplify the calculation of the correlation matrix by using a kind of factorisation. However, as will see in the next chapter, factorisation is sensitive to the ordering of the series when estimating the variance-covariance matrix[14]. Secondly, the SMC method relies on a (time invari-

[13] For a discussion of the SMC see Riskmetrics (1995). For a discussion of other Monte Carlo Methods and their use in VaR see Pritsker (1996) (grid Monte Carlo) and Owen and Tavela (1997) (quasi Monte Carlo).

[14] Changing that order will affect the simulation results.

ant) correlation structure of the data. But as we have seen, security co-variances are changing over different periods and the betas tend to be higher during volatile periods like that of the 1987 crash. Hence, correlations in the extremes are higher and the WCS will underestimate the risk. Finally, the use of a correlation matrix requires returns in the Monte-Carlo method to follow an arbitrary distribution. In practice the empirical histogram of returns is "smoothed" to fit a known distribution. However, the WCS is highly dependent on a good prediction of uncommon events, or catastrophic risk and the smoothing of the data leads to a cover up of extreme events, thereby neutralising the catastrophic risk.

Univariate Monte Carlo methods can be employed to simulate directly various sample paths of the value of the current portfolio holdings. Hence, once a stochastic process for the portfolio returns is specified, a set of random numbers, which conform to a known distribution that matches the empirical distribution of portfolio returns, are added to form various sample paths of portfolio return. The portfolio VaR is then estimated from the corresponding density function. Nevertheless, this method is still exposed to a major weakness. The probability density of portfolio residual returns is assumed to be known[15].

In this study, to further the acceptance of the VaR methodology, we will assess its reliability under conditions likely to be uncorrelated in financial markets. The logical method to investigate this issue is through the use of historical simulation. Historical simulation relies on a uniform distri-

[15] A second limitation arises if the (stochastic) model that describes portfolio returns restricts portfolio variance to be constant over time.

bution to select innovations from the past[16]. These innovations are applied to current asset prices to simulate their future evolution. Once a sufficient number of different paths has been explored, it is possible to determine a portfolio VaR without making arbitrary distributional assumptions. This is especially useful in the presence of abnormally large portfolio returns.

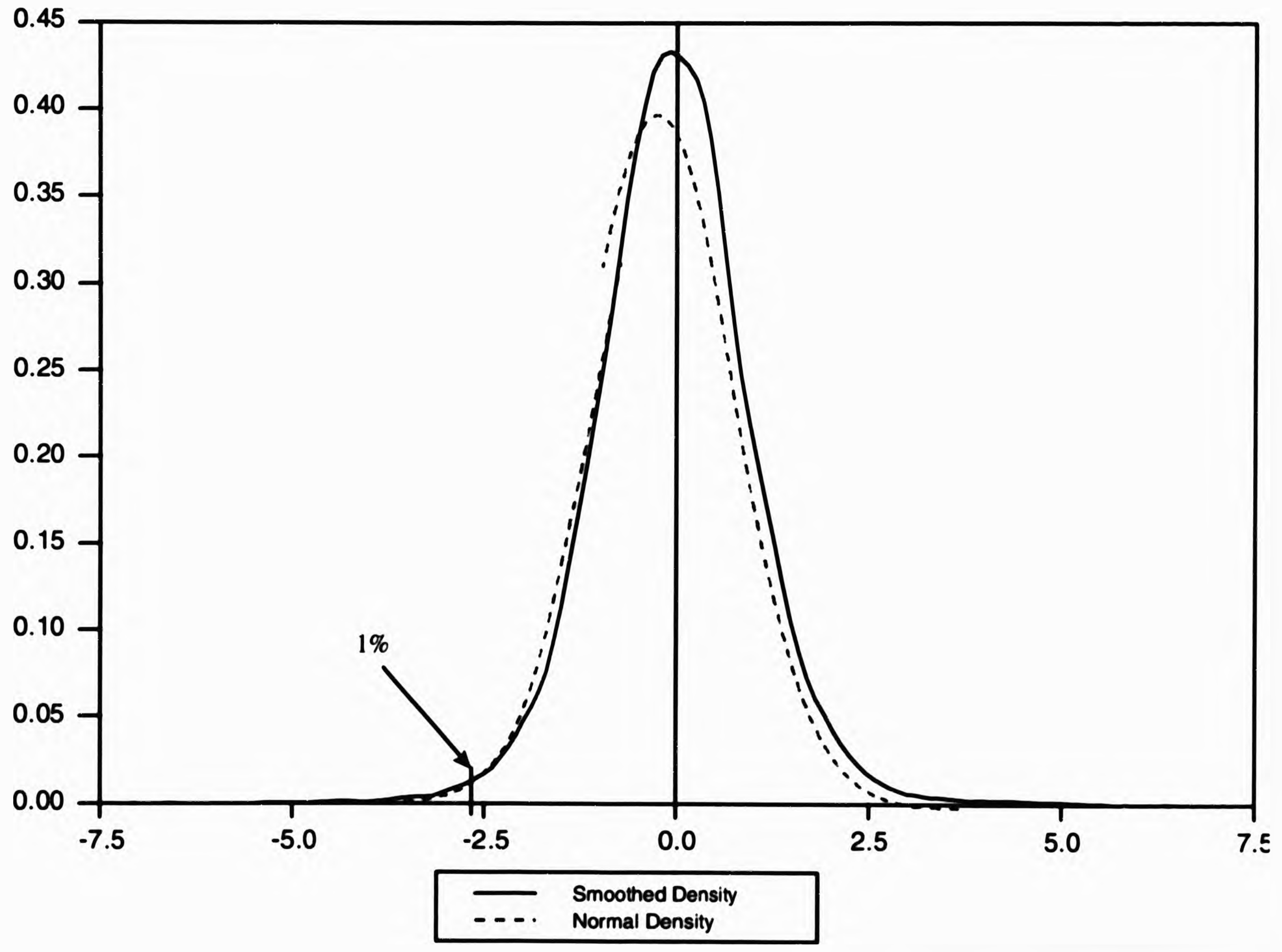
To make historical simulation consistent with the clustering of large returns, we will employ the GARCH volatility estimates of (4.4) to scale randomly selected past portfolio residual returns. First, the past daily portfolio residual returns are divided by the corresponding GARCH volatility estimates to obtain standardised residuals. Hence, the residual returns used in the historical simulation are i.i.d. which ensures that the portfolio simulated returns will not be biased. A simulated portfolio return for tomorrow is obtained by multiplying randomly selected standardised residuals by the GARCH volatility to forecast the next day's volatility. This simulated return is then used to update the GARCH forecast for the following days, that is it as multiplied by a newly selected standardised residual to simulate the return for the second day. This recursive procedure is repeated until the VaR horizon (*i.e.* 10 days) is reached, generating a sample path of portfolio volatilities and returns. A batch of 10 thousand sample paths of portfolio returns is computed and a confidence band for the portfolio return is built by taking the first and the ninety-ninth percentile of the frequency distribution of returns at each time. The lower percentile identifies the VaR over the next 10 days.

[16] Historical simulation is better known as bootstrapping simulation. For a detailed discussion about this simulation technique see Efron and Tibshirani (1993).

To illustrate our methodology we use the standardised conditional residuals for our portfolio over the entire 1986-1995 period as shown in fig 4.2. We then construct interactively the daily portfolio volatility that these returns imply according to (4.4). We use this volatility to re-scale our returns. The resulting returns reflect current market conditions rather than historical conditions associated with the returns in figure 4.1.

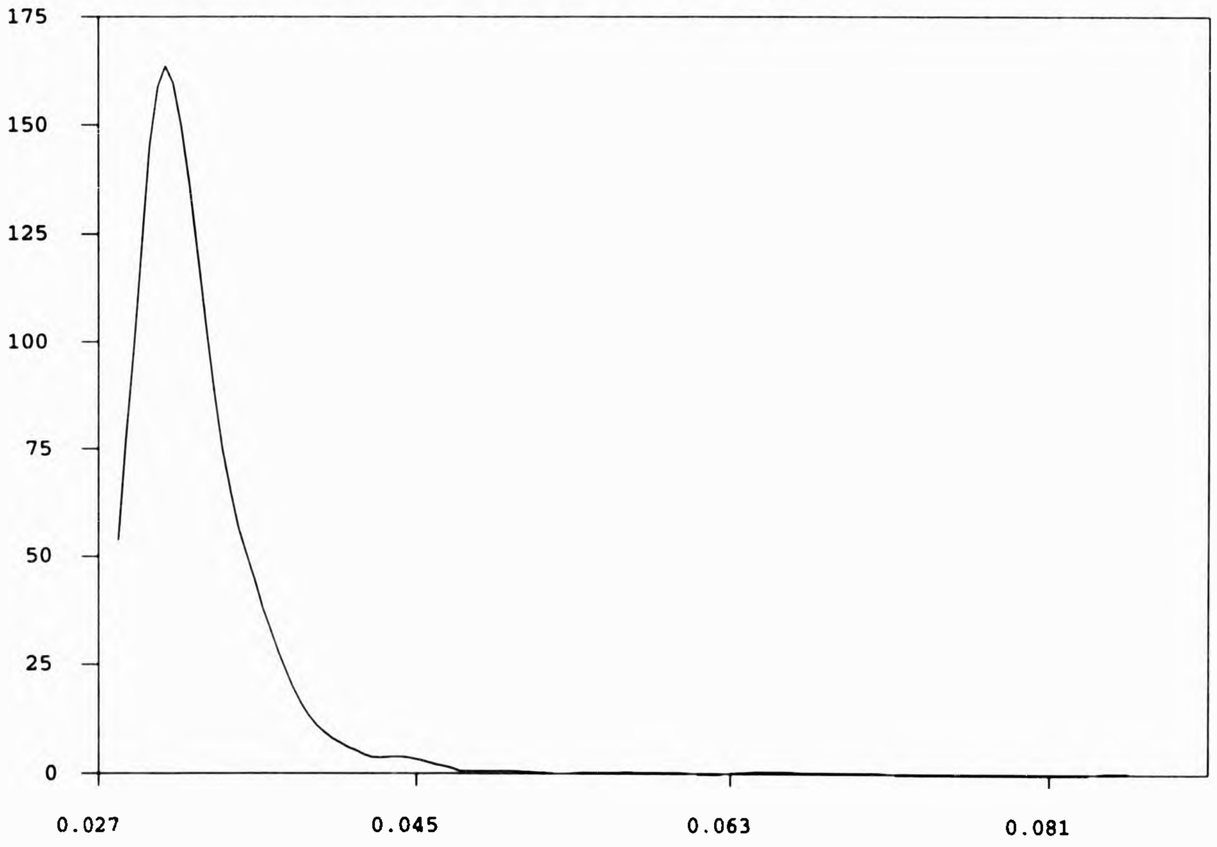
To obtain the distribution of our portfolio returns we replicated the above procedure 10,000 times. The resulting -normalised- distribution is shown in figure 4.7. The normal distribution is shown in the same figure for comparison.

Fig 4.7 : Normalized Estimated Distribution of Returns in 10 days versus the normal density (10,000 Simulations)



Not surprisingly, simulated returns on our well-diversified portfolio are almost normal, except for their steeper peaking around zero and some clustering in the tails. The general shape of the distribution supports the validity of the usual measure of VaR for our portfolio. However, a closer examination of our simulation results shows how even our well-diversified portfolio may depart from normality under worst case scenarios. There are in fact several occurrences of very large negative returns, reaching a maximum loss of 7.22%. Our empirical distribution implies (under the WCS) losses of at least 3.28% and 2.24% at confidence levels of 1% and 5% respectively[17].

Fig 4.8 Estimated Distribution of VaR



[17] Note that the empirical distribution has asymmetric tails and is kurtotic. Our methodology ensures that the degree of asymmetry is consistent with the statistical properties of portfolio returns over time.

The reason for this departure is the changing portfolio volatility and thus portfolio VaR, shown in figure 4.8. Portfolio VaR over the next 10 days depends on the random returns selected in each simulation run. Its pattern is skewed to the right, showing how large returns tend to cluster in time. These clusters provide realistic WCS consistent with historical experience. Of course our methodology may produce more extreme departures from normality for less diversified portfolios.

4.6 Conclusions

While portfolio holdings aim at diversifying risk, this risk is subject to continuous changes. The GARCH methodology allows us to estimate past and current and predicted future risk levels of our current position. However, the correlation based VaR, which employed GARCH variance and covariance estimates, failed the diagnostic tests badly. The VaR model used in this chapter is a combination of historical-simulation and GARCH volatility. It only relies on historical data for securities prices but applies the most current portfolio positions to historical returns. The use of historical returns of portfolio components and current weights can produce accurate estimates of current risk for a portfolio of traded securities. Information on the time series properties of returns of the portfolio components is transformed into a conditional estimate of the current portfolio volatility with no need for using complex multivariate time series procedures. Our approach leads to a simple formulation of stress analysis and correlation risk.

There are three useful products of our methodology. The first one is a simple and accurate measure for the volatility of the current portfolio from which an accurate assessment of current risk can be made. This is achieved without using computationally intensive multivariate methodologies. The second is the possibility of comparing a series of volatility patterns similar to figures 4.5 with the historical volatility pattern of the actual portfolio with its changing weights. This comparison allows for an evaluation of the managers' ability to "time" volatility. Timing volatility is an important component of performance, especially if expected security returns are not positively related to current volatility levels. Finally, the possibility of using the GARCH residuals on the current portfolio weights allows for the implementation of meaningful stress testing procedures. Stress testing and the evaluation of correlation risk are important criteria in risk management models.

To test our simplified approach to VaR we employed the same hypothetical portfolio used in chapter three. We fitted an asymmetric GARCH on the portfolio returns and we forecasted portfolio volatility and VaR. The results indicate that this approach to estimating VaR is superior to the correlation based model used in the previous chapter. This is implied by the GARCH model yielding unbiased estimators for the portfolio conditional variance. Furthermore, this conditional variance estimate can now predict, on average, one third of the next day's square price movement.

We then applied the concept of correlation stability which we argue is a very useful tool in risk management in that it measures the proportion of an increase or decrease in the portfolio VaR caused by changes in asset correlations. In comparing the conditional volatility of our diversified and undiversified hypothetical portfolio, the effects of changes in correlations can be highlighted. While we found that the volatility of the diversified portfolio is lower than the undiversified portfolio, the use of correlation stability has the useful property of acting as an early warning to risk managers in relation to the effects of a negative shock, such as that of a stock market crash, on the riskiness of our portfolio. This is appealing to practitioners because it can be used to determine the ability of risk managers to diversify portfolio risk. Correlation stability is appealing to practitioners because it can be used, both in working with the portfolio selection and assessing the ability of risk managers to diversify portfolio risk.

Thereafter, we show how "Worst Case" scenarios (WCS) for stress analysis may be constructed by applying the largest outliers in the innovation series to the current GARCH parameters. While the VaR estimated previously considers the market risk of a portfolio in relation to the frequency that a specific loss will be exceeded, it does not determine the size of the loss. Our exercise simulates the effect of the largest historical shock on current market conditions and evaluates the likelihood of a given loss occurring over the VaR horizon.

In conclusion, our simulation methodology allows for a fast evaluation of VaR and WCS for large portfolios. It takes into account current market conditions and does not rely on the knowledge of the correlation matrix of security returns.

Chapter 5

The Dynamic Mechanisms of Volatility Transmission Among National Stock Markets

In chapter three we have shown that both the systematic and specific volatility on national equity markets changes each day. The changes in conditional betas among national markets neither seems to be independent or to occur simultaneously. The existence of any dependency in the way conditional betas across different markets are changing can be seen as market interdependency. The dynamic mechanisms of volatility transmission across markets is the subject of this chapter. These mechanisms affect VaR measurements by introducing time-varying, possibly asynchronous components of portfolio volatility that are ignored in the original static framework of portfolio theory. In this framework, VaR measurements will depend on the recent history of other markets. A benchmark for the empirical relevance of this problem may be obtained from the error variance decomposition technique. If the recent history of other variables explains a large portion of portfolio variance over the portfolio horizon, these conditioning variables need to be included if an accurate measure of risk is desired. The resulting augmented model is Markovian and produces VaR estimates conditional on recent history. The ability to obtain estimates of volatility as transmitted across markets with a lag provide us with a warning to close down positions and reduce the VaR. We investigate the need for such an extension across six national equity markets. Our results suggest that, in most cases, only the first lagged return of a foreign market, mostly the US market, contributes to the volatility of our tested portfolio.

5.1 Previous Work on Interlinks Among Equity Markets

Since the early seventies numerous studies have examined the interdependence among national stock markets. *e.g.* Granger and Morgensten (1970), Grubel and Fadner (1971), Hilliard (1979). Although these studies used a variety of statistical methods to support their conclusions they all agreed on one fact, price movement interdependence in international portfolios is much smaller than in domestic portfolios. The objective of most of these studies is to provide evidence for benefits arising from diversifying internationally. Therefore, they focused their analysis on the correlation matrix of a set of national equity indices. However, they used unconditional second moments of returns and the correlation coefficients were restricted to be constant over the estimation period. Consequently, any conclusion they made regarding national market interdependence was conditional on this assumption. Other studies investigated the mechanisms through which price movements are transmitted from one market to the other. This issue has been of major interest since the crash of October 1987. Many investors and academics have observed that equity price movements around the globe are linked via short run dependencies, Roll (1989), Eun and Shim (1989), Von Furstenberg and Jeon (1989).

Eun and Shim (1989) searched for links across nine major national equity markets using the vector autoregressive analysis of Sims (1980), where they investigated the mechanisms for transmitting price movements from one national market to the rest. They used daily prices in local currencies covering the period 31 December 1979 till 20 December 1985. They argued that impulse response analysis and variance decomposition, part of the vector autoregressive methodology, are ideal for answering questions such as "how much movement in one stock market can be explained by innovations in other markets?", or "how rapidly are the price movements in one market transmitted to other markets?", (see Eun and Shim 1989 p. 242). Eun and Shim, among others, report that "innovations in the United States are rapidly transmitted to the other markets in a clearly recognisable fashion, whereas no single foreign market can significantly explain the US market movements", (p. 241).

In another study, Von Furstenberg and Jeon (1989) employed vector autoregressive models and principal components analysis to investigate the links between daily price changes among four markets; Japan, Germany, Great Britain and United States. They split the sample into 1987 pre-crash and post-crash periods. The pre-crash sample covered the period from January 7 (6 for Japan) 1986 to October 14 (13) 1987, a total of 462 observations. The post-crash period sample covered the period November 22 (21 for Japan) 1987 to November 24 (23) 1988. Thus, the few days prior and after the crash were considered to be outliers and excluded from both samples. They found that the correlations among those markets increased substantially after October 1987. This is not surprising since the after-crash period was characterised by high volatility and the data sample was relatively short, less than a year. As we have seen in chapter three, during highly volatile periods, and in particular during the 1987 crash, both national betas and the proportion of the systematic risk tend to increase.

The findings of Von Furstenberg and Jeon (1989) and Eun and Shim (1989) have two additional weaknesses. First, they did not recognise that national market movements might be linked via their variances and co-variances as well as linked through their means. Being linked by their second moments implies that markets do not necessarily have to react always in the same direction to "news" from a leading market. For example a drop in US prices might sometimes be followed by a fall and at other times by a rise on the far east markets. Secondly, they did not recognise that national market interdependence may not be constant over time. By contrast, if markets are linked through their second moments and if those moments are time-varying, as we have shown in the GARCH methodology in chapter three, then there might be reasons to believe that this market dependence may vary as well. This might explain why during, and immediately after, the crash of 1987 the correlation coefficients increased well above historical levels. Moreover, by treating the volatilities as time series, it will be possible to reveal any existing non-synchronous patterns in the way shocks are transmitted among markets. This can help in developing various *market timing* strategies for trading or hedging purposes. Clearly, historical models will fail to recognise these changes in volatility levels and capture any asynchronies that might occur.

Hammao *et al.* (1990) were the first to investigate the transmission mechanisms in volatilities across international stock markets. Using daily stock returns for Japanese, UK and US equities measured from close-to-open and open-to-close, they examined how volatility surprises in one market affect the other two. They used a GARCH parameterisation which allows the current volatility of a single market to depend upon the volatility surprises of the previously open foreign market. They found that the spillover effects from the US and UK stock markets to the Japanese market are significant. However, the spillover effects on the UK and US markets are not. The major weaknesses in their analysis was their focus on short run dependencies only. They failed to answer questions like what are the longer run implications for other markets after a volatility surprise in one market.

Chan *et al.* (1991) investigated the intraday volatility transmission among the S&P 500 stock index returns and the S&P500 stock index futures returns. By using five minute return data for the two series they estimated the conditional volatility of each index as a bivariate AR(1)-GARCH(1,3) process with constant correlation. They then employed impulse response analysis between the squared error of one market (origin of a shock) and the conditional volatility of the other in order to search for lagged volatility spillovers. They concluded that there is a strong dependence in both directions in the volatility of returns between the cash and futures markets. Previous studies found evidence that "news" disseminates in the futures market before the cash market. Chan *et al.* found that "news" disseminates in both futures and cash markets. However, earlier studies have distinguished between "good" and "bad" news, while Chan *et al.* treat both types of news as equal.

In another study, Karolyi (1995), employed a combination of vector autoregressive and bivariate GARCH techniques to investigate how innovations are transmitted from US to Canadian markets and vice-versa. He specified the two return equations as vector autoregressive processes which allow for lagged innovations in one market to have an impact on its own as well as on those of the other markets' conditional mean and variance. The forecast errors was modelled as a bivariate GARCH process, similar to that of (2.16). Hence, the vector autoregressive innovations are purified from volatility clusters and lagged return spillovers. He then employed impulse response analysis on these standardised residuals to simulate the way innovations are transmitted from the US to Canada and vice-versa. Nevertheless, this study investigated market transmission of "news" through the means rather than through the variance of returns. Furthermore, no confidence bands were shown so any market interdependence studied was lacking appropriate statistical tests. In addition Karolyi's study suffers from a restrictive parameterisation that he imposed on the GARCH process[1].

Most of the previous work has searched for linkages across different national markets by examining changes in price levels. This study will follow the same line of investigation as Hamao *et al.* (1990) by searching for market interdependencies in their conditional second moments. However, we will investigate volatility transmission mechanisms over a larger number of equity markets, six. Further, this study differs from others in two further aspects. Firstly, price interdependence will be investigated via each market's systematic risk rather the overall volatility. Secondly, confidence bands will be computed for each impulse response function.

We believe that it is very important to distinguish between systematic and specific volatility, since stocks (national markets) are inter-linked only through the systematic counterpart of their risk, Sharpe (1963). The

1 The conditional variance-covariance matrix was specified as in Baba *et al.* (1990) (BEKK).

specific counterpart is due to idiosyncratic (domestic) factors affecting a particular stock only (national market) and has nothing in common with the volatility governing other stocks (national markets). Focusing the analysis on markets' common risk is both consistent with finance theory and is expected to strengthen the significance of the results.

5.2 Methodology

This study will search for national market interdependencies in the changes of their time-varying betas against the world index. The vector autoregressive methodology will be employed both to examine dependencies in beta changes and to simulate the way a local market's beta will respond when a shock to another markets' beta occurs[2]. The vector autoregressive methodology is ideal for studying the "*impulse response function*" of the conditional betas. The "*forecast error variance decomposition*" of a unit increase in a local market's beta will disclose to what extent this is due to innovations in each of the other markets. The variables used in the vector autoregressive model are the six series of time-varying betas computed in chapter three with the bivariate GARCH system.

This study intends to address, and seeks an answer to a number of questions with regard to the mechanisms that govern volatility spillovers between national markets. In addition, it will demonstrate the use of the impulse response analysis and variance decomposition as valuable tools for studying market timing. This will enable us to foresee how markets might behave after a major shock has occurred in one of them. It will

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2 An increase (decrease) in a local market's beta is synonymous with an increase (decrease) in this market systematic risk.

also help us to identify which national markets are active leaders and which one are followers when there are strong cross border price movements. Monte Carlo simulation will be used to appraise the significance of the results.

5.2.1 The Vector Autoregressive Analysis

To explore the dynamic impact on the current level of a market's beta of past changes in its own and other betas, it is necessary to set-up a system of simultaneous equations with at least as many equations as dependent variables. Considering the complicated nature of national markets' interrelationships it will almost be impossible to specify a large scale structural model. Hence, the vector autoregressive analysis is an ideal tool for this case. It estimates a dynamic simultaneous equation system without imposing any restrictions on the structure of the relationships among the variables. Hence, "..the VAR (vector autoregressive) model can be seen as a flexible approximation to the reduced form of the correctly specified but unknown model of the actual variable structure"., (Eun and Shim (1989) p. 242). Another great advantage of vector autoregressive models is the flexibility they offer for policy simulation. Once the vector autoregressive system has been estimated, the moving average representation can be used both to calculate the impulse response function and to forecast variance decomposition. The first can be used to simulate the dynamic responses of a national market's beta to innovations in the beta of some other market while the latter can be used to measure the relative importance of each market in generating unexpected variations in other markets' (systematic) volatility. Hence, the vector autoregressive analysis has many appealing properties for investigating the dynamic links between national markets.

To model a N variable system, an unrestricted vector autoregressive parameterisation will be employed here. The vector autoregressive model is formulated as:

$$Y_t = K + A_1Y_{t-1} + A_2Y_{t-2} + .. + A_sY_{t-s} + e_t \qquad (5.1)$$

where Y_t is a Nx1 column vector of weekly conditional betas for each of the thirteen markets, K is Nx1 vector of intercept coefficients, $A_1,..,A_s$ are NxN matrices of coefficients, s is the system's order, and e_t is a Nx1 column vector of forecast errors of the Y_t using all past Y_s. Hence, on the left hand side of each equation there is a time series variable with a local market's sensitivity to the world factor while on the right hand side there are its own and the other markets' past sensitivity values. The right hand side of each equation contains the same terms. The ij*th* element of A_s measures the impact that a change in the beta risk to the j*th* market would have on the i*th* market in s periods. A positive (negative) but statistically significant coefficient implies that j*th* market's volatility is expected to increase (decrease) as a result of a rise in the i*th* market's beta occurring s periods earlier. Thus, the column $A_{i,s}$ expresses the direction and magnitude of lagged volatility spillovers that originate from the i*th* market s weeks before and go towards the vector of markets Y.

The model in (5.1) is often referred to as the unrestricted vector autoregressive model since it allows all the variables in the system to interact linearly with their own and other variables' past values and it uses only historical values to forecast the quantitative effect that each variable has on its own and the other variables values. Because each variable is a function of lagged values of all other variables, this vector autoregression can be seen as a general dynamic specification. The unrestricted vector autoregression of (5.1) is estimated by ordinary least squares (OLS); Zellner (1962) proved that OLS estimates of such a system are consistent and efficient if each equation has exactly the same set of explanatory variables.

Provided that the process is stable, (5.1) can be written as a vector moving average (VMA) process:

$$Y_t = E(Y) + \sum_{s=0}^{\infty} \Phi_s \; e_{t-s} \tag{5.2}$$

where the i*th* element of the Nx1 vector Y represents the i*th* national market's systematic volatility at period t as a linear least squares projection on the past periods' systematic volatility of all markets on the system; e_{t-s} is a Nx1 vector with unexpected changes in those volatilities at period t-s and, Φ_s is a NxN symmetric matrix (variance-covariance) with the sensitivity coefficients to the unexpected innovations on e_{t-s}. The matrix Φ_s can be interpreted as

$$\frac{\partial Y_{t+s}}{\partial e_t'} = \phi_s \tag{5.3}$$

that is, the ij*th* element of Φ_s measures the sensitivity of the i*th* market's beta value at time t+s to a one unit shock in the j*th* market's beta, ($e_{j,t}$), at time t, holding all other betas in the system constant. A numerical simulation of (5.3) will provide us with an understanding of how the rest of the markets react to a shock in the *ith* market's common volatility.

One way of exploring the dynamics governing volatility spillovers is by simulation. This is done by setting $e_{j,t}=1$ and $e_{i,t} = 0$ for $i \neq j$, as well as $Y_{t-1} = Y_{t-2} = .. = Y_{t-s} = 0$. This is repeated for $j=1,..,n$ to obtain the realisations of the Φ matrix for the s periods. Thus, a sequence of s realisations is obtained for the *ijth* element of Φ. This is called the *impulse-response function* or *dynamic multiplier* of the *jth* beta. The impulse response function describes response of the *ith* beta to a single impulse (shock) in the *jth* (at time t) beta with all others dated at time t or earlier held constant. In other words, $\Phi_{ji,s}$ represents the reaction of the *jth* market's systematic risk to a unit shock in the *ith* systematic risk s periods ago, provided of course the effect is not contaminated by shocks in any other market's risk (common or domestic) included in the vector autoregressive system.

5.2.2 Orthogonalised Shocks

To isolate the impact that innovations in one national market have in the vector autoregressive system from innovations in any other market, it is necessary for $E(e_t e_t^T) = \Omega$ to be a diagonal matrix, *i.e.* the innovation processes contained in e_t should be orthogonal to each other. Although the vector elements of e_t are serially uncorrelated by construction, there is no guarantee that the contemporaneous components will be uncorrelated as well. It is however possible to transform the e_t into a new vector u_t with $E(u_{i,t}\, u_{j,t})=0$, for $i \neq j$.

One popular method of transforming the variance-covariance matrix of the vector autoregressive residuals into a vector of orthogonal innovations, u_t, is to use the Choleski factorisation. This consists in finding a NxN lower triangular matrix V that satisfies:

$$u=V^{-1}e \quad \text{and} \quad V V^T = \Omega$$

$$\text{or} \quad V^{-1}\Omega(V^T)^{-1} = I \tag{5.4}$$

where I is the identity matrix. By replacing e_t with u_tV and after omitting the mean term $E(Y)$, equation (5.2) can be rewritten as

$$Y_t = \sum_{s=0}^{\infty} \Phi_s \, V \, u_{t-s} \tag{5.5}$$

By defining $Q_s=\Phi_s V$ (5.5) can be written as

$$Y_t = \sum_{s=0}^{\infty} Q_s \, u_{t-s} \tag{5.6}$$

The mean term, $E(Y)$, is dropped since it is of no interest in the simulation process[3]. The elements of the NxN matrix Q_s can be used to generate the effect of a shock in e_i at period s=0 on the entire time paths of the vector of variables Y. Hence, the elements of Q_s are impact multipliers. The sequence Q_s from s=0,1,2.., shows the dynamic responses of Y_t to unit shocks in $u_{t=s}$. The ij*th* element of Q_s is the impulse response of the j*th* national market's betas to a shock of one standard deviation in the

3 Every variable Y_t in the system has two components, its best linear predictor based on past values of all variables and an innovation term e_t. We are interested in simulating how Y_t will react to unpredictable changes of any other variable where those unpredictable changes are expressed by e_t.

ith beta occurring s periods before. If the variables in Y are stationary then the impulse responses should tend toward zero as s becomes large.

Therefore, it is possible to trace the likely response in each of the six markets' systematic volatility to the innovations in the beta in one market alone. This can provide insights in how common volatility originating in one particular national stock market is transmitted to the other countries. It should be noted however that the Choleski factorisation imposes a "Wold causal chain" in the vector autoregressive system implying that a shock in the first variable has contemporaneous effects on all the other variables, a shock in the second variable has contemporaneous effects on all following variables but not the first one, and etc. Hence, the Choleski factorisation is not unique but depends on the ordering of the variables.

5.2.3 Forecast Error Variance Decomposition

The vector autoregressive models, as well as being useful for forecasting, can also be used to disclose properties of the forecast errors, e_t, and to desegregate further the relationships that govern the variables in the system. It enables us to simulate how an artificial shock in one variable will affect the forecast error variance of itself and each other variable in the system in s=0,1,.., periods.

Let us presume that we have consistent estimates of the K and A_1, ..,A_s coefficients in (5.1) and we want to forecast the values of the various national market betas s periods ahead, Y_{t+s}, conditional on the known beta values at time t. This can be done by updating (5.1) recursively s periods in the future. The variance decomposition function (VDF) tells us in what proportion the variable k is accounting for the variance of the variable v in s steps ahead; the VDF is given by:

$$VDF(v,k,s) = \frac{\sum_{i=0}^{s-1} Q_{ivk}^2 \sigma_k^2}{\sum_{i=0}^{s-1}\sum_{j=1}^{N} Q_{ivj}^2 \sigma_j^2} \times 100 \tag{5.7}$$

Hence, each period's forecast error variance is decomposed into N components, each of them associated with innovations in one of the variables of the system at time t. This kind of analysis is often referred to as *innovation accounting.*

5.3 The Data and Preliminary Analysis

To capture any potential interactions which may arise from the systematic risk, it is necessary to have accurate estimates of the markets' betas at time t. In the rest of this analysis we will employ the conditional beta estimates of the each national markets against the word index described in chapter 3[4]. In our vector autoregressive analysis we will employ the time varying betas both in local currency, shown in appendix 3A, and 3B respectively. Beta risk is synonymous with systematic risk, therefore the conditional beta series contains information about local markets' common risk only. Thus, the vector autoregressive analysis of the conditional betas will provide us with information about the path and pattern of volatility transmission among markets. But in this analysis the volatility variables entering the vector autoregressive system will not be al-

[4] The beta series bivariate GARCH model in that of (2.16) and the estimation and diagnostics are described in chapter 3.

lowed to be contaminated by any components that are due to idiosyncratic factors and have no impact on any other market.

Another reason for employing the GARCH model of (2.16) is the fact that the estimated conditional betas form the best available estimator for systematic volatility at each measurement period and thus provide indispensable information about the dynamic changes in local portfolios' market risk. The vector autoregressive modelling of those conditional betas will explore further any market linkage and volatility spillovers.

We will first investigate the market interlinks with returns which are expressed in local currency and that any impact on the results due to foreign exchange fluctuations is excluded[5]. On a daily or weekly basis, returns are subject to domestic and international economic and political news. Among others Eun and Shim (1989) and Lin *et al.* (1994) use market indices expressed in local currency units.

The rationale for not expressing returns in a common currency is to segregate market risk from currency risk. It is well known that these two risks are not additive[6] and that expressing the various domestic portfolio returns in a common currency will have an adverse impact on their conditional volatility. Likewise, when the prices of the aggregated stocks are expressed in a common currency, then will have a similar impact on the world index's volatility. Furthermore, the pair-wise correlation coefficients between a set of domestic portfolios tend to be smaller when the

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[5] Exchange rate fluctuations have an impact on stock price movements but, ceteris paribus, prices adjust to exchange rate differentials over a longer period.

[6] If σ_d^2 is the foreign market risk in the local currency, σ_f^2 the exchange rate volatility, σ_p^2 the total risk of the portfolio in the investor's currency, and r the correlation coefficient between the two risks, then ignoring non-linear terms, $\sigma_p^2 = \sigma_d^2 + \sigma_f^2 + 2r\sigma_d\sigma_f$, implying that $\sigma_p < \sigma_d + \sigma_f$, since $r<1$.

currency factor is added to their returns. Thus, caution is needed to interpret the results of such an analysis since often, in empirical studies, the various market returns are translated into a common currency. Many studies, when examining the cross correlation among different countries' financial data, do not dissociate these two risks.

To investigate the robustness of our results, we divide the data-set in two equal sub-samples and repeat the analysis[7]. This will provide some insight as to whether our conclusions regarding the sources that generate volatility in the global markets are sample biased.

Since in the VaR analysis, the potential overall losses of the portfolio must be expressed in common currency, investors, when studying volatility spillovers, distinguish currency from local risk. Therefore, we repeat the vector autoregressive analysis with the conditional betas estimated when all domestic portfolio returns are translated into US dollars.

5.3.1 The Variables in the System

Both the *impulse response* analysis and *forecast error variance decomposition* will require that the residuals, e, are separated into orthogonal innovations by calculating the Choleski decomposition described in section 5.2.2. However, this statistical decomposition depends on the sequence in which variables are ordered in the vector Y. Therefore, the orthogonalisation of Ω requires a "Wold causal chain" among the current elements of all variables in the system[8]. To limit the choice of arbitrary

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7 See p 198 for more details.

8 An alternative method to decompose the estimated vector autoregressive innovations into orthogonal shocks is the structural vector autoregressive approach, i.e. Bernanke (1986) and Blanchard and Quah (1989). These approaches are based on identifying long run restrictions derived from a structural economic

settings, it is necessary to establish the direction of causation among the variables in the vector autoregression. It is therefore necessary to determine a pyramid of causality order on the variables in the system so that shocks in any orthogonalised innovation will only affect variables below that ranking.

To determine the direction of causation, the Granger causality test has been applied on the conditional beta series for all possible pair-wise combinations. We found that the results are sensitive to the number of days in the lead-lag relationship. Volatility in country A may cause volatility in country B when a small number of lags is used but not when a larger lag is used. This will cause problems in interpreting the causality relationships. Since, in our study, the causality relationships are used to find the rank order in the vector autoregressive system, we set the causality lags equal to the lags in the vector autoregressive system. Hence, we selected the lag order using the two criteria described in section 5.3.2. We found that one day lag minimises both criteria.

The causality test helps to detect the presence of volatility spillovers among the thirteen local markets contained in our sample. To determine the order of causality, it is necessary to adjust for different time zones. Difference in the closure of the markets will play a role when we study volatility transmission from west to east[9]. While the US market is open, both the European and Asian markets will be closed[10]. Consequently, this leads to testing the following *lead-lag* relationships:

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model. However because financial variables are lacking such restrictions, this approach cannot be implemented here.

9 Throughout this analysis we will assume that all markets on the same continent close at the same time.

10 For a detailed discussion on the opening times of major equity markets see Malliaris and Urrutia (1992).

H1: If the US leads the European and Asian markets, a shock from the US markets at time t will not be reflected in the returns on these two until the next trading day[11].

H2: If the European markets lead the US and Asian markets, a shock from the European markets at time t will be reflected in the returns of the US at the same period, but on the Asian markets at period t+1.

H3: If the Asian markets lead the US and European markets, then a shock in the first market at time t will have an impact on returns in both the European and US markets on the same day.

Given the importance of adjusting causality for time zone differences, Malliaris and Urrula (1992) propose a Granger regression model that captures the lead-lag relationship between different markets. Consequently, if it is hypothesised that if the US leads European markets (as most studies suggests) then the Granger regression model would be as follows:

$$\beta^{EU}_{t+1} = \pi_0^{EU} + \pi_1^{EU}\beta^{US}_{t-1} + \pi_2^{EU}\beta^{EU}_{t} + \xi^{EU}_{t} \qquad (5.8.a)$$

where β^{EU}_t, β^{US}_t are the conditional betas of the European and US markets observed at period t. Similarly, to test the hypothesis that a European market leads the US, the Granger regression will be

11 Since our data consists of end of day prices, we measure volatility as at the end of daily trading. Hence we will assume that the impact of any news in a specific country will be reflected in the next closing prices of that market.

$$B^{US}_t = \pi_0^{US} + \pi_1^{US} B^{US}_{t-1} + \pi_2^{US} B^{EU}_{t-1} + \xi^{US}_t \quad (5.8.b)$$

To investigate the causality relationship between the European or American markets on the one side and Asian markets on the other, we followed a similar set up in the Granger equations. The terms -EU- have been replaced with the term -AS- (Asia) and the terms -US- with the term -EU- when the Asia/Europe causality hypothesis was tested for, or left unchanged if the Asia/US was studied.

Table 5.1 reports the F-statistics only for those significant, at 0.10 probability, pair-wise causality relationships. We find that the US and Japan, and to a lesser extent the UK market, plays a leading role in all three continents. Volatility originated in US is transmitted to Asia (Japan and Hong Kong) and Europe (Sweden, Spain and UK). Similarly, volatility originating in Japan is transmitted in the next trading day to both the European (Netherlands and Spain) and US markets. We found bi-directional volatility spillovers between the UK and US markets but not between the UK and Japan[12]. We also found that Sweden, Switzerland and Spain play a dominant role, but their influence is regional, (central Europe and Singapore and Hong Kong). News in these countries, however, is not transmitted to the major markets[13].

12 However, tests of higher lag order revealed that there is a causality relationship between these two markets.

13 As we have said above, these results are to determine the rank order of the vector autoregressive system and not to conduct a rigorous analysis of the causality relationships among the different markets.

Table 5.1 Granger causality tests on conditional betas

From	To	F-test	From	To	F-test
US	Hong Kong	4.82	SPAIN	Denmark	8.81
	Japan	9.80*		France	18.78
	Sweden	14.04		Germany	8.21
	Spain	10.37		Hong Kong	8.96
	UK	3.78*		Italy	13.86
UK	Germany	5.94		Netherland	3.41
	Singapore	2.94*	SWEDEN	France	26.04
	US	5.94*		Germany	5.78
JAPAN	Netherl.	4.35*		Hong Kong	11.95
	Sweden	7.97		Italy	4.10
	Spain	2.93		Netherl.	3.28
	US	11.12*		Singapore	8.74
FRANCE	Japan	2.74		Switzerl.	5.22
	Singapore	3.96	SWITZERL	Denmark	3.52
	US	2.93		France	14.22
GERMANY	France	4.53		Germany	4.94
	Honk Kong	2.79		Netherl.	3.89
	Singapore	3.29		Singapore	3.72
HONG KONG	Denmark	4.85		Spain	7.10
	Switzerl.	3.68	SINGAPORE	Japan	3.39
	Singapore	4.45		UK	3.41
NETHERL	France	3.74			
	Japan	5.32*			
	Singapore	5.44			

The symbol * indicate bi-directional causality.

Although there are many strong unidirectional causality relationships, there are also a number of bi-directional or feedback causalities which make the interpretation of the causality matrix rather complex. For example there is a feedback effect between the US and Japan, the US and the UK and the Netherlands and Japan. While France transmits volatility to the US, other central European stock markets (Germany, Netherlands, Sweden, Switzerland and Spain) also have an impact on the volatility of French stocks. Therefore, this makes it difficult to determine which country will be in the top of the causality pyramid.

Therefore, a ranking of all the thirteen national markets on a solid pyramid pattern based upon their volatility causalities and spillovers cannot be formed. Given the importance that causality plays within the vector autoregressive modelling, we decided to restrict the empirical analysis to six markets only. These are: France, Hong Kong, Japan, Switzerland, UK and US[14].

UK, US and Japanese equities are included because of their large share in the world market capitalisation. Also, the 1987 crash originated from the US which affected stock markets world-wide. The capitalisation of the Japanese stock market is comparable to that of US market. In addition Japan forms the centre of the pacific region economy. The UK stock market is the third largest in terms of capitalisation and holds a leading position in supporting bilateral investments between Europe and US. In addition, because of the liberal legislation and the time zone differences, London has developed into a major arbitrage trading centre. The French stock market is chosen because the causality test reveals that shocks originating there are transmitted to US and Japanese markets. In addition, because of the way transactions are settled (once a month) the Paris bourse attracts many arbitrageurs from around the world, which makes the French stock market an integral part of the World market. Hong Kong is included in the vector autoregressive system because it has influence on two European and one Asian market. Also, as we have found, Hong Kong is linked with a number of European markets

14 The Granger causality test shows that perhaps two or three other markets had the same importance as the Swiss and Hong Kong markets, and therefore also should be included in the system. Because the computational complexity grows at an exponential rate as more variables are added in the system, we had to restrict to six the number of markets included in the vector autoregressive model. However, when a few markets are replaced, the general verdict of this analysis remains unaltered.

(Germany, Sweden, Spain) since the causality test reveals that changes in the volatility of these markets is transmitted, the following trading day, to Hong Kong. The choice for the last market in the system is among the remaining central European markets, Germany, Netherlands, Spain, Sweden and Switzerland. They are all medium sized markets and play a role in Europe and the Far East and possibly represent one common industrial central European factor. We preferred to include in our vector autoregressive system Switzerland because of the its foreign exchange stability and the accessibility that it offers to foreign investors.

5.3.2 Selecting the Lag Order

To set the correct lag order s in the vector autoregressive system, the Akaike information criterion (AIC) and Schwarz Bayesian criterion (SBC) were used. The two criteria were calculated as follows:

$$AIC = T\,ln(\text{residual sum of squares}) + 2n$$

$$SBC = T\,ln(\text{residual sum of squares}) + n\,ln(T)$$

where T is the number of observations used and n is the number of parameters estimated in each vector autoregressive equation. This is equal to the number of lags s, times the number of variables in the system, plus the constant term. The above test is computed for different values of s and the correct lag order is the one with the lowest statistic. Obviously, to compare adequately the various lag options, T should be kept fixed. Both tests have been applied to the six variable vector autoregressive system for lags one to eight and the corresponding statistics appear in table 5.2. As can be seen from the table, both tests indicate a lag of order one.

Table 5.2 Testing for lag order

lag :	1	2	3	4	5	6
AIC	-32.52	-32.15	-32.14	-32.13	-32.11	-32.10
SBC	-32.05	-31.98	-31.90	-31.81	-31.71	-31.62

5.4 Estimating the Vector Autoregressive System

On the basis of the above analysis, a six variable vector autoregressive system of lag order one has been formulated. The causal ordering of the variables in the system are as follows: US, Japan, UK, France, Switzerland and Hong Kong. The US has been positioned at the top of the system hierarchy because of its share in market capitalisation and also for the leading role that it plays during world-wide volatility fluctuations, *e.g.* the crash of 1987, and the mini crash at the end of 1989. Japan has been placed second before the UK market because of its large market capitalisation and the feedback effect that it has with US stocks, since volatility generated in the former is transmitted to the latter but not vice-versa. The UK market has been placed third; it only influences three other markets, among them the US. The UK stock market is also third in the world market capitalisation ranking over the sample period.

The conditional betas for France, Switzerland and Hong-Kong form the remaining three variables in the system. The order for these last three places in the vector autoregressive system has been chosen according to their order in the world index capitalisation. The results however were not sensitive to changes of order in the last three variables in the system. The coefficient estimates from the vector autoregressive model are reported in table 5.3.

As expected in each country equation, the coefficient of its own lagged beta is large and very significant. In addition to that, in several equations, there are large coefficients for other countries' lagged values. For example, there is a significant negative impact from lagged French beta to the US beta. Other statistically significant impacts arise from lagged UK to French beta and vice versa. The adjusted coefficient of determination is quite high for most of the regressions, thereby validating the methodology used to explain intermarket volatility spillovers.

Table 5.3 Vector autoregressive estimates for dynamic betas

	const	US	JP	UK	FR	SW	HK	R^2-adj	DW
US	0.114	0.93	-0.015	-0.011	-0.021	0.008	-0.003	0.894	1.911
	(7.62)	(106.77)	(3.31)	(0.82)	(1.87)	(0.82)	(0.56)		
Japan	0.06	-0.023	0.974	-0.018	0.022	-0.001	-0.004	0.969	2.103
	(4.02)	(2.69)	(213.28)	(1.44)	(1.97)	(0.09)	(0.79)		
UK	0.054	-0.006	-0.006	0.939	0.009	0.007	-0.004	0.901	2.015
	(5.13)	(0.98)	(1.80)	(105.56)	(1.13)	(1.01)	(1.05)		
France	0.049	-0.02	-0.002	0.031	0.916	0.03	0.02	0.901	2.06
	(3.72)	(2.70)	(0.53)	(2.74)	(94.27)	(3.44)	(0.45)		
Switzt	0.029	-0.005	0.001	0.006	-0.005	0.942	0.009	0.895	2.056
	(2.34)	(0.67)	(0.28)	(0.59)	(0.53)	(114.54)	(1.98)		
Hong Kong	0.049	-0.017	-0.009	-0.025	0.019	0.008	0.966	0.941	2.134
	(3.35)	(1.95)	(1.91)	(1.94)	(1.68)	(0.83)	(183.98)		

-t statistics in parenthesis

Unfortunately, other studies cited earlier, which followed a similar approach to explain market inter-linkages, did not report the results of their vector autoregressive analysis. This makes it impossible to compare our results. However, in one study, Bos *et al.* (1995), reported more detailed results. The highest adjusted coefficient of determination Bos *et* *al.* estimated was 0.18 while the lowest only 0.01. However, they used monthly returns for three countries, Finland, Sweden and the US, to study the international co-movement of Finish stocks. If we compare their study with the above results, we notice that markets are linked more strongly through their second moments than their first.

5.4.1 Impulse Response Analysis

After estimating the vector autoregressive system, impulse response analysis and variance decomposition can be computed. The variance-covariance matrix of the vector autoregressive residuals has been orthogonalised as in section 5.2.2. The orthogonal matrix V in (5.4) was used to simulate the dynamic responses to an artificial shock of one unit in each of the six markets. The results appear in table 5.4.

Table 5.4 Impulse response to a unit shock in a national market beta.

shock in	week		impulse	response	in		
		US	Japan	UK	France	Swiss	Hong Kong
US	5	0.0619	-0.0307	0.0213	0.0153	0.0048	0.0143
	10	0.0434	-0.0338	0.0153	0.0082	0.0029	0.0079
	20	0.0233	-0.0347	0.084	0.0012	0.0007	0.0019
FRANCE	5	-0.0040	0.0704	-0.0031	-0.0019	-0.0008	0.0045
	10	-0.0069	0.0621	-0.0040	-0.0019	-0.0001	0.0016
	20	-0.0090	0.0496	-0.0043	-0.0013	0.0006	-0.0013
UK	5	-0.0035	-0.0010	0.0411	0.0271	0.0154	0.0107
	10	-0.0061	-0.0014	0.0315	0.0245	0.0123	0.0077
	20	-0.0076	0.0000	0.0189	0.0185	0.0078	0.0052
JAPAN	5	-0.0038	0.0036	0.0022	0.0471	0.0182	0.0121
	10	-0.0065	0.0076	0.0034	0.0332	0.0133	0.0145
	20	-0.0080	0.0112	0.0035	0.0178	0.0079	0.0148
SWISS	5	0.0013	-0.0003	0.0014	0.0061	0.0495	0.0099
	10	0.0016	-0.0002	0.0025	0.0096	0.0369	0.0104
	20	0.0003	0.0005	0.0031	0.0103	0.0206	0.0103
HONG KONG	5	-0.0007	-0.0011	-0.0010	0.0006	0.0024	0.0674
	10	-0.0011	-0.0020	-0.0016	0.0013	0.0042	0.0570
	20	-0.0012	-0.0025	-0.0018	0.0021	0.0055	0.0414

As can be seen from the findings in table 5.4, an innovation in the systematic risk in each national market is rapidly transmitted to all the other markets in the system. Moreover, the velocity with which the innovations are transmitted and also their duration and the speed of decay vary across markets. Most importantly, dynamic responses may be inversely related, *i.e.* an increase in volatility in one market might lead to a reduction in volatility in another.

5.4.2 Simulated Confidence Intervals

It is premature to interpret the results in table 5.4 until confidence bands around these responses are computed. A confidence band for a statistical estimator quantifies its uncertainty and allows for correct interpretation and use of measurement information. Large confidence bands discourage decision making based on inadequate measurement and call into question their credibility. Runkle (1987) says "..supplying impulse responses or variance decomposition without confidence intervals is tantamount to using regression coefficients without *t* statistics", p 438.

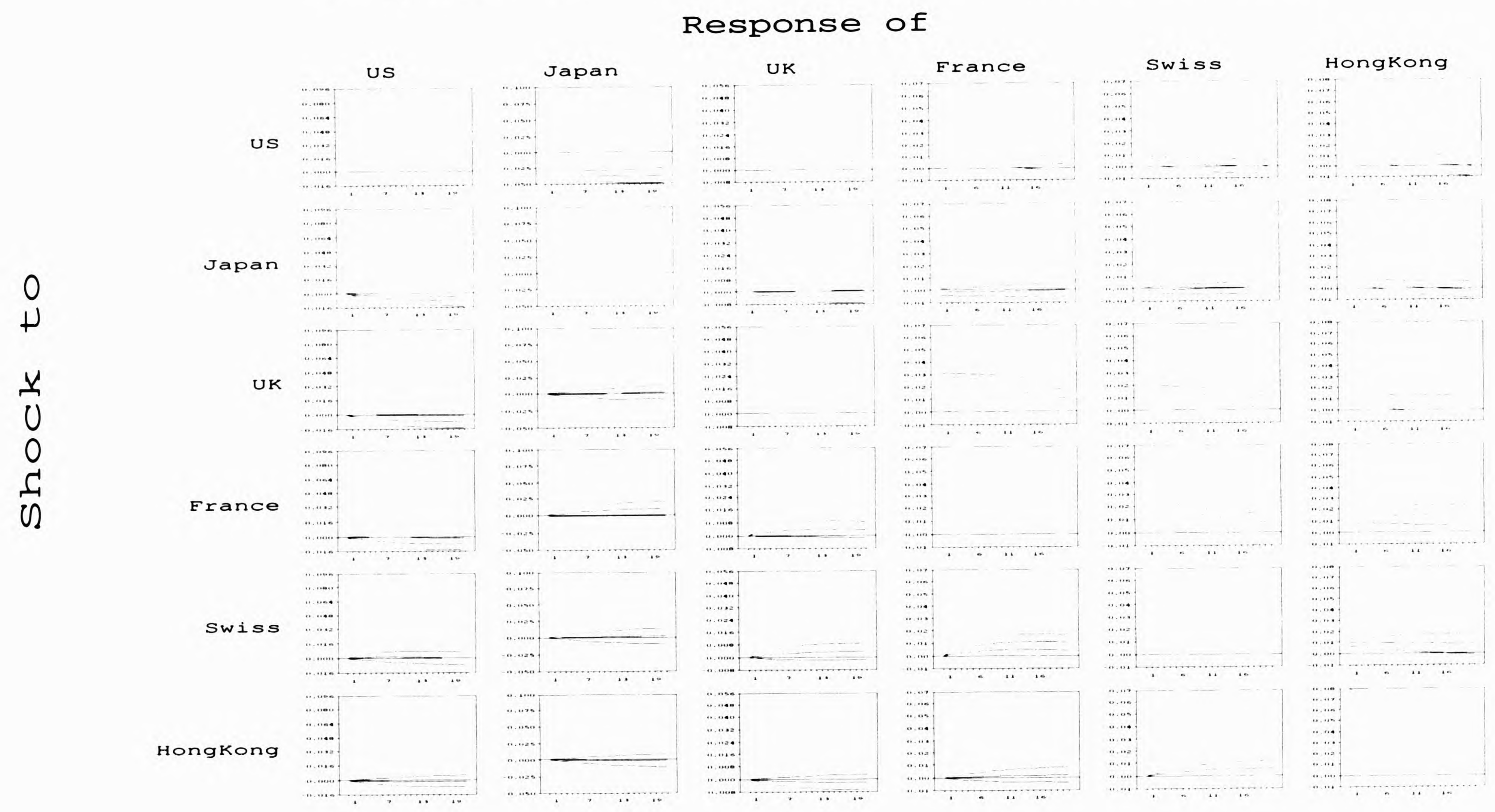
Hence, confidence bands for the dynamic responses and innovation accounting will provide a statistical foundation for any verdict we reach regarding the timing and magnitude of volatility transmitted across markets. Standard errors for the impulse responses, and innovation accounting, can be calculated using Monte Carlo simulation. This is based on the method of Kloek and Van Dijk (1978) as implemented in RATS, see Doan (1992).

In figure 5.1, the time paths of each markets' dynamic responses to each market's innovations are shown. There are thirty six smaller charts, arranged in six columns by six rows. The label on the vertical axis of the first column of the charts indicates the local market where an innovation has occurred. The plots in the same row show the dynamic response of each local market to this innovation. The order in which these responses are displayed appears above in the first row of plots.

Hence, each graph displays the dynamic response of one local portfolio to a shock in one variable in the system, which could also be a shock to itself. The dynamic response is presented with a middle line, while the other two lines form the upper and lower confidence band, of two standard errors around that response at each time.

since we are interested in examining impulse values that are different from zero with probability of at least 95%, for the rest of this analysis, the term impulse function or dynamic response we refer to the confidence band that is closer to the horizontal axis. Hence, if at any period the horizontal axis is positioned inside the upper and lower confidence bands then the impulse response is zero. That is, the impulse values are not different from zero with a probability of 0.95. On the other hand if the horizontal axis is below the lower band then value of the impulse response is equal to the lower band. Therefore, an innovation in one national market will lead to a significant increase in the beta of the other market. Conversely, if the horizontal axis is above the upper band then the impulse response will be equal to the upper band. Consequently, any innovation from one market will have a negative impact on the beta of the other market. Under these scenarios, the impulse response values will be different from zero.

Figure 5.1 Impulse Responses



The first row of charts displays how an innovation in the US beta is transmitted to the each national market. It is apparent that "news"[15] in US equities affects the volatility of all other national markets included in the system. The extent to which US "news" influences foreign stocks is not uniform. For all markets with the exception of Japan, there is a statistically significant increase in their beta. On the other hand, for the Japanese market, the beta responds negatively to the US innovation. Although this might seem to contradict our ex-ante beliefs, there is nothing in our model preventing one country's sensitivity against the world from increasing when the sensitivity of another country does so[16].

The dynamic responses also differ in duration and timing. There is an instantaneous volatility transmission from the US to all other countries, but the number of days that these countries need to return to normal differs. The longest shock are observed in Japan and UK. It takes at least a month for the Japanese and UK betas to return to their pre-shock levels. On the other hand, the Swiss equity market seems to discount US market related news in only three days. The time it takes for local markets to discount common volatility can be seen as a test of efficiency. The more rapid the response to systematic news and the quicker the beta returns to normal levels, the more efficient the comparative local market is.

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15 This term refers to the news that affect jointly that particular market and its covariance with the world. This type of news is the innovation in the local market's beta.

16 In the analysis that follows, we may not always distinguish between positive and negative responses and the reader is advised to refer to the corresponding chart in figure 5.1 to identify the sign of the response.

US stocks are affected only by news in Japan. Although the impact is small, it lasts for an entire one month period in our simulation. News in Japan also has a significant impact on Hong Kong equities.

Shocks in the UK equity market are transmitted to all markets ranked behind it in the vector autoregressive system. In all three, France, Switzerland and Hong Kong, there is an immediate positive response to an increase in the UK stock's beta. Hong Kong's volatility is re-established at pre-shock levels in only few days and the Swiss one in a month. For French stocks, it takes a little longer to return to normal levels.

Shocks in the French beta are quickly transmitted to Swiss and Hong Kong equities. However, the impact in the Hong Kong market decays quickly and vanishes after three weeks. No other market is significantly affected by French innovations. Innovations in the Hong Kong beta are not transmitted outside this market.

5.4.3 Forecast Error Variance Decomposition

As has been explained earlier, the forecast error variance in the systematic volatility for each national market can be partitioned according to its sources. These sources are past innovations in its own conditional beta or in that of any other national market beta included in the system. The method used to identify these sources of error variance is called *variance decomposition* or *innovation accounting* and is described in section 5.2.3.

To avoid any innovation accounting being contaminated by disturbances occurring in more than one market, the vector autoregressive residuals, e, have been transformed into orthogonal innovations, **u**, using the Choleski factorisation, explained in section 5.2.2. The variance decomposition in table 5.5 shows the average amount of the variance in each variable, after five, ten and twenty days, attributable to each shock.

The leading role that US equities play in transmitting volatility across national markets is now more evident. US innovations are a cause of error variance in almost all other national markets examined here. The US stock price movements have more influence in Japan and the UK where they account up to 21% of the error variance in the first and up to 22% in the latter market's beta. US news are less responsible for changes in the French and Hong Kong stocks and are not responsible for changes in the betas of Swiss stocks. On the other hand, innovations in "foreign" market betas also have almost no repercussions in US equities. In a five day horizon, innovations which occurred in all foreign markets together account for less than 0.4% in the error variance of the US beta. However, this number rises to 5% after 20 days of trading. This variation is explained by innovations in the three largest markets, UK, Japan, and France.

According to the results, the UK seems to be another influential market. UK innovations can explain up to 26.2%, 8.16% and 2.6% of the changes in the betas of the French, Swiss and Hong Kong markets respectively. The influence of French news is responsible only for changes in the betas of nearby Swiss stocks and to a lesser extent Hong Kong. French news is also the most influential foreign news in the US market after Japan. The most important foreign news that influences the volatility of French stocks is the UK, followed by US and Swiss. Paradoxically, news in Japan has no impact on the French stocks.

Table 5.5 Accounting innovations in the national markets' beta.

market explained	weeks	by US	innovation Japan	in UK	France	Swiss	Hong Kong
US	5	99.652	0.122	0.094	0.112	0.015	0.004
	10	98.543	0.530	0.405	0.467	0.040	0.016
	20	95.341	1.774	1.315	1.478	0.052	0.041
JAPAN	5	13.094	86.828	0.006	0.064	0.000	0.008
	10	16.244	83.345	0.018	0.364	0.001	0.028
	20	21.022	77.704	0.017	1.185	0.001	0.070
UK	5	21.933	0.235	77.716	0.074	0.028	0.015
	10	21.057	0.469	78.035	0.269	0.119	0.056
	20	19.899	0.976	77.945	0.643	0.386	0.151
FRANCE	5	9.066	0.071	18.426	72.072	0.360	0.004
	10	7.662	0.103	21.847	69.038	1.330	0.019
	20	6.112	0.134	26.237	64.019	3.409	0.088
SWISS	5	0.883	0.041	7.296	11.336	80.387	0.057
	10	0.785	0.029	7.687	11.033	80.222	0.244
	20	0.662	0.025	8.167	10.865	79.455	0.825
HONG KONG	5	5.364	0.650	2.616	1.911	1.574	87.885
	10	4.201	0.455	2.295	2.975	1.954	88.120
	20	2.972	0.305	1.982	4.811	2.673	87.258

The above results agree with the findings of Eun and Shim (1989) who report that the US stock market is the most influential in the word. Eun and Shim also found that innovations in foreign markets exert an influence on US stocks, but mostly in the longer run. However, although our study shares a common objective with that of Eun and Shim (to investigate the international market transmission mechanism among stock market movements) a detailed comparison between them is not possible because of the prevailing differences in the frequency and the nature of the variables used in the vector autoregressive analysis.

5.4.4 Confidence Intervals for Forecast Variance Error Decomposition

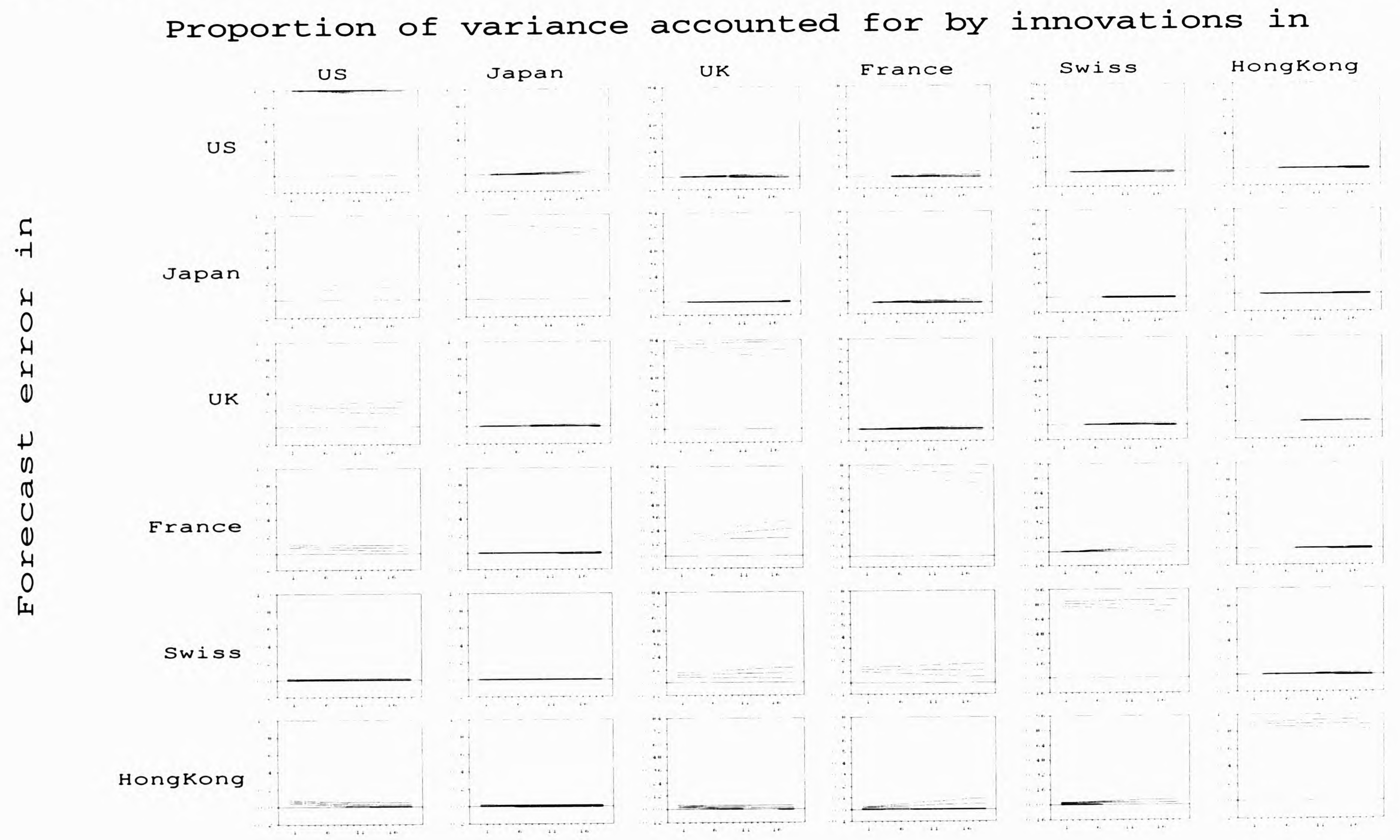
Like the impulse response analysis, any verdict regarding the sources of variation in the national portfolio's unexpected beta changes needs to be supported by confidence levels. Monte Carlo simulation, similar to that used with the impulse response analysis, has been employed to generate randomly Nx1 vector of residuals. In that way we estimated the VDF as in (5.7). The process is repeated for 1000 simulations to get the upper and lower band which contains the 95% of the paths for each markets innovation accounting.

The time paths of the simulated accounting national market innovations together with the upper and lower confidence bands are shown in Figure 5.2. Each single chart displays the time path of innovation accounting of one market to one shock in a variable in the system. Hence there are 36 single charts arranged in six rows by six columns. The way the graphs are organised is identical with that of figure 5.1 and the methodology is explained fully in section 5.4.2

In each of the 36 graphs, the line in the centre shows the average value of the simulated time path for the innovation accounting corresponding to a particular market. The other two lines, above and below, bound the 95% of the simulation outcomes[17].

[17] Although innovation accounting can never have negative values, in order to facilitate the observation we display the two confidence bands as symmetrical, plus or minus two standard deviations from the means. Consequently, some of them are shown as having negative values.

Figure 5.2 Forecast Error Variance Decomposition



Like the impulse response analysis, simulated confidence intervals tend to be wide and undermine many of the central tendency values estimated. Hence, for most innovation accounting estimates, in particular the lower values associated with innovations in a foreign market, the simulation has generated standard errors that make them statistically not different from zero. As a result, out of the thirty innovation accounting of foreign market news, only half a dozen are significantly greater than zero. By contrast, the confidence bands for innovation accounting of domestic news are well above zero for the entire four weeks horizon in all six cases.

The simulated results once more shows that US is by far the most influential market in the system. We can now say with certainty that US news accounts for a significant variation in the unexpected changes in Japanese volatility; on average 15% during the first few days after a US news announcement, and up to 40% in the following four weeks. US equities are also responsible for a very large part in the UK beta error variation where they account for between 20% and 40%. US news can also explain a small but significant part of the unexpected changes in the French and Hong Kong stock markets' volatility. However, the US news does not exert any significant impact on the volatility of the Swiss stocks. On the other hand, we cannot reject the hypothesis, at the 95% confidence level, that US news fully accounts for its own volatility during the first few weeks. That is because the upper band of simulated US accounting innovations is one. Four weeks after a US news announcement, the upper band's values diminish and leave more space for foreign markets to explain US error variation.

We also identify a second factor, UK news, but its influence is restricted to Europe. UK news explains between 12% and 35% of the variation in the French stock's beta. Furthermore, UK news is also responsible for a small but significant share of the total systematic volatility of Swiss stocks.

Apart from a strong US factor and a regional UK factor, we can scarcely identify any other common source of news which acts as a driving force in cross-markets price movements. In fact, in only one of the remaining markets (France) the lower confidence bands, associated with a foreign market's innovation accounting (Switzerland), can be distinguished from zero. On the other hand, at 95% confidence level we cannot reject the hypothesis that the US volatility is exogenous to UK or French news.

Finally, for the remaining three markets, Japan, Switzerland and Hong Kong, the results suggest that they act like followers rather than active players in the international volatility transmission mechanism. There is no additional evidence that internationally relevant news in any of them will change the beta value of another market. On the other hand, foreign markets have a small but significant feed into the error variance in each of those six markets. It is impossible to identify with precision each individual external source that accounts for innovations in each market systematic volatility changes. Most of the findings agree with those of Eun and Shim but disagree in several other aspects.

5.5 Investigating the Robustness of the Results

To investigate the robustness of our results we divided the conditional beta series into two equal samples and repeated the vector autoregression analysis. The first sub-sample covers the period from the beginning of 1986 until the end of 1990. This period is characterised by higher than usual volatility which consists of the sharp rise in world equity prices along with the dramatic events of October 1987. Further, it also contains the mini crash of January 1990 and the Gulf invasion of the summer 1990. On the other hand, the second period is characterised by stable growth in the world equity prices. Indeed, the annual historical volatility of the world index during the first period is almost twice as much of the second period volatility (13.5% and 8.75% respectively).

The Akaike information and Schwarz Bayesian criteria indicate that, for both data samples, a lag order of one was appropriate. Tables 5.6.a and 5.6.b. report the F statistics from the Granger causality test of (5.8) used to identify the order of the variables in each of the two vector autoregressive systems.

There are a number of similarities as well as differences among the two tables. During the 1986-90 period Japan was found to be very influential in the movement of global equity prices. Volatility originating in Japan is transmitted to four other markets, among them the US and UK. The European markets, with the exception of the UK, are also very influential but their impact is confined to within Europe and the smaller Asian markets. It is a surprise to us that shocks from the UK are not found to influence any other countries with the exception of Sweden. On the other hand no market other than Japan influences UK stocks.

During the second period, 1991-95, we notice a diminishing role for Japan and increasingly dominating role for the US and the UK. In fact the results suggest that Japan has become very passive during that period when volatility originating from the UK and the other smaller European markets are quickly transmitted to Japanese stocks.

Table 5.6.a Granger causality tests on conditional betas (1986-1990)

From	To	F-test	From	To	F-test
US	Japan	11.47*	SPAIN	France	5.54
	Sweden	4.84		Germany	7.00
UK	Sweden	3.54		Hong Kong	6.35
JAPAN	Netherl.	5.68		Switzerl.	6.15*
	Singapore	5.63*	SWEDEN	France	8.54
	UK	6.35		Germany	8.53
	US	3.66*		Hong Kong	8.10
FRANCE	Italy	2.99		Singapore	6.87*
	Singapore	4.66		Switzerl.	3.53
	US	5.92	SWITZERL	France	11.90
GERMANY	France	4.79		Germany	6.52
	Singapore	3.29		Netherl.	3.58
DENMARK	Singapore	3.35		Singapore	4.85
HONG KONG	Germany	4.14		Spain	3.22*
	Singapore	3.31	SINGAPORE	Japan	8.03
ITALY	Singapore	3.10		Netherl.	5.32*
NETHERL	France	6.88		Sweden	3.13*
	Singapore	7.35*			

The symbol (*) indicates bi-directional causality.

Table 5.6.b Granger causality tests on conditional betas (1991-1995)

From	To	F-test	From	To	F-test
US	Germany	4.01	SPAIN	Denmark	3.25
	Sweden	4.60		France	4.96
	Spain	4.30		Germany	10.35
UK	Japan	3.01*		Japan	7.13
	Spain	4.69*		Italy	8.18
JAPAN	France	5.08		Netherl.	9.12
	UK	4.42*		UK	3.14*
GERMANY	France	4.23	SWEDEN	France	12.83
	Japan	7.50		Germany	7.44
	Italy	3.68		Hong Kong	4.58
ITALY	Denmark	7.27		Italy	4.08
	France	4.00		Netherl.	5.39
	Japan	4.30	SWITZERL	Spain	3.14
NETHERL	France	8.95	DENMARK	Japan	4.53
	Japan	10.74		UK	4.18
	UK	4.27			

The symbol (*) indicates bi-directional causality.

When comparing the causality relationships reported in tables 5.6.a and 5.6.b with those in table 5.1, we notice a number of differences. The most remarkable is the UK-Japan relationship. During the two sub-periods, changes in volatility originating in Japan "cause" changes in the UK market. For the overall ten year sample period, however, the F statistic rejects the hypothesis that Japan plays a role in the volatility of the UK market. A closer examination of the causality test reveals that, although during the two sub periods the coefficient for Japan is significant, for the overall period it becomes insignificant. This could be attributable to structural changes in the dynamics that govern the conditional betas. One limitation of the Granger causality analysis is that it does not allow such shifts. This problem could partially be solved if we allow for more lags in each of the regression pairs used to test for causality. As the aim of this study is to investigate the volatility transmission mechanism globally, the Granger causality test is employed to identify the order of the variables in the system rather than single pair-wise relationships.

Therefore, we use the same lag order of one in all the causality regressions because this was found to be the most appropriate one for the vector autoregressive system.

In order to study the robustness of our impulse response analysis presented in sections 5.4.2, we repeat the Monte Carlo simulation for each of the two sub-periods. The results, shown in figures 5.3 and 5.4, indicate some changes in the mean values for the impulse responses and in some instances, *i.e.* Japan, the confidence bands are wider. Simulated confidence bands for the forecast error variance decomposition for the period 1985-90 and 1991-95 are shown in figures 5.5 and 5.6 respectively.

Figure 5.3 Impulse Responses (1985-90)

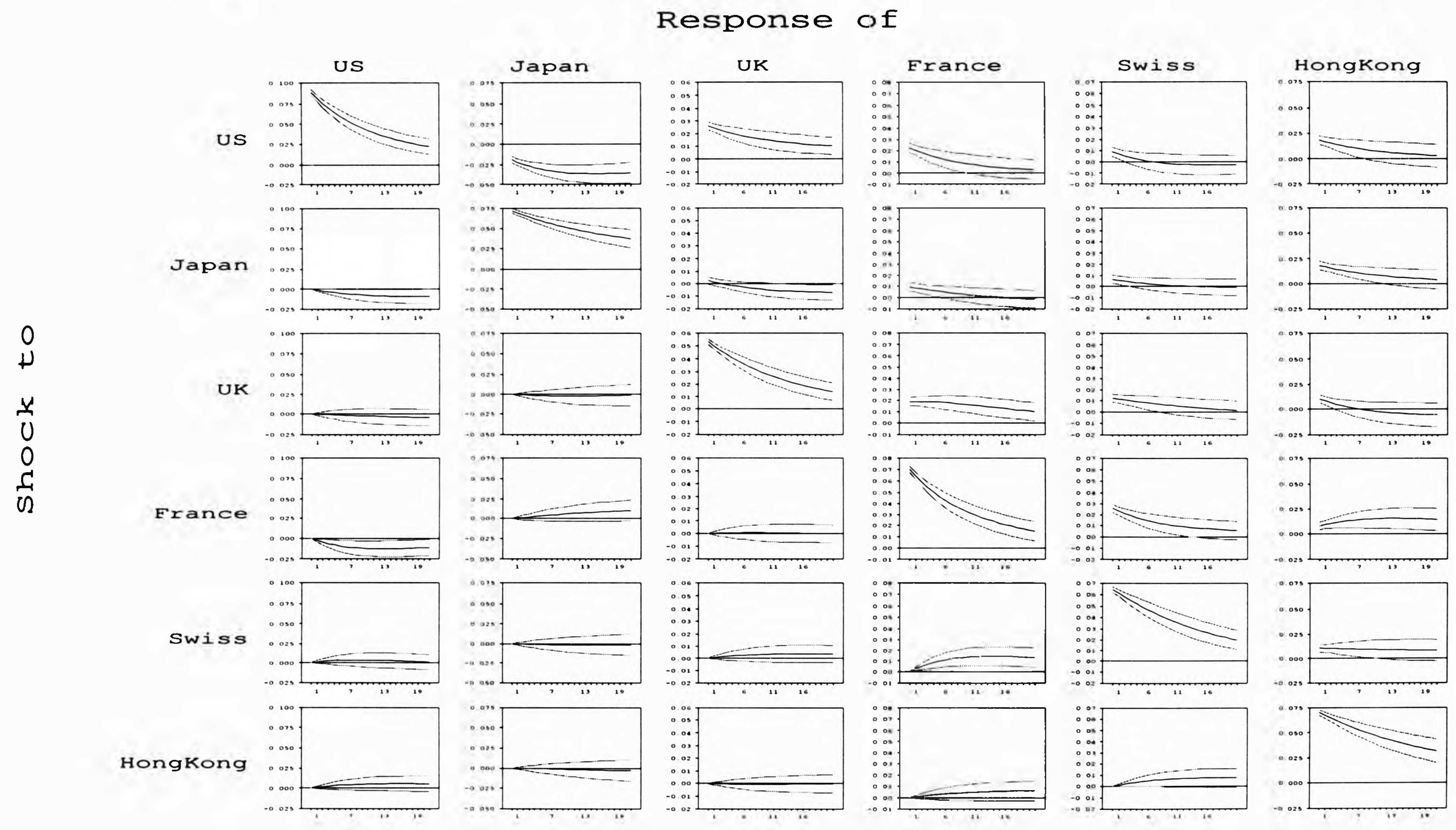


Figure 5.4 Impulse Responses (1991-95)

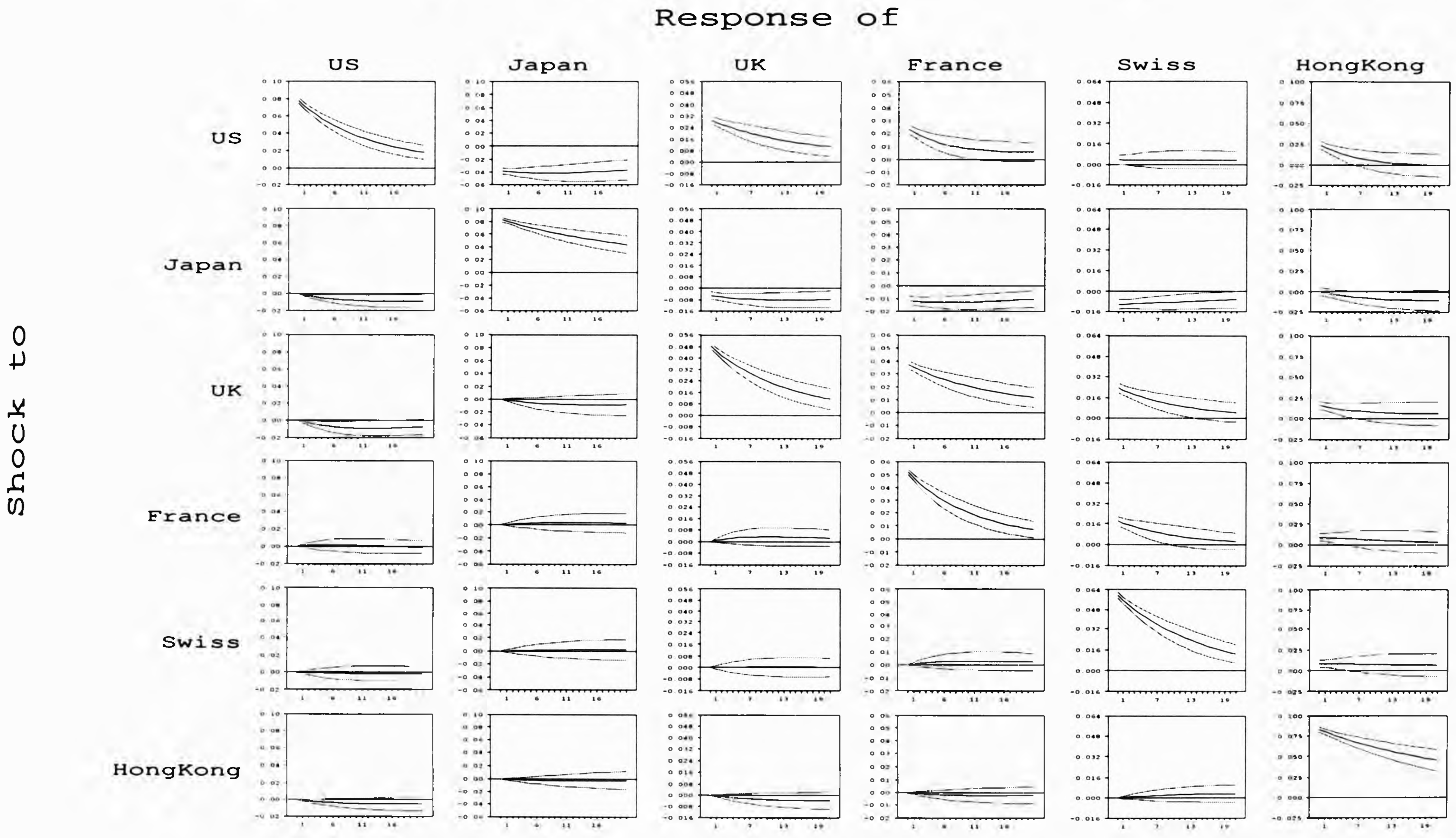


Figure 5.5 Error Variance Decomposition (1985-90)

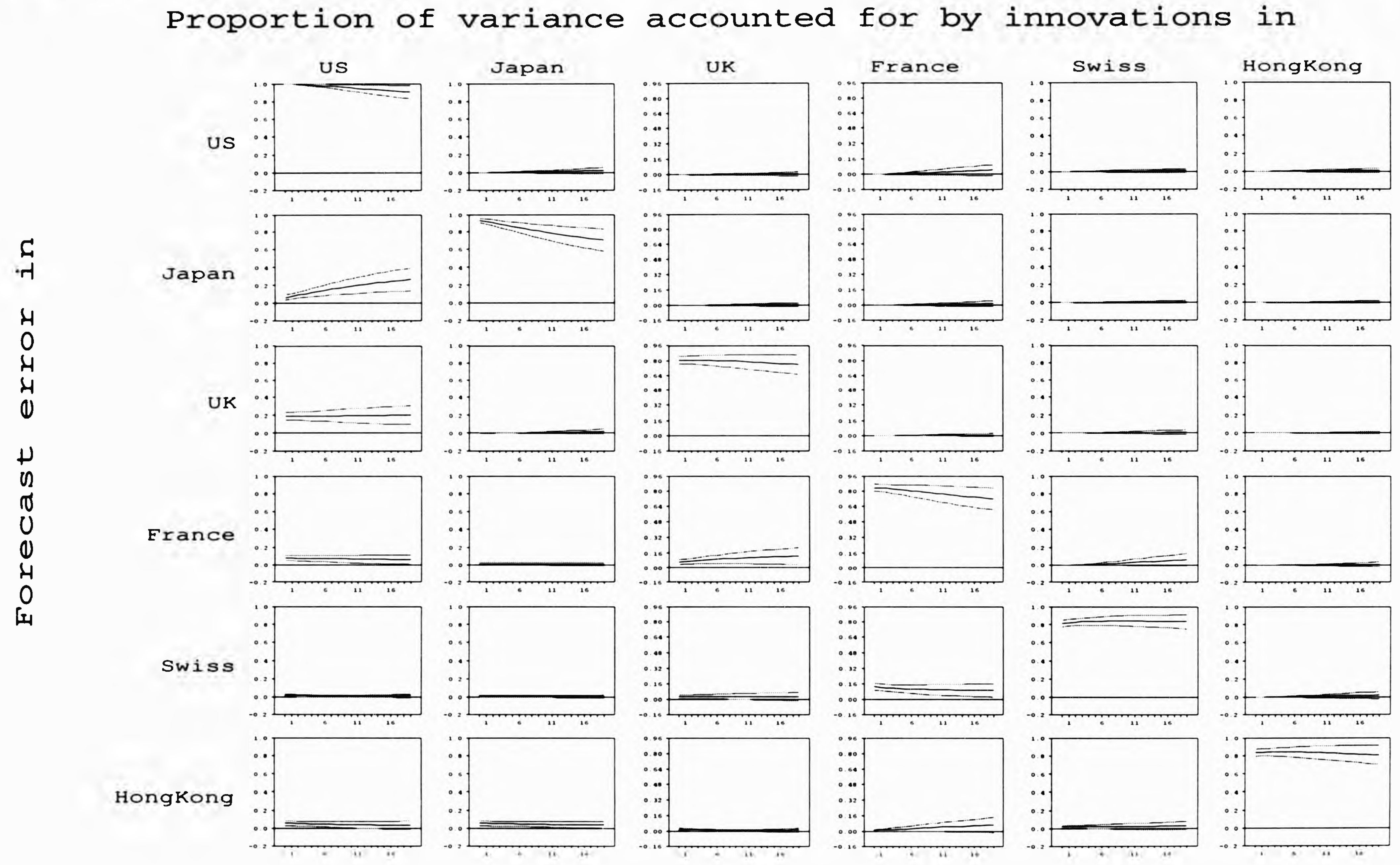
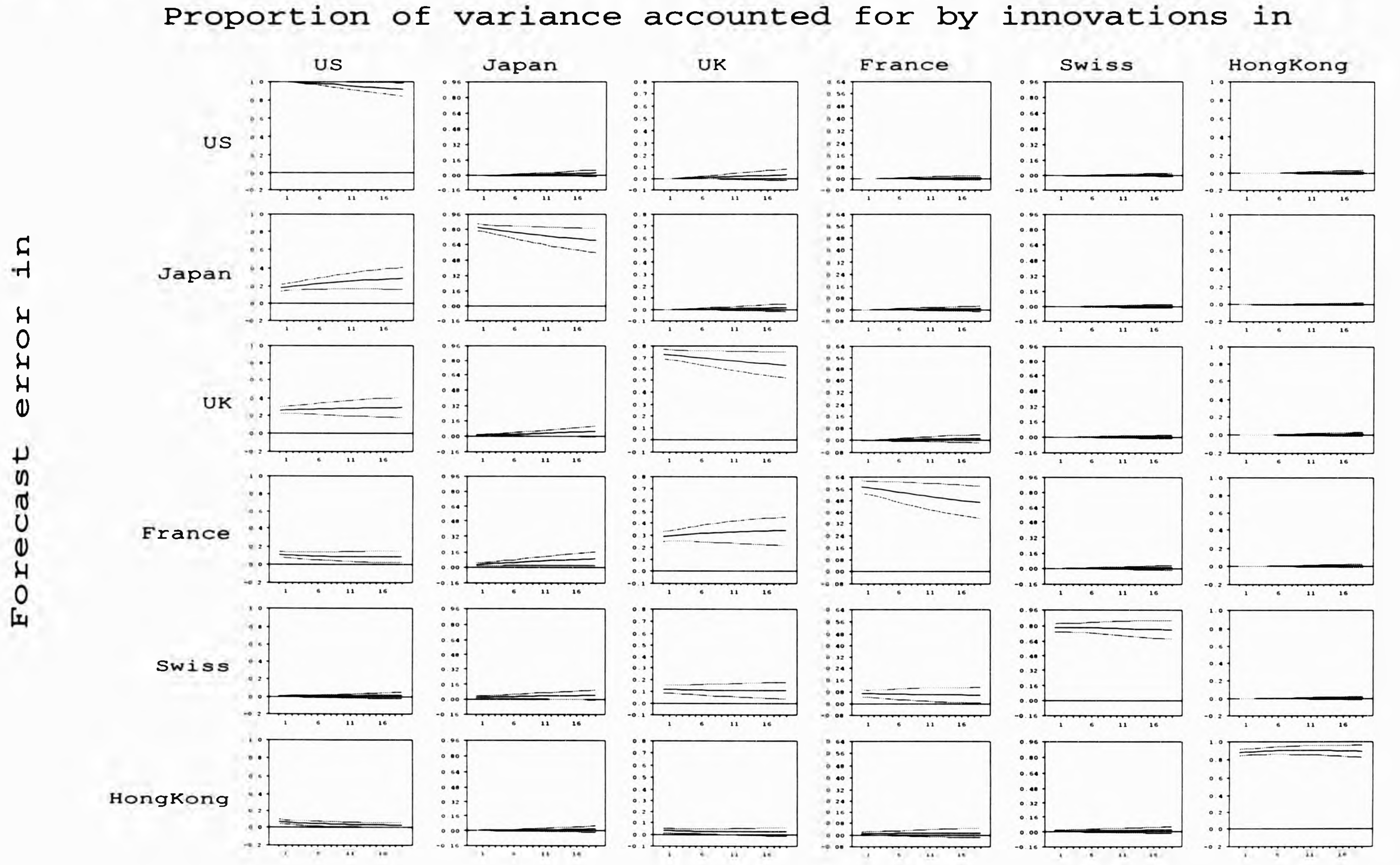


Figure 5.6 Error Variance Decomposition (1991-95)



We notice that US innovations play an increasingly important role in Japan during the second sub-period. We also notice during the same sub-period, the UK innovations account for about a quarter of the changes in the French betas and for one tenth in the changes of Swiss betas. The general conclusions, however, when compared with the overall set of figures 5.1-5.2 remain unchanged.

5.6 Volatility Transmission When Returns are in US Dollars

In the previous sections we applied the vector autoregressive analysis to the conditional betas estimated from domestic returns. Hence, the only risks to which investors are exposed are the losses that they may face in the value of foreign equity as measured in local currency. In VaR analysis, however, any potential losses must be estimated in a common currency, e.g. US dollars. Therefore, when currency exposure is not perfectly hedged, investors, in addition to losses in local currency, are exposed to foreign currency losses. When investors study the way volatility is transmitted from one market to other, they may want to know what is the aggregate (foreign stock and currency movement) effect on their portfolio. To account for the currency as well as the local risk in our volatility transmission mechanism, we employed conditional betas estimated from returns expressed in US dollars to conduct the vector autoregressive analysis.

The F statistics from the Granger causality (only those significant) are reported in table 5.7. The results indicate that there are now more unilateral and instantaneous causality relationships than when returns are expressed in the local currency. Further, the role of the dominant markets in the global volatility transmission has also changed. Now Japan plays the leading role in determining global equity prices. Indeed, any changes in Japanese equities affects half of the national markets included in our sample; however, it has no impact on either the US or the UK stocks. We also observe that the Singapore and most European markets are sources of volatility for large part of the global market. Given that each series is a combination of equity returns and dollar exchange rates, in each cross-pair for which we test for Granger causality, we have in effect four series. Hence, when we interpret the results, we should be cautious. It may be that the Granger causality is attributable to the cross exchange rate movements rather than the changes in the prices of two foreign stock markets.

Figure 5.7 shows the Monte-Carlo simulation for the impulse response function for the six markets when returns are expressed in US dollars[18].

When comparing the results with those of figure 5.1, we notice a number of changes in the way some markets are reacting to "news" in Japan and the UK. Three markets, the UK, France and Switzerland, are found now to be reacting to "news" in the Japan stocks-dollar/Yen series. For France, Switzerland and Hong Kong, the effect of news in Japan lasts for the entire month. When returns are in local terms, this has a minor impact, which last few days only on the US and Hong Kong markets. Furthermore, we notice some changes in the French forecast error variance decomposition. The results also indicate that although French prices are not responding to innovations in the UK stocks the French stocks-dollar/franc combination reacts positively to news in the UK stocks-dollar/sterling.

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[18] To be able to compare the results with those of section 5.4.2 and 5.4.4., we used the same variables and kept the same order in the vector autoregressive system.

Table 5.7. Granger causality tests on conditional betas (returns in US$)

From	To	F-test	From	To	F-test
US	Hong Kong	2.96	NETHERL	France	2.95*
	Singapore	3.45		Singapore	29.00*
JAPAN	Germany	4.22*	SWEDEN	France	8.38
	France	7.66		Germany	14.40*
	Sweden	10.95		Hong Kong	14.30*
	Switzerl	5.55		Italy	7.99
	Spain	7.47		Netherl.	7.96
	Singapore	18.52		Switzerl.	3.48
FRANCE	Germany	4.56*		Spain	10.69
	Netherl.	3.05*	SWITZERL	Germany	3.28
	Spain	6.82		France	5.64
	Singapore	3.72*		Netherl.	4.29
GERMANY	France	2.84*		Singapore	9.26
	Japan	4.58*		Spain	8.76
	Sweden	3.22*		UK	4.29
	Spain	4.51		US	3.46
	Singapore	14.74*	SINGAPORE	Germany	9.41*
HONG KONG	Sweden	3.85*		Hong Kong	7.91
	Spain	2.99		France	4.72*
ITALY	Japan	5.03		Italy	2.94
DENMARK	Singapore	14.92		Netherl.	10.75*
SPAIN	Singapore	4.24*		Spain	9.11*

The symbol (*) indicates bi-directional causality.

Figure 5.8 we show the forecast error variance decomposition with foreign market returns measured in US dollars. There are a few differences with the results of figure 5.2; the most remarkable ones is that domestic innovations in any of the markets, with the exception of Switzerland, account for a greater proportion of the variance. This is due to a non perfect correlation between exchange rates and stock price changes. In the case of Switzerland, we can see that domestic innovations explain to a lesser extent changes in its beta. About a fifth of the variance is now explained by French innovations.

Figure 5.7 Impulse Responses (returns in US$)

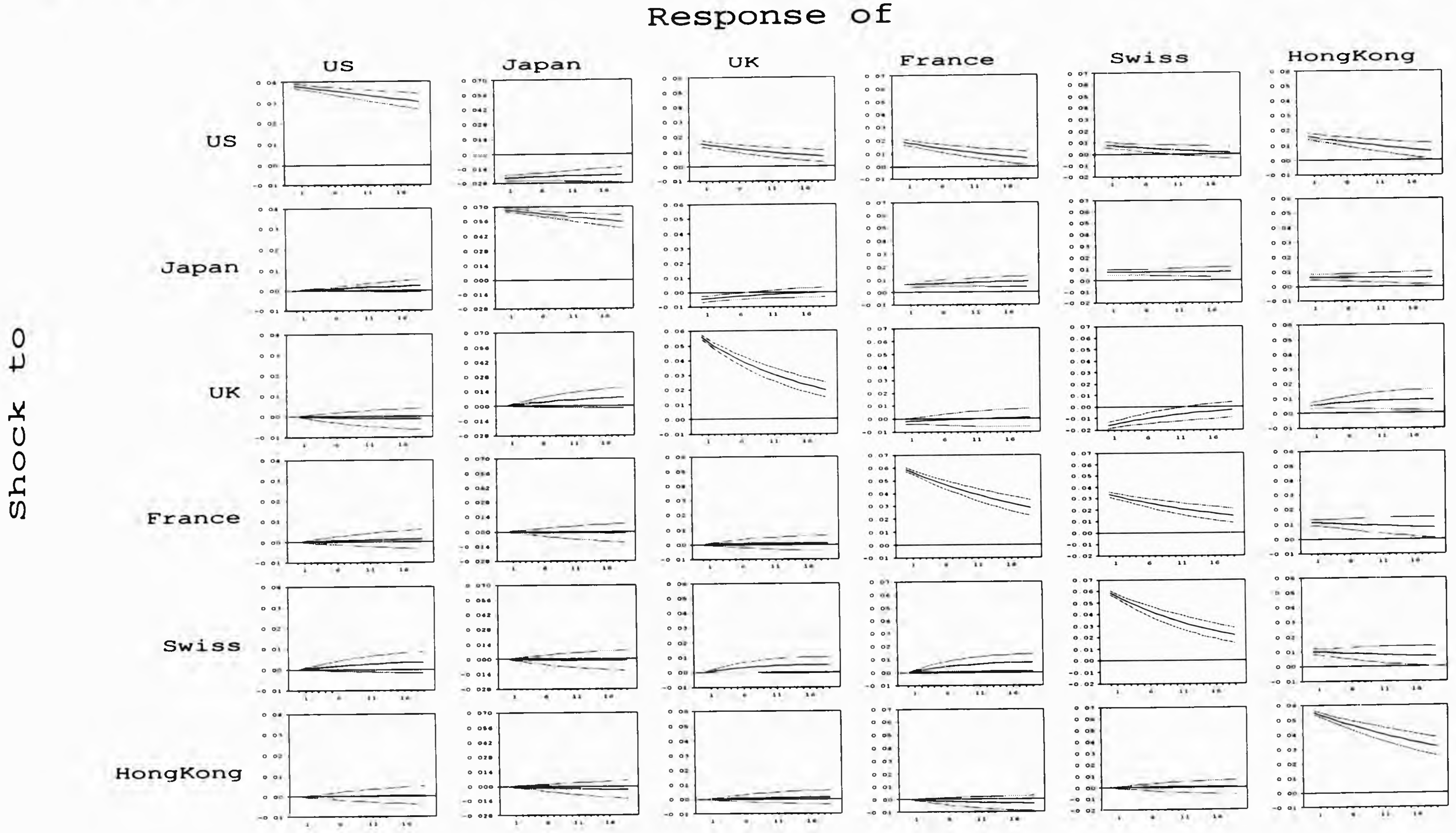
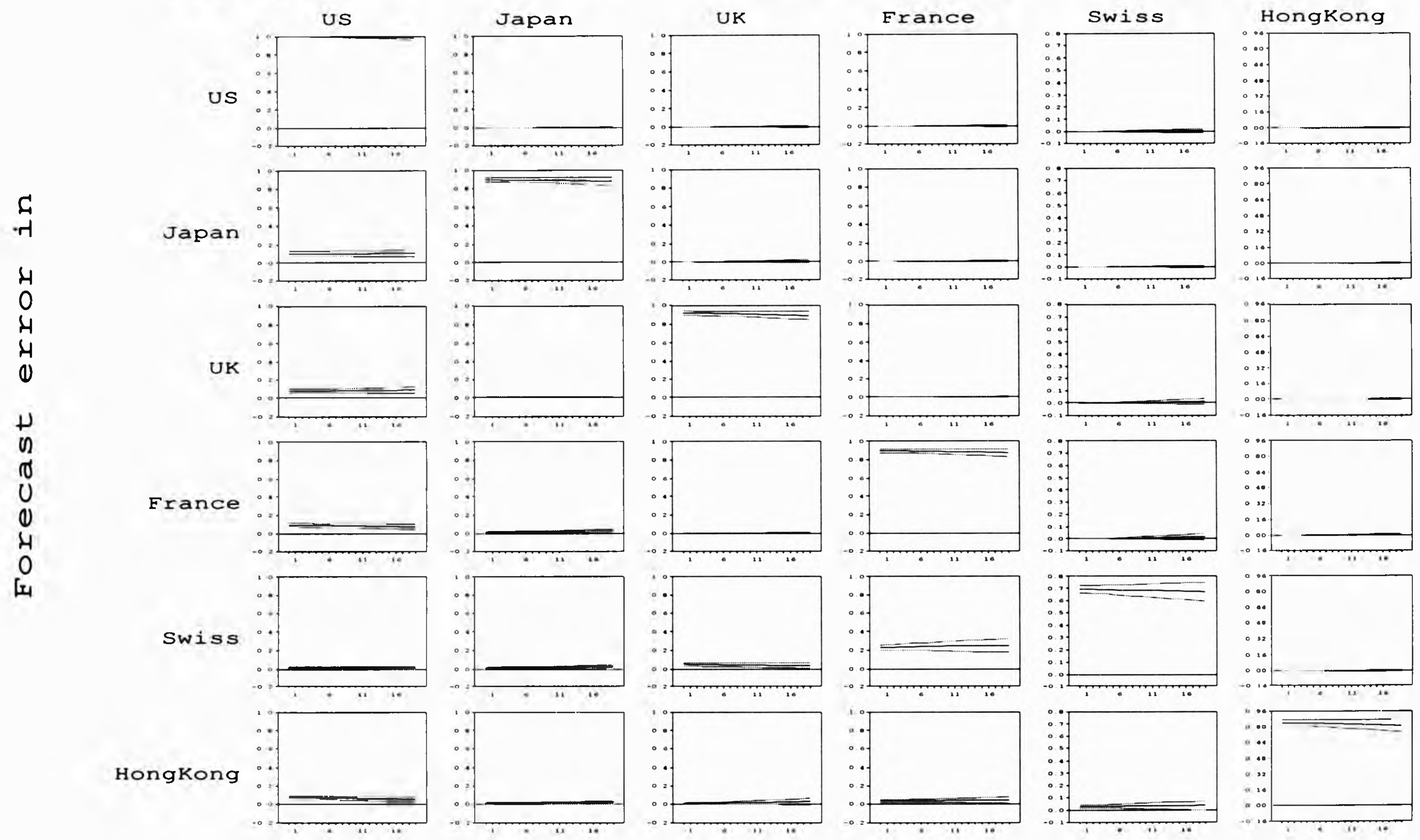


Figure 5.8 Error Variance Decomp. (returns in US$)

Proportion of variance accounted for by innovations in



It is evident that the link between the Swiss and French Franc to the German Mark helps us to explain most of the two conditional beta changes. We also notice that the US innovations account for less in the variance of UK stocks.

5.7 Conclusions

In this chapter we first outlined the basic theory behind vector autoregressive models and then we used time-varying betas to search for linkages across national equity markets. Our approach differs from that of other studies because we allow for market interdependencies through both first and second moments of returns. In contrast, most of the previous studies have searched for market interdependencies only in the first moments of returns.

We used conditional second moments, as obtained with the bivariate GARCH model of chapter three, to analyse market interdependencies and search for volatility spillovers. Our approach to the search for volatility spillovers differs from that of Hamao *et al.* (1990) because we cautiously restrict our analysis to each national market's systematic risk rather than to the overall risk. We believe that it is essential to distinguish the systematic from the specific risk since the latter is the result of domestic news whose impact is contained within the national market and which does not affect foreign stock prices. As we have seen in chapter three, for many markets the systematic (conditional) volatility can be as low as a few percentage points below its total volatility, the rest being idiosyncratic volatility. In those cases, using the overall (conditional) volatility is contradictory with the SIM and may lead to a wrong verdict.

We used the vector autoregressive methodology to simulate the way national news (*systematic* news only) influences foreign stock prices. We used Monte Carlo simulation to compute standard errors for the impulse responses and innovation accounting. This allows us to draw a confidence band around the central values of these estimates and to consolidate our findings about the volatility transmission mechanism that generates national markets' inter-linkages. By contrast, many of the studies published so far do not show any standard error for the impulse response or variance decomposition estimates. It is true that simulated standard errors for impulse responses and variance decomposition tend to be relatively large (see Runkle 1987), but average estimates alone can not be used to ground any type of acceptable conclusions.

Using the vector autoregressive methodology, we uncover a strong US factor which dominates all but one of the foreign stock price movements examined. US news contributes to a larger share of systematic volatility in Japanese and UK stocks. This can possibly be explained by the fact that the dependence of the UK and Japanese economies on the US is greater than that of the other markets examined. We also identify a second factor, the UK, which affects the other two European markets in the vector autoregressive system. However, the UK plays no role in US or Asian stocks' volatility and hence, UK news can probably be seen as an European factor.

To test the robustness of the above results, we re-estimated the vector autoregressive system and we repeated the Monte Carlo simulation on the conditional beta series divided into two equal sub-samples; 1986-90 and 1991-95. The results from the Granger causality test provide evidence that Japanese stocks play a significant role in generating global volatility. Nevertheless, other findings, including these from the impulse response analysis and variance decomposition, do not support the hypothesis that the volatility transmission mechanisms during any of the two sub-periods differ from those found when the Monte Carlo simulation is applied to the entire (10 year) data sample.

Finally, we focused on the relationship between stock indices using returns in US dollars as opposed to the local currency. The usefulness of this approach lies in its relevance to the VaR analysis. Under VaR, potential losses are measured in a common currency. Consequently, when the foreign currency is not perfectly hedged, investors will be faced with a combination of risks; any depreciation of the value of the foreign asset in local terms and any losses in the foreign currency market. Thus, investors are more interested in knowing what the two combined losses they may have to face are.

Upon explaining the results from the Granger causality tests we found that Japan is the dominant market. However, Japan has no influence in the US and UK stocks. The Monte Carlo simulation confirms the findings about Japan. News in Japan is quickly transmitted to the UK, France, Switzerland and Hong Kong. Nevertheless, the Granger causality test is only employed to help us to identify the ordering of the variables in the system; these results need be interpreted with caution. Indeed, while UK stocks do not react to Japanese news, according to the causality test, we found the opposite to be true when using Monte Carlo simulation.

Furthermore, we found differences in the results based on local currency returns when using Monte Carlo simulation to estimate confidence intervals for the forecast error variance decomposition. Most notably, with the exception of Switzerland, most of the changes in systematic volatility is explained by the domestic innovations.

The vector autoregressive analysis we conducted aims to shed light on the way volatility (of the combined with foreign exchange returns) is transmitted from one market to others. This can help risk managers in anticipating losses and so hedge risk before these losses incur. In addition, understanding the way national stock prices respond to news occurring in a third country may help in the estimation of portfolio VaR, *i.e.* in a Markovian way.

Chapter 6

General Conclusions

VaR is an estimate of the minimum possible loss an investment or portfolio of investments could face at a given probability over a period of time. The VaR methodology is essentially an extension of concepts from modern portfolio theory. Therefore, its effectiveness depends on the validity of the assumptions of the underlying portfolio theory, or their relaxation where necessary to fit the empirical evidence. The crucial element upon which modern portfolio theory is based is the assessment of probabilities about the likely outcome of future security prices. These probabilistic assessments are better known as the mean-variance tradeoff because the unconditional first and second moments of the distribution of historical returns are used in their valuation. It is true that modern portfolio theory relies on many assumptions, the strongest of which perhaps is that expected returns, variances and covariances remain constant over the holding period. Any changes in asset variances and covariances will affect the portfolio's riskiness and its potential for losses.

Traditional VaR models are based either on the unconditional (historical) or the weighted (ES) estimate of portfolio volatility (*e.g.* Riskmetrics

1995). The motivation for this thesis relates to the need to establish a more accurate method of modelling portfolio volatility for being employed in the VaR analysis. This is of particular interest for practitioners given that a more accurate estimation of VaR is of paramount importance for the reliability of risk management systems and hence reducing the likelihood of their failure in extreme market conditions. The traditional approach to estimating portfolio risk is based on historical returns, variances and covariances that rely on the implicit assumptions of normality, independence and homoskedasticity. However, the stylised facts about daily returns point to the contrary since the distribution of speculative price changes is rather non-normal, *i.e.* is leptokurtotic, and has second moments that change over time. The implications of using unconditional variances in VaR analysis is that it underestimates the possibility of incurring a predifined amount of loss. For this reason, this thesis has highlighted the inappropriateness of the traditional based approach to estimating portfolio risk and introduced an alternative approach that makes a more efficient use of available information regarding the dynamics that govern investment holdings. This was achieved through a combination of historical-simulation and GARCH volatility which avoids the use of computationally intensive multivariate methodologies in the traditional approach of correlation based VaR.

To capture any change in the variances and covariances, one can employ conditional volatility models, such as GARCH and SV. By removing the heteroskedasticity from the returns, which may account for much of the excess kurtosis, we seek to get residual returns close to normal. The focus of this thesis has been the investigation of the effectiveness and applicability of non-linear statistical techniques in portfolio risk analysis. In chapter three we investigated empirically the effectiveness of multivariate GARCH models in estimating portfolio risk. We constructed a hypothetical portfolio invested across thirteen countries with investment weights that match the capitalisation of the MSCI world index of December 1995. We employed multivariate GARCH analysis to estimate the volatility of the portfolio and calculate its VaR at 29 December 1995 (last trading day of 1995). Since for the joint estimation of multivariate GARCH models of even a moderate size portfolio (*e.g.* half a dozen assets) is prohibitive, we used the SIM of Sharpe (1963). The SIM allows asset price dependencies by linking the conditional mean of returns with the market index. We, therefore, estimated thirteen bivariate GARCH systems; in each system, one variable represents each local market and the other is the world index common to all thirteen bivariate models. To fit the GARCH model to this hypothetical portfolio we used the last ten years of daily historical returns of all assets.

Our results show that the variances and covariances of all assets included in the portfolio are subject to daily changes. The GARCH methodology used here is efficient in capturing a large part (*i.e.* about one third) of these changes and removing most of the non-linearity present in the unconditional distribution of the returns. Nevertheless, as stress analysis has shown, the VaR of the dummy portfolio when calculated using the above variances and covariances is biased and has little power to predict the portfolio's losses. When we compared the above VaR estimates (based on the multivariate GARCH) with those of the simpler ES we found no evidence that there is a significant improvement in measuring portfolio risk.

The reason for the poor performance in the measurement of portfolio risk relates to the fundamental limitations of the multivariate GARCH approach, which was used in chapter three. First, simplifying the computation using the SIM factorisation requires the existence of a common factor that is capable of explaining a large portion of the assets variance whilst leaving their residual risks being orthogonal. The second limitation acknowledged relates to the way that the variance-covariance matrix is partitioned. Using the bivariate GARCH does not guarantee that the resulting variance-covariance matrix will maintains the joint multivariate properties of the series. Consequently portfolio variance estimates are likely to be biased and/or have little explanatory power.

Therefore, one of the innovations of this thesis is to remedy this problem through a simplified approach to estimating portfolio risk. This is approach, which estimates VaR without the use of a correlation matrix, considered in chapter four. Assets' past returns are multiplied by current weights to create an univariate time series of historical portfolio returns. This series, which contains all current and past information about the asset co-movements, is used to estimate current and forecast future values of portfolio volatility. This method has the appealing property that it reduces portfolio VaR to a univariate time series analysis. Hence, it overcomes the dimentionality problems arising from the estimation of the variance-covariance matrix, while it permits greatest flexibility in modelling the volatility conditionally, therefore accounting for clusters and leptokurtosis.

Securities with strong non-linearities such as options may be included by substituting them with the products of the current *delta* (*i.e.* the ratio of the change in the price of an option to the change in the price of the underlying asset) of the option multiplied by the volatility of their notional underlying assets. Thereafter, the resulting time series of portfolio returns is analysed to identify the best fitting time series model. Accurate point estimates of current volatility are then produced and VaR is computed from them.

The same hypothetical portfolio from chapter three was used to test the proficiency of this simplified approach to portfolio VaR. The result suggests that this univariate (simplified) approach to estimating VaR is superior to the correlation based model estimated previously. The simplified VaR yields unbiased estimates for portfolio variance and it can predict about one third of next day's squared price movement. Hence, it uses (the information set available) in a more efficient way than the bivariate GARCH.

Another innovation in this thesis, not considered in previous studies, is the probabilistic approach to estimate the Worst Case Scenario or WCS thereafter. While the VaR only considers the market risk of a portfolio with respect to the frequency that a specific loss will be exceeded, it does not measure the size of the biggest loss possible in the event of an extreme shock. This is investigated using the WCS. We show how the WCS for stress analysis may be constructed using the largest outliers in the innovation series scaled by current GARCH volatility. This approach enables us to simulate the impact of the largest historical shocks on current market conditions. Further, we ascertained the likelihood that a loss of this magnitude is likely to occur over the VaR horizon.

Finally, we extended the analysis to focus on the issue of correlation stability. We defined this as the proportion of an increase or decrease in the portfolio VaR that is attributable to changes in asset variances or correlations and can be used to measure the risk managers ability to diversify portfolio risk. To ascertain the effects of changes in correlations, the conditional volatility of our diversified and undiversified hypothetical portfolio were compared. While it was found that the volatility of the diversified portfolio was lower than that of the undiversified portfolio, we found that the use of correlation stability has the useful property of acting as an early warning to risk managers regarding the effects of an extreme shock, such as a crash, on the riskiness of our portfolio.

This thesis also differed from previous research through the investigation in the way volatility is transmitted amongst national stock markets. Several studies in the past searched for market interdependencies in first moments. They used correlation analysis and principal components on price changes to identify market interlinks. Whatever their conclusions, these studies suffer from two major weaknesses. They either assume that market relationships remain constant over time or they distinguish "good" from "bad" news. We feel that national markets respond only to "common" news and in an interchangeable manner. What is good news for the world market may be viewed as bad news for a national market, and vice-versa. We also feel that the responses to world news are not necessarily contemporaneous.

An innovation of this thesis is that we use the systematic risk as opposed to overall risk to ascertain the interrelationships amongst national markets. Contrary to the approach adopted in earlier investigations that focus on the first moments of returns, we allowed for market interdependencies through both first and second moments of returns. We know from the SIM that a national market's *beta* with the world index is synonymous with its systematic risk. Hence, the time variation in these *betas* can be seen as variations in the corresponding national market's systematic risk. If markets are interrelated through their first moments they will also be related through their second moments. The time varying volatility series produced in chapter three are ideal tools for investigating the dynamic patterns that might govern world-wide volatility transmission mechanisms. Conditional *betas* estimated using the bivariate GARCH model contain important information about the time path of national markets' systematic volatility.

The degree to which national markets are integrated has important implications for a number of investment decisions. Those investing internationally are interested in the extent to which a market is correlated with others. Investment decisions based on market timing, *i.e.* dynamic hedgers and active traders, are also interested in understanding how and when markets will react to news. We believe that in this thesis we have provided an answer to these questions. Firstly, we found that national market integration may change in each period. Our results show that during international turmoil, for example the 1987 crash or the Gulf war, all national markets respond with a change in their systematic volatility. But these responses are neither synchronised nor they have an uniform magnitude. Studying the past behaviour of markets' conditional *betas* can provide us with important information about the extent to which each national market is integrated with the others during different news scenarios.

We employed the vector autoregression methodology on the time-varying *betas* to simulate how national markets react to each other's news. Because internationally relevant news will always be announced in one country first, typically it will first affect domestic equity prices. News can then be interpreted as national or regional factors that can affect other national markets. Studying the way national markets respond to those factors will provide valuable information for a number of investment decisions which are based on market timing. Contrary to the findings of past research, the use of conditional betas revealed strong interlinks amongst national markets as suggested by the high R^2 for all countries in table 5.3. Further, unlike previous studies, we use Monte Carlo simulation to estimate the confidence intervals and determine the timing and magnitude of volatility transmitted across markets. We found that there is a strong US factor that influences all stock prices across the globe. A second factor, UK news, has also been identified but its influence is limited to Europe.

To test the robustness of our result, we divided the data sample into two sub-periods dictated by periods where volatility was most profound and periods characterised by stable growth in the world equity prices. Using the Granger causality test to identify the order of the variables in each of the two vector autoregressive systems, we found that in the first subsample 1986 to 1990, Japan played the dominant role, however, in the second subsample 1991 to 1995, this role diminished. On the other hand, in the first subsample, the UK and US plays an less important role whereas in the second sample, both markets are more influential.

However, these findings were based on returns expressed in local currency. Another innovation in this thesis is that we considered interlinks across markets when returns are expressed in both local currency and in US dollars. Unlike previous investigations, this invited a unique opportunity to determine whether the nature of the relationship amongst markets is attributable to movements in equity prices or due to exchange rate changes. Consequently we argued the implication for VaR analysis is that where currency exposure is not perfectly hedged, investors, in addition to loses in domestic terms, are exposed to foreign currency losses. To investigate this, Monte-Carlo simulation for the impulse response function were conducted when returns are expressed in US dollars. The results differed considerably when compared with earlier analysis based on returns expressed in local currency in relation to the role of Japan. Most notably, innovations originating from Japan was found to have a significant effect on other markets which lasted longest in France, Switzerland and Hong Kong. This is attributable to exchange rate movements as opposed to movements in equity prices.

Our results suggest that VaR models should be subject to empirical testing before regulators approve them. Back-testing on a trial period may ratify this requirement. A reliable VaR model, not only satisfies regulatory requirements, but it becomes a useful device for practical portfolio risk management. In particular, the incorporation of intermarket volatility transmission mechanisms is a promising venue for future research because it has the potential to allow for a reduction in risk exposure before foreign shocks work their way into domestic markets.

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